Computer algebra independent integration tests

1-Algebraic-functions/1.2-Trinomial-products/1.2.4-Improper/1.2.4.2-d-x-^m-a-x^q+b-x^n+c-x^-2-n-q-^p

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [140]. This is test number [50].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100.00 (140)	% 0.00 (0)
Mathematica	% 99.29 (139)	% 0.71 (1)
Maple	% 97.14 (136)	% 2.86 (4)
Maxima	% 17.14 (24)	% 82.86 (116)
Fricas	% 92.14 (129)	% 7.86 (11)
Sympy	% 37.86 (53)	% 62.14 (87)
Giac	% 75.71 (106)	% 24.29 (34)
Mupad	% 51.43 (72)	% 48.57 (68)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

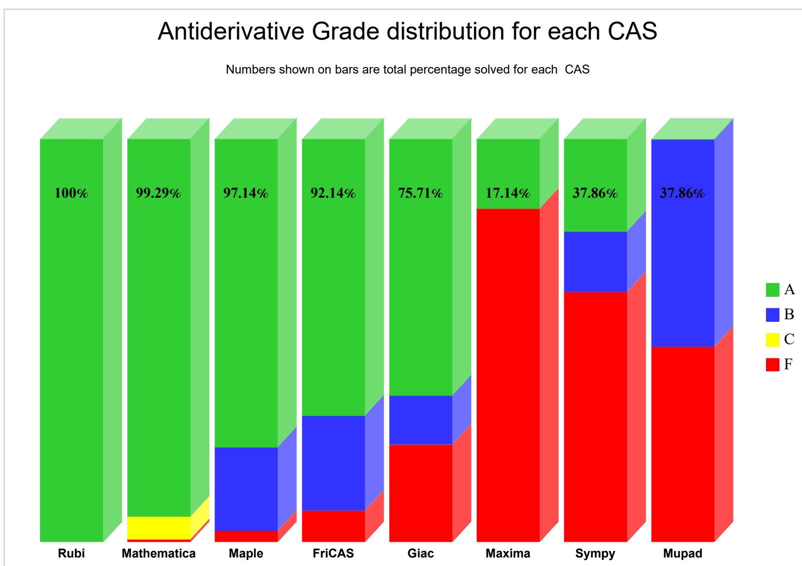
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

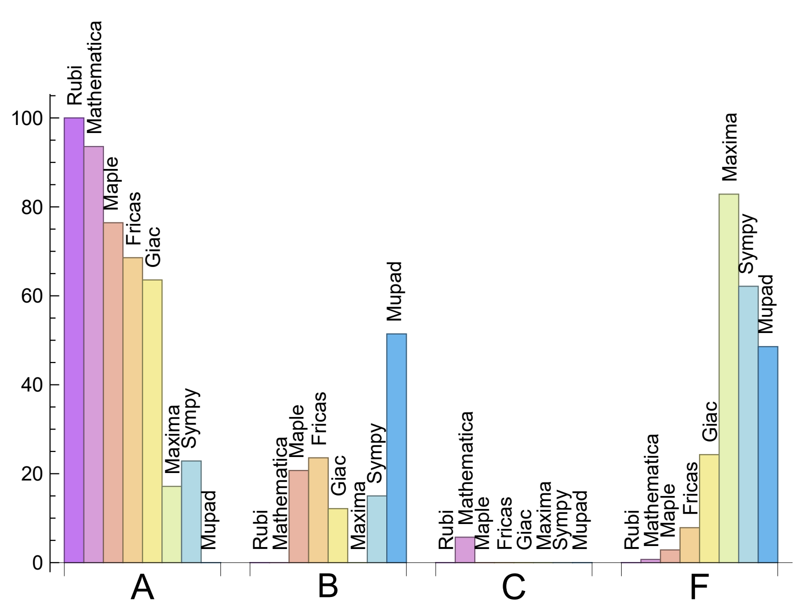
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	93.57	0.00	5.71	0.71
Maple	76.43	20.71	0.00	2.86
Maxima	17.14	0.00	0.00	82.86
Fricas	68.57	23.57	0.00	7.86
Sympy	22.86	15.00	0.00	62.14
Giac	63.57	12.14	0.00	24.29
Mupad	0.00	51.43	0.00	48.57

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	1	100.00 %	0.00 %	0.00 %
Maple	4	100.00 %	0.00 %	0.00 %
Maxima	116	80.17 %	0.00 %	19.83 %
Fricas	11	90.91 %	0.00 %	9.09 %
Sympy	87	71.26 %	28.74 %	0.00 %
Giac	34	50.00 %	20.59 %	29.41 %
Mupad	68	100.00 %	0.00 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

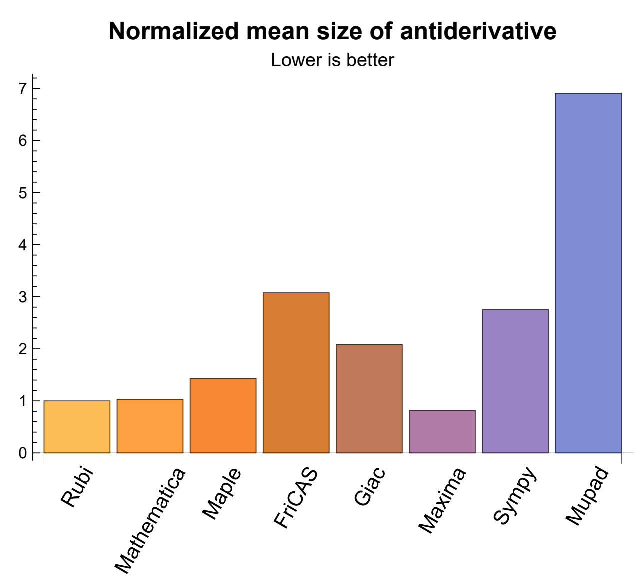
1.3 Performance

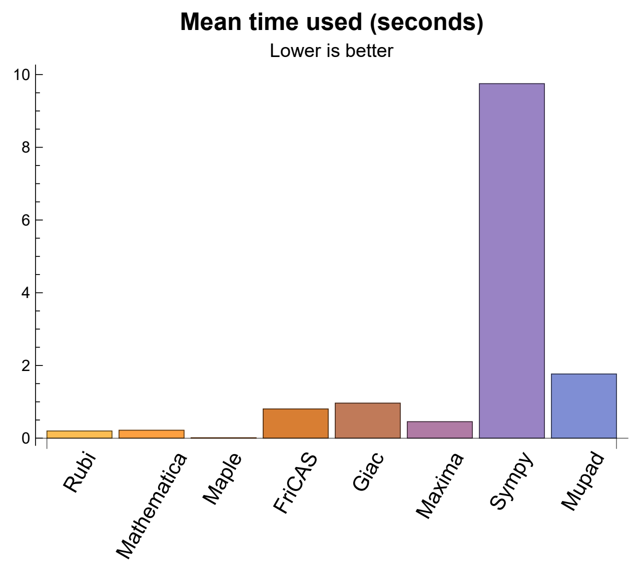
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.20	139.59	1.00	103.50	1.00
Mathematica	0.22	135.97	1.03	109.00	0.99
Maple	0.01	236.37	1.43	143.00	1.33
Maxima	0.45	32.71	0.81	28.50	0.81
Fricas	0.80	470.89	3.07	272.00	2.74
Sympy	9.75	233.81	2.75	148.00	1.25
Giac	0.96	362.54	2.08	76.00	1.09
Mupad	1.77	1266.07	6.91	172.00	2.35

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {104, 122}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
```

```
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

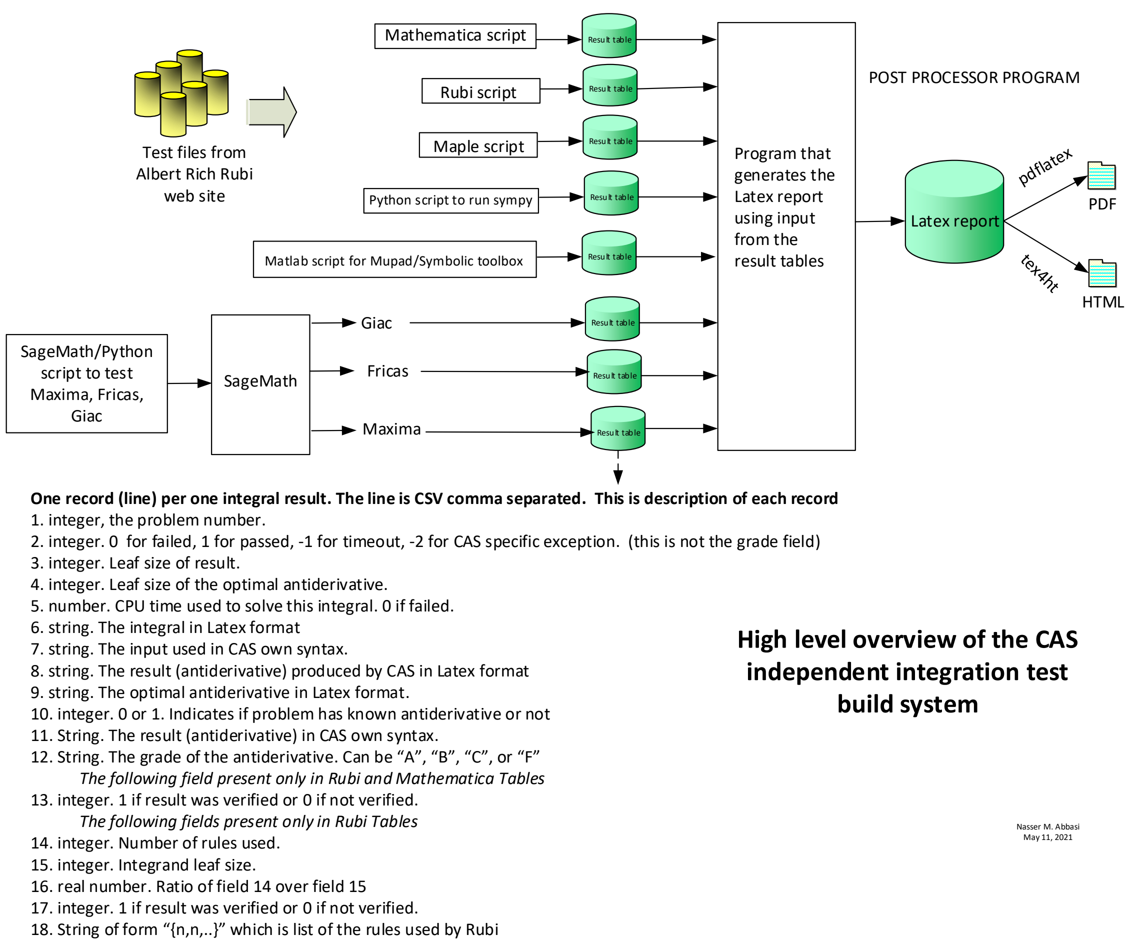
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 106, 108, 109, 111, 113, 115, 118, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139 }

B grade: { }

C grade: { 105, 107, 110, 112, 114, 116, 117, 119 }

F grade: { 140 }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 20, 21, 22, 23, 29, 30, 31, 32, 33, 34, 36, 38, 39, 40, 41, 42, 43, 44, 45, 46, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 73, 74, 75, 76, 77, 78, 80, 82, 83, 84, 85, 86, 87, 88, 91, 93, 95, 96, 97, 106, 107, 108, 109, 111, 113, 114, 115, 116, 117, 120, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139 }

B grade: { 19, 24, 25, 26, 27, 28, 35, 37, 47, 48, 65, 72, 79, 81, 89, 90, 92, 94, 98, 99, 100, 101, 102, 103, 105, 110, 112, 118, 119 }

C grade: { }

F grade: { 104, 121, 122, 140 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 137 }

B grade: { }

C grade: { }

F grade: { 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 138, 139, 140 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 73, 74, 75, 76, 77, 78, 80, 82, 84, 86, 88, 106, 108, 109, 111, 113, 115, 120, 121, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139 }

B grade: { 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 60, 72, 79, 81, 83, 85, 87, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 118 }

C grade: { }

F grade: { 104, 105, 107, 110, 112, 114, 116, 117, 119, 122, 140 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 79, 81, 83, 85, 87, 92, 94, 96, 98 }

B grade: { 11, 12, 13, 14, 15, 19, 20, 21, 22, 23, 78, 80, 82, 84, 86, 88, 89, 91, 93, 95, 97 }

C grade: { }

F grade: { 16, 17, 18, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 90, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 38, 39, 40, 41, 42, 50, 51, 52, 56, 57, 58, 59, 66, 67, 68, 69, 70, 71, 73, 74, 75, 76, 77, 78, 80, 82, 84, 86, 88, 89, 91, 93, 95, 97, 99, 101, 103, 106, 113, 115, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139 }

B grade: { 65, 72, 79, 81, 83, 85, 87, 90, 92, 94, 96, 98, 100, 102, 109, 111, 118 }

C grade: { }

F grade: { 33, 34, 35, 36, 37, 43, 44, 45, 46, 47, 48, 49, 53, 54, 55, 60, 61, 62, 63, 64, 104, 105, 107, 108, 110, 112, 114, 116, 117, 119, 120, 121, 122, 140 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 58, 59, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 129, 133, 137 }

C grade: { }

F grade: { 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 60, 61, 62, 63, 64, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 130, 131, 132, 134, 135, 136, 138, 139, 140 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	19	19	19	19
normalized size	1	1.00	1.00	0.80	0.76	0.76	0.76	0.76	0.76
time (sec)	N/A	0.007	0.002	0.000	0.440	0.563	0.062	0.542	0.032

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	19	19	19	19
normalized size	1	1.00	1.00	0.80	0.76	0.76	0.76	0.76	0.76
time (sec)	N/A	0.007	0.002	0.000	0.436	0.469	0.062	0.454	0.032

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	19	19	19	19
normalized size	1	1.00	1.00	0.80	0.76	0.76	0.76	0.76	0.76
time (sec)	N/A	0.004	0.000	0.002	0.429	0.628	0.061	0.536	0.032

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	19	19	19	19
normalized size	1	1.00	1.00	0.80	0.76	0.76	0.76	0.76	0.76
time (sec)	N/A	0.006	0.001	0.001	0.445	0.760	0.064	0.361	0.031

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	16	16	15	16	16
normalized size	1	1.00	1.00	0.85	0.80	0.80	0.75	0.80	0.80
time (sec)	N/A	0.006	0.001	0.001	0.426	0.580	0.061	0.490	0.025

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	45	44	46	48	46	45
normalized size	1	1.00	1.00	0.83	0.81	0.85	0.89	0.85	0.83
time (sec)	N/A	0.054	0.008	0.000	0.427	0.428	0.075	0.576	0.034

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	45	44	46	49	46	45
normalized size	1	1.00	1.00	0.83	0.81	0.85	0.91	0.85	0.83
time (sec)	N/A	0.029	0.007	0.001	0.436	0.593	0.075	0.484	0.024

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	45	48	46	48	46	45
normalized size	1	1.00	1.00	0.83	0.89	0.85	0.89	0.85	0.83
time (sec)	N/A	0.027	0.006	0.000	0.447	0.476	0.072	0.396	0.022

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	45	44	44	49	46	45
normalized size	1	1.00	1.00	0.83	0.81	0.81	0.91	0.85	0.83
time (sec)	N/A	0.032	0.007	0.002	0.428	0.674	0.076	0.493	0.025

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	45	44	44	48	46	45
normalized size	1	1.00	1.00	0.83	0.81	0.81	0.89	0.85	0.83
time (sec)	N/A	0.032	0.009	0.002	0.425	0.581	0.078	0.482	0.024

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	84	132	0	297	381	86	112
normalized size	1	1.00	0.94	1.48	0.00	3.34	4.28	0.97	1.26
time (sec)	N/A	0.094	0.109	0.005	0.000	0.772	0.841	0.453	0.141

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	73	101	0	235	306	67	172
normalized size	1	1.00	1.04	1.44	0.00	3.36	4.37	0.96	2.46
time (sec)	N/A	0.059	0.063	0.003	0.000	0.885	0.623	0.548	2.033

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	57	56	0	185	216	55	112
normalized size	1	1.00	1.02	1.00	0.00	3.30	3.86	0.98	2.00
time (sec)	N/A	0.040	0.030	0.003	0.000	0.926	0.326	0.421	0.133

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	38	35	0	120	124	34	46
normalized size	1	1.00	1.12	1.03	0.00	3.53	3.65	1.00	1.35
time (sec)	N/A	0.026	0.006	0.001	0.000	0.755	0.222	0.497	0.035

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	61	62	0	211	564	62	213
normalized size	1	1.00	0.98	1.00	0.00	3.40	9.10	1.00	3.44
time (sec)	N/A	0.047	0.065	0.007	0.000	0.946	4.358	0.505	2.298

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	77	112	0	269	0	79	339
normalized size	1	1.00	0.95	1.38	0.00	3.32	0.00	0.98	4.19
time (sec)	N/A	0.098	0.081	0.007	0.000	0.918	0.000	0.325	2.504

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	102	150	0	358	0	105	447
normalized size	1	1.00	0.98	1.44	0.00	3.44	0.00	1.01	4.30
time (sec)	N/A	0.146	0.136	0.007	0.000	0.977	0.000	0.508	0.587

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	131	214	0	445	0	136	524
normalized size	1	1.00	0.96	1.56	0.00	3.25	0.00	0.99	3.82
time (sec)	N/A	0.203	0.104	0.010	0.000	0.874	0.000	0.457	2.596

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	132	352	0	837	842	161	261
normalized size	1	1.00	0.88	2.35	0.00	5.58	5.61	1.07	1.74
time (sec)	N/A	0.159	0.196	0.010	0.000	0.694	1.792	0.567	2.456

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	109	209	0	635	729	125	279
normalized size	1	1.00	0.96	1.83	0.00	5.57	6.39	1.10	2.45
time (sec)	N/A	0.103	0.146	0.009	0.000	0.619	1.366	0.595	2.491

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	81	97	0	387	280	88	135
normalized size	1	1.00	1.21	1.45	0.00	5.78	4.18	1.31	2.01
time (sec)	N/A	0.039	0.091	0.007	0.000	0.675	0.600	0.493	2.129

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	69	70	0	338	253	76	110
normalized size	1	1.00	1.05	1.06	0.00	5.12	3.83	1.15	1.67
time (sec)	N/A	0.038	0.067	0.003	0.000	0.586	0.564	0.383	2.183

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	70	68	0	341	265	76	119
normalized size	1	1.00	1.06	1.03	0.00	5.17	4.02	1.15	1.80
time (sec)	N/A	0.035	0.071	0.004	0.000	0.593	0.588	0.459	0.084

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	107	237	0	781	0	126	620
normalized size	1	1.00	0.99	2.19	0.00	7.23	0.00	1.17	5.74
time (sec)	N/A	0.148	0.184	0.014	0.000	0.545	0.000	0.505	2.871

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	131	328	0	975	0	171	775
normalized size	1	1.00	0.89	2.22	0.00	6.59	0.00	1.16	5.24
time (sec)	N/A	0.198	0.264	0.015	0.000	0.859	0.000	0.454	2.833

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	175	418	0	1226	0	229	914
normalized size	1	1.00	0.87	2.07	0.00	6.07	0.00	1.13	4.52
time (sec)	N/A	0.250	0.343	0.016	0.000	1.030	0.000	0.441	2.956

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	218	515	0	1407	0	282	1120
normalized size	1	1.00	0.87	2.04	0.00	5.58	0.00	1.12	4.44
time (sec)	N/A	0.323	0.318	0.017	0.000	1.320	0.000	0.550	3.064

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	318	318	272	619	0	1640	0	347	1260
normalized size	1	1.00	0.86	1.95	0.00	5.16	0.00	1.09	3.96
time (sec)	N/A	0.392	0.378	0.020	0.000	2.067	0.000	0.439	3.143

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	257	180	310	0	390	0	283	-1
normalized size	1	1.00	0.70	1.21	0.00	1.52	0.00	1.10	-0.00
time (sec)	N/A	0.588	0.237	0.009	0.000	0.848	0.000	0.873	0.000

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	150	265	0	326	0	230	-1
normalized size	1	1.00	0.73	1.29	0.00	1.59	0.00	1.12	-0.00
time (sec)	N/A	0.370	0.161	0.009	0.000	0.825	0.000	0.746	0.000

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	119	167	0	260	0	166	-1
normalized size	1	1.00	0.73	1.02	0.00	1.60	0.00	1.02	-0.01
time (sec)	N/A	0.058	0.213	0.008	0.000	0.712	0.000	0.881	0.000

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	100	146	0	220	0	125	-1
normalized size	1	1.00	0.84	1.23	0.00	1.85	0.00	1.05	-0.01
time (sec)	N/A	0.078	0.133	0.005	0.000	0.607	0.000	0.946	0.000

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	134	126	0	638	0	0	-1
normalized size	1	1.00	0.77	0.73	0.00	3.69	0.00	0.00	-0.01
time (sec)	N/A	0.126	0.095	0.006	0.000	0.672	0.000	0.000	0.000

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	131	173	0	653	0	0	-1
normalized size	1	1.00	0.76	1.00	0.00	3.77	0.00	0.00	-0.01
time (sec)	N/A	0.124	0.117	0.006	0.000	0.788	0.000	0.000	0.000

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	112	207	0	226	0	0	-1
normalized size	1	1.00	0.98	1.82	0.00	1.98	0.00	0.00	-0.01
time (sec)	N/A	0.148	0.096	0.006	0.000	0.772	0.000	0.000	0.000

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	131	234	0	272	0	0	-1
normalized size	1	1.00	0.85	1.51	0.00	1.75	0.00	0.00	-0.01
time (sec)	N/A	0.256	0.123	0.008	0.000	0.742	0.000	0.000	0.000

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	160	387	0	336	0	0	-1
normalized size	1	1.00	0.78	1.89	0.00	1.64	0.00	0.00	-0.00
time (sec)	N/A	0.386	0.168	0.011	0.000	0.994	0.000	0.000	0.000

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	422	422	236	649	0	664	0	521	-1
normalized size	1	1.00	0.56	1.54	0.00	1.57	0.00	1.23	-0.00
time (sec)	N/A	1.203	0.384	0.010	0.000	0.948	0.000	1.461	0.000

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	364	364	197	479	0	558	0	429	-1
normalized size	1	1.00	0.54	1.32	0.00	1.53	0.00	1.18	-0.00
time (sec)	N/A	1.039	0.246	0.010	0.000	0.734	0.000	1.351	0.000

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	180	431	0	474	0	365	-1
normalized size	1	1.00	0.62	1.50	0.00	1.65	0.00	1.27	-0.00
time (sec)	N/A	0.519	0.219	0.009	0.000	0.682	0.000	0.994	0.000

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	163	289	0	384	0	284	-1
normalized size	1	1.00	0.82	1.46	0.00	1.94	0.00	1.43	-0.01
time (sec)	N/A	0.178	0.173	0.007	0.000	0.832	0.000	0.925	0.000

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	132	265	0	320	0	232	-1
normalized size	1	1.00	0.80	1.61	0.00	1.94	0.00	1.41	-0.01
time (sec)	N/A	0.128	0.060	0.005	0.000	0.742	0.000	1.013	0.000

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	166	222	0	791	0	0	-1
normalized size	1	1.00	0.73	0.98	0.00	3.48	0.00	0.00	-0.00
time (sec)	N/A	0.255	0.224	0.006	0.000	0.898	0.000	0.000	0.000

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	158	254	0	757	0	0	-1
normalized size	1	1.00	0.72	1.16	0.00	3.46	0.00	0.00	-0.00
time (sec)	N/A	0.242	0.180	0.006	0.000	0.710	0.000	0.000	0.000

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	162	338	0	757	0	0	-1
normalized size	1	1.00	0.74	1.54	0.00	3.46	0.00	0.00	-0.00
time (sec)	N/A	0.238	0.190	0.007	0.000	0.904	0.000	0.000	0.000

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	257	175	435	0	815	0	0	-1
normalized size	1	1.00	0.68	1.69	0.00	3.17	0.00	0.00	-0.00
time (sec)	N/A	0.351	0.289	0.009	0.000	0.811	0.000	0.000	0.000

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	141	501	0	332	0	0	-1
normalized size	1	1.00	0.72	2.54	0.00	1.69	0.00	0.00	-0.01
time (sec)	N/A	0.363	0.115	0.007	0.000	0.997	0.000	0.000	0.000

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	249	177	534	0	394	0	0	-1
normalized size	1	1.00	0.71	2.14	0.00	1.58	0.00	0.00	-0.00
time (sec)	N/A	0.504	0.166	0.010	0.000	0.989	0.000	0.000	0.000

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	105	144	0	226	0	0	-1
normalized size	1	1.00	0.73	1.01	0.00	1.58	0.00	0.00	-0.01
time (sec)	N/A	0.174	0.093	0.009	0.000	0.499	0.000	0.000	0.000

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	89	88	0	188	0	108	-1
normalized size	1	1.00	0.86	0.85	0.00	1.83	0.00	1.05	-0.01
time (sec)	N/A	0.078	0.052	0.006	0.000	0.782	0.000	0.915	0.000

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	66	65	0	129	0	37	-1
normalized size	1	1.00	0.93	0.92	0.00	1.82	0.00	0.52	-0.01
time (sec)	N/A	0.037	0.035	0.006	0.000	0.665	0.000	0.912	0.000

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	70	66	0	130	0	59	-1
normalized size	1	1.00	1.56	1.47	0.00	2.89	0.00	1.31	-0.02
time (sec)	N/A	0.016	0.019	0.005	0.000	0.789	0.000	0.919	0.000

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	89	88	0	194	0	0	-1
normalized size	1	1.00	1.16	1.14	0.00	2.52	0.00	0.00	-0.01
time (sec)	N/A	0.054	0.051	0.007	0.000	0.681	0.000	0.000	0.000

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	112	152	0	232	0	0	-1
normalized size	1	1.00	0.94	1.28	0.00	1.95	0.00	0.00	-0.01
time (sec)	N/A	0.149	0.084	0.007	0.000	0.763	0.000	0.000	0.000

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	183	283	0	616	0	0	-1
normalized size	1	1.00	0.70	1.08	0.00	2.35	0.00	0.00	-0.00
time (sec)	N/A	0.506	0.230	0.009	0.000	0.816	0.000	0.000	0.000

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	141	199	0	486	0	195	-1
normalized size	1	1.00	0.70	0.99	0.00	2.42	0.00	0.97	-0.00
time (sec)	N/A	0.305	0.152	0.009	0.000	0.881	0.000	0.888	0.000

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	112	166	0	414	0	110	-1
normalized size	1	1.00	0.73	1.08	0.00	2.71	0.00	0.72	-0.01
time (sec)	N/A	0.175	0.119	0.007	0.000	0.768	0.000	0.983	0.000

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	37	53	0	73	0	45	75
normalized size	1	1.00	0.92	1.32	0.00	1.82	0.00	1.12	1.88
time (sec)	N/A	0.040	0.075	0.004	0.000	0.620	0.000	0.916	2.122

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	36	52	0	72	0	45	75
normalized size	1	1.00	0.92	1.33	0.00	1.85	0.00	1.15	1.92
time (sec)	N/A	0.040	0.024	0.003	0.000	0.659	0.000	0.765	2.034

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	109	166	0	411	0	0	-1
normalized size	1	1.00	1.16	1.77	0.00	4.37	0.00	0.00	-0.01
time (sec)	N/A	0.068	0.128	0.008	0.000	0.926	0.000	0.000	0.000

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	138	201	0	496	0	0	-1
normalized size	1	1.00	0.96	1.40	0.00	3.44	0.00	0.00	-0.01
time (sec)	N/A	0.163	0.100	0.009	0.000	0.622	0.000	0.000	0.000

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	181	292	0	630	0	0	-1
normalized size	1	1.00	0.87	1.40	0.00	3.01	0.00	0.00	-0.00
time (sec)	N/A	0.287	0.157	0.009	0.000	0.917	0.000	0.000	0.000

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	225	340	0	716	0	0	-1
normalized size	1	1.00	0.83	1.25	0.00	2.64	0.00	0.00	-0.00
time (sec)	N/A	0.452	0.203	0.010	0.000	1.016	0.000	0.000	0.000

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	343	343	272	446	0	866	0	0	-1
normalized size	1	1.00	0.79	1.30	0.00	2.52	0.00	0.00	-0.00
time (sec)	N/A	0.621	0.249	0.011	0.000	1.306	0.000	0.000	0.000

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	34	77	37	71	280	107	89
normalized size	1	1.00	0.92	2.08	1.00	1.92	7.57	2.89	2.41
time (sec)	N/A	0.013	0.026	0.003	0.446	0.663	1.176	0.485	2.084

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	19	19	19	19
normalized size	1	1.00	1.00	0.80	0.76	0.76	0.76	0.76	0.76
time (sec)	N/A	0.008	0.002	0.002	0.428	0.456	0.061	0.382	0.029

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	19	19	19	19
normalized size	1	1.00	1.00	0.80	0.76	0.76	0.76	0.76	0.76
time (sec)	N/A	0.007	0.002	0.002	0.428	0.519	0.061	0.391	0.031

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	19	19	19	19
normalized size	1	1.00	1.00	0.80	0.76	0.76	0.76	0.76	0.76
time (sec)	N/A	0.004	0.000	0.002	0.423	0.613	0.061	0.378	0.028

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	16	16	15	16	16
normalized size	1	1.00	1.00	0.85	0.80	0.80	0.75	0.80	0.80
time (sec)	N/A	0.006	0.001	0.000	0.425	0.557	0.066	0.363	0.027

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	17	17	17	20	17
normalized size	1	1.00	1.00	0.86	0.81	0.81	0.81	0.95	0.81
time (sec)	N/A	0.008	0.002	0.001	0.426	0.599	0.095	0.415	0.025

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	16	20	12	16	16
normalized size	1	1.00	1.00	0.94	0.89	1.11	0.67	0.89	0.89
time (sec)	N/A	0.007	0.002	0.003	0.428	0.606	0.095	0.585	0.030

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	69	300	85	241	1377	399	271
normalized size	1	1.00	0.91	3.95	1.12	3.17	18.12	5.25	3.57
time (sec)	N/A	0.046	0.071	0.005	0.436	0.673	4.287	0.516	2.195

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	45	44	46	51	46	45
normalized size	1	1.00	1.00	0.83	0.81	0.85	0.94	0.85	0.83
time (sec)	N/A	0.036	0.007	0.002	0.424	0.410	0.079	0.405	0.034

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	48	45	44	46	46	46	45
normalized size	1	1.00	0.89	0.83	0.81	0.85	0.85	0.85	0.83
time (sec)	N/A	0.054	0.008	0.000	0.446	0.502	0.076	0.384	0.024

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	45	48	46	51	46	45
normalized size	1	1.00	1.00	0.83	0.89	0.85	0.94	0.85	0.83
time (sec)	N/A	0.026	0.007	0.001	0.430	0.554	0.074	0.357	0.024

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	48	45	44	44	46	46	45
normalized size	1	1.00	0.89	0.83	0.81	0.81	0.85	0.85	0.83
time (sec)	N/A	0.046	0.008	0.001	0.429	0.613	0.077	0.524	0.024

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	42	41	41	48	43	42
normalized size	1	1.00	1.00	0.86	0.84	0.84	0.98	0.88	0.86
time (sec)	N/A	0.027	0.005	0.001	0.430	0.595	0.080	0.507	0.023

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	93	142	0	313	391	92	842
normalized size	1	1.00	0.93	1.42	0.00	3.13	3.91	0.92	8.42
time (sec)	N/A	0.122	0.085	0.004	0.000	0.586	2.946	0.467	2.203

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	250	467	0	1564	194	2457	4127
normalized size	1	1.00	1.23	2.30	0.00	7.70	0.96	12.10	20.33
time (sec)	N/A	0.602	0.155	0.025	0.000	0.652	4.254	2.037	2.718

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	78	111	0	254	316	75	655
normalized size	1	1.00	0.96	1.37	0.00	3.14	3.90	0.93	8.09
time (sec)	N/A	0.087	0.044	0.004	0.000	0.677	1.950	0.633	2.438

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	202	343	0	1059	129	2109	3026
normalized size	1	1.00	1.13	1.92	0.00	5.92	0.72	11.78	16.91
time (sec)	N/A	0.227	0.108	0.014	0.000	0.653	2.177	1.500	2.584

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	62	60	0	197	223	59	118
normalized size	1	1.00	0.98	0.95	0.00	3.13	3.54	0.94	1.87
time (sec)	N/A	0.067	0.023	0.003	0.000	0.791	0.919	0.423	0.166

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	165	208	0	559	75	503	416
normalized size	1	1.00	1.10	1.39	0.00	3.73	0.50	3.35	2.77
time (sec)	N/A	0.094	0.082	0.011	0.000	0.758	0.819	1.805	2.209

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	39	36	0	129	131	35	41
normalized size	1	1.00	1.08	1.00	0.00	3.58	3.64	0.97	1.14
time (sec)	N/A	0.043	0.008	0.003	0.000	0.624	0.495	0.489	2.040

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	129	116	0	613	87	1026	763
normalized size	1	1.00	0.86	0.77	0.00	4.09	0.58	6.84	5.09
time (sec)	N/A	0.080	0.079	0.013	0.000	0.765	1.185	1.863	2.479

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	113	66	0	223	253	68	1014
normalized size	1	1.00	1.64	0.96	0.00	3.23	3.67	0.99	14.70
time (sec)	N/A	0.072	0.061	0.007	0.000	0.583	4.284	0.428	2.701

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	191	232	0	1116	148	1839	2997
normalized size	1	1.00	1.10	1.33	0.00	6.41	0.85	10.57	17.22
time (sec)	N/A	0.195	0.401	0.018	0.000	1.192	2.623	1.883	2.859

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	135	119	0	293	345	94	2033
normalized size	1	1.00	1.52	1.34	0.00	3.29	3.88	1.06	22.84
time (sec)	N/A	0.133	0.120	0.007	0.000	1.048	123.748	0.432	3.910

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	151	383	0	868	877	161	1473
normalized size	1	1.00	0.91	2.31	0.00	5.23	5.28	0.97	8.87
time (sec)	N/A	0.223	0.186	0.013	0.000	0.732	112.278	1.985	0.528

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	331	327	844	0	2856	0	3339	7599
normalized size	1	1.00	0.99	2.55	0.00	8.63	0.00	10.09	22.96
time (sec)	N/A	0.703	0.663	0.036	0.000	0.907	0.000	3.707	3.756

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	121	222	0	663	745	152	1336
normalized size	1	1.00	0.92	1.68	0.00	5.02	5.64	1.15	10.12
time (sec)	N/A	0.152	0.168	0.012	0.000	0.647	19.764	1.951	2.943

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	282	602	0	2257	379	2736	6293
normalized size	1	1.00	1.04	2.22	0.00	8.33	1.40	10.10	23.22
time (sec)	N/A	0.526	0.513	0.029	0.000	1.059	33.307	4.025	3.858

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	93	104	0	407	282	96	187
normalized size	1	1.00	1.19	1.33	0.00	5.22	3.62	1.23	2.40
time (sec)	N/A	0.072	0.086	0.010	0.000	0.866	1.517	2.000	2.197

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	235	452	0	1668	296	2132	4973
normalized size	1	1.00	0.99	1.91	0.00	7.04	1.25	9.00	20.98
time (sec)	N/A	0.358	0.407	0.026	0.000	0.955	4.547	3.389	3.635

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	79	77	0	360	269	82	178
normalized size	1	1.00	1.05	1.03	0.00	4.80	3.59	1.09	2.37
time (sec)	N/A	0.069	0.066	0.006	0.000	0.867	1.360	2.053	0.144

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	222	342	0	1680	298	1970	4854
normalized size	1	1.00	1.00	1.55	0.00	7.60	1.35	8.91	21.96
time (sec)	N/A	0.240	0.434	0.069	0.000	0.904	13.168	3.080	3.363

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	79	75	0	361	267	82	172
normalized size	1	1.00	1.07	1.01	0.00	4.88	3.61	1.11	2.32
time (sec)	N/A	0.065	0.080	0.006	0.000	0.802	1.277	2.132	2.160

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	243	733	0	2309	394	2682	6404
normalized size	1	1.00	0.96	2.91	0.00	9.16	1.56	10.64	25.41
time (sec)	N/A	0.461	0.418	0.057	0.000	1.058	165.893	3.818	3.847

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	207	253	0	813	0	166	5048
normalized size	1	1.00	1.70	2.07	0.00	6.66	0.00	1.36	41.38
time (sec)	N/A	0.188	0.331	0.017	0.000	1.016	0.000	2.322	6.312

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	308	308	302	712	0	2912	0	3087	7555
normalized size	1	1.00	0.98	2.31	0.00	9.45	0.00	10.02	24.53
time (sec)	N/A	1.352	0.616	0.034	0.000	1.239	0.000	2.433	2.639

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	248	352	0	1007	0	182	5491
normalized size	1	1.00	1.53	2.17	0.00	6.22	0.00	1.12	33.90
time (sec)	N/A	0.250	0.264	0.020	0.000	1.229	0.000	2.407	6.766

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	361	361	344	913	0	3435	0	3651	8739
normalized size	1	1.00	0.95	2.53	0.00	9.52	0.00	10.11	24.21
time (sec)	N/A	3.077	0.715	0.038	0.000	1.501	0.000	3.916	4.907

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	328	443	0	1242	0	274	5999
normalized size	1	1.00	1.50	2.02	0.00	5.67	0.00	1.25	27.39
time (sec)	N/A	0.312	0.375	0.023	0.000	1.408	0.000	2.040	7.473

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	170	0	0	0	0	0	-1
normalized size	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.171	0.090	0.040	0.000	0.882	0.000	0.000	0.000

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	380	380	486	1042	0	0	0	0	-1
normalized size	1	1.00	1.28	2.74	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.287	1.481	0.053	0.000	0.883	0.000	0.000	0.000

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	126	157	0	232	0	127	-1
normalized size	1	1.00	0.98	1.22	0.00	1.80	0.00	0.98	-0.01
time (sec)	N/A	0.093	0.077	0.010	0.000	0.823	0.000	0.971	0.000

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	347	347	452	508	0	0	0	0	-1
normalized size	1	1.00	1.30	1.46	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.224	0.989	0.016	0.000	0.922	0.000	0.000	0.000

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	155	136	0	666	0	0	-1
normalized size	1	1.00	0.80	0.70	0.00	3.43	0.00	0.00	-0.01
time (sec)	N/A	0.209	0.060	0.013	0.000	0.957	0.000	0.000	0.000

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	192	369	0	396	0	662	-1
normalized size	1	1.00	0.79	1.51	0.00	1.62	0.00	2.71	-0.00
time (sec)	N/A	0.357	0.204	0.014	0.000	0.712	0.000	2.112	0.000

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	487	487	609	1878	0	0	0	0	-1
normalized size	1	1.00	1.25	3.86	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.457	2.291	0.020	0.000	0.890	0.000	0.000	0.000

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	152	295	0	332	0	518	-1
normalized size	1	1.00	0.86	1.67	0.00	1.88	0.00	2.93	-0.01
time (sec)	N/A	0.138	0.111	0.023	0.000	0.835	0.000	1.583	0.000

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	425	425	540	1394	0	0	0	0	-1
normalized size	1	1.00	1.27	3.28	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.449	1.736	0.020	0.000	0.880	0.000	0.000	0.000

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	82	72	0	135	0	60	-1
normalized size	1	1.00	1.00	0.88	0.00	1.65	0.00	0.73	-0.01
time (sec)	N/A	0.062	0.018	0.011	0.000	0.933	0.000	0.625	0.000

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	193	177	0	0	0	0	-1
normalized size	1	1.00	1.60	1.46	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.046	0.118	0.015	0.000	0.892	0.000	0.000	0.000

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	83	72	0	137	0	56	-1
normalized size	1	1.00	1.63	1.41	0.00	2.69	0.00	1.10	-0.02
time (sec)	N/A	0.029	0.019	0.018	0.000	0.830	0.000	0.545	0.000

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	330	330	303	508	0	0	0	0	-1
normalized size	1	1.00	0.92	1.54	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.184	0.471	0.018	0.000	0.906	0.000	0.000	0.000

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	391	391	463	533	0	0	0	0	-1
normalized size	1	1.00	1.18	1.36	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.248	1.027	0.019	0.000	0.861	0.000	0.000	0.000

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	126	179	0	424	0	193	-1
normalized size	1	1.00	1.22	1.74	0.00	4.12	0.00	1.87	-0.01
time (sec)	N/A	0.073	0.079	0.015	0.000	0.802	0.000	0.662	0.000

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	468	468	519	1136	0	0	0	0	-1
normalized size	1	1.00	1.11	2.43	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.409	1.347	0.045	0.000	1.039	0.000	0.000	0.000

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	160	220	0	508	0	0	-1
normalized size	1	1.00	1.04	1.43	0.00	3.30	0.00	0.00	-0.01
time (sec)	N/A	0.175	0.074	0.023	0.000	0.948	0.000	0.000	0.000

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	46	0	0	83	0	0	-1
normalized size	1	1.00	0.90	0.00	0.00	1.63	0.00	0.00	-0.02
time (sec)	N/A	0.050	0.086	0.079	0.000	0.704	0.000	0.000	0.000

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	239	0	0	0	0	0	-1
normalized size	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.399	5.125	0.038	0.000	0.800	0.000	0.000	0.000

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	73	58	0	55	0	60	-1
normalized size	1	1.00	1.62	1.29	0.00	1.22	0.00	1.33	-0.02
time (sec)	N/A	0.009	0.020	0.011	0.000	0.865	0.000	0.437	0.000

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	73	58	0	55	0	60	-1
normalized size	1	1.00	1.62	1.29	0.00	1.22	0.00	1.33	-0.02
time (sec)	N/A	0.012	0.003	0.007	0.000	0.800	0.000	0.725	0.000

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	73	58	0	55	0	60	-1
normalized size	1	1.00	1.62	1.29	0.00	1.22	0.00	1.33	-0.02
time (sec)	N/A	0.012	0.004	0.004	0.000	0.872	0.000	0.492	0.000

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	70	81	0	70	0	69	-1
normalized size	1	1.00	0.81	0.94	0.00	0.81	0.00	0.80	-0.01
time (sec)	N/A	0.041	0.033	0.010	0.000	0.849	0.000	0.393	0.000

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	70	81	0	70	0	69	-1
normalized size	1	1.00	0.81	0.94	0.00	0.81	0.00	0.80	-0.01
time (sec)	N/A	0.042	0.010	0.006	0.000	0.867	0.000	0.511	0.000

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	70	81	0	70	0	69	-1
normalized size	1	1.00	0.81	0.94	0.00	0.81	0.00	0.80	-0.01
time (sec)	N/A	0.041	0.003	0.004	0.000	0.592	0.000	0.381	0.000

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	37	35	0	111	0	35	34
normalized size	1	1.00	0.97	0.92	0.00	2.92	0.00	0.92	0.89
time (sec)	N/A	0.016	0.020	0.003	0.000	0.909	0.000	0.386	0.080

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	70	64	0	130	0	59	-1
normalized size	1	1.00	1.56	1.42	0.00	2.89	0.00	1.31	-0.02
time (sec)	N/A	0.022	0.024	0.009	0.000	0.897	0.000	0.466	0.000

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	72	64	0	131	0	53	-1
normalized size	1	1.00	1.53	1.36	0.00	2.79	0.00	1.13	-0.02
time (sec)	N/A	0.076	0.034	0.014	0.000	1.052	0.000	0.493	0.000

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	74	66	0	139	0	53	-1
normalized size	1	1.00	1.51	1.35	0.00	2.84	0.00	1.08	-0.02
time (sec)	N/A	0.090	0.028	0.008	0.000	0.984	0.000	0.490	0.000

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	39	0	124	0	38	44
normalized size	1	1.00	1.00	0.89	0.00	2.82	0.00	0.86	1.00
time (sec)	N/A	0.036	0.007	0.006	0.000	0.858	0.000	0.449	2.226

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	81	72	0	135	0	62	-1
normalized size	1	1.00	1.65	1.47	0.00	2.76	0.00	1.27	-0.02
time (sec)	N/A	0.015	0.017	0.007	0.000	0.946	0.000	0.436	0.000

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	83	72	0	137	0	56	-1
normalized size	1	1.00	1.63	1.41	0.00	2.69	0.00	1.10	-0.02
time (sec)	N/A	0.067	0.018	0.011	0.000	0.926	0.000	0.399	0.000

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	85	74	0	145	0	56	-1
normalized size	1	1.00	1.60	1.40	0.00	2.74	0.00	1.06	-0.02
time (sec)	N/A	0.077	0.019	0.010	0.000	0.925	0.000	0.551	0.000

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	31	20	47	0	55	33
normalized size	1	1.00	1.00	0.78	0.50	1.18	0.00	1.38	0.82
time (sec)	N/A	0.027	0.008	0.004	0.945	0.795	0.000	0.472	0.427

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	73	58	0	55	0	60	-1
normalized size	1	1.00	1.62	1.29	0.00	1.22	0.00	1.33	-0.02
time (sec)	N/A	0.012	0.008	0.000	0.000	0.709	0.000	0.427	0.000

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	62	50	0	49	0	47	-1
normalized size	1	1.00	1.44	1.16	0.00	1.14	0.00	1.09	-0.02
time (sec)	N/A	0.047	0.020	0.013	0.000	0.815	0.000	0.331	0.000

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.067	0.395	0.138	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [86] had the largest ratio of [.5000]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	1	1.00	20	0.050
2	A	2	1	1.00	18	0.056
3	A	1	0	1.00	16	0.000
4	A	2	1	1.00	20	0.050
5	A	2	1	1.00	20	0.050
6	A	3	2	1.00	22	0.091
7	A	3	2	1.00	20	0.100

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
8	A	3	2	1.00	18	0.111
9	A	3	2	1.00	22	0.091
10	A	3	2	1.00	22	0.091
11	A	7	6	1.00	22	0.273
12	A	6	6	1.00	22	0.273
13	A	5	5	1.00	22	0.227
14	A	3	3	1.00	22	0.136
15	A	7	7	1.00	20	0.350
16	A	8	7	1.00	18	0.389
17	A	8	7	1.00	22	0.318
18	A	8	7	1.00	22	0.318
19	A	8	7	1.00	22	0.318
20	A	7	7	1.00	22	0.318
21	A	4	4	1.00	22	0.182
22	A	4	4	1.00	22	0.182
23	A	4	4	1.00	22	0.182
24	A	8	7	1.00	22	0.318
25	A	8	7	1.00	22	0.318
26	A	8	7	1.00	20	0.350
27	A	8	7	1.00	18	0.389
28	A	8	7	1.00	22	0.318
29	A	8	6	1.00	24	0.250
30	A	7	6	1.00	22	0.273
31	A	5	5	1.00	20	0.250
32	A	4	4	1.00	24	0.167
33	A	7	6	1.00	24	0.250
34	A	7	6	1.00	24	0.250
35	A	5	5	1.00	24	0.208
36	A	6	5	1.00	24	0.208
37	A	7	5	1.00	24	0.208
38	A	10	7	1.00	22	0.318
39	A	10	7	1.00	20	0.350
40	A	8	7	1.00	24	0.292
41	A	6	5	1.00	24	0.208
42	A	5	4	1.00	24	0.167
43	A	8	7	1.00	24	0.292

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
44	A	8	7	1.00	24	0.292
45	A	8	7	1.00	24	0.292
46	A	9	8	1.00	24	0.333
47	A	7	6	1.00	24	0.250
48	A	8	6	1.00	24	0.250
49	A	6	6	1.00	24	0.250
50	A	4	4	1.00	24	0.167
51	A	3	3	1.00	22	0.136
52	A	2	2	1.00	20	0.100
53	A	3	3	1.00	24	0.125
54	A	5	5	1.00	24	0.208
55	A	8	6	1.00	24	0.250
56	A	7	6	1.00	24	0.250
57	A	6	6	1.00	24	0.250
58	A	1	1	1.00	24	0.042
59	A	1	1	1.00	24	0.042
60	A	3	3	1.00	24	0.125
61	A	5	5	1.00	22	0.227
62	A	6	5	1.00	20	0.250
63	A	7	5	1.00	24	0.208
64	A	8	5	1.00	24	0.208
65	A	2	1	1.00	18	0.056
66	A	2	1	1.00	18	0.056
67	A	2	1	1.00	16	0.062
68	A	1	0	1.00	14	0.000
69	A	2	1	1.00	18	0.056
70	A	2	1	1.00	18	0.056
71	A	2	1	1.00	18	0.056
72	A	3	2	1.00	20	0.100
73	A	3	2	1.00	20	0.100
74	A	4	3	1.00	18	0.167
75	A	3	2	1.00	16	0.125
76	A	4	3	1.00	20	0.150
77	A	3	2	1.00	20	0.100
78	A	8	7	1.00	20	0.350
79	A	6	5	1.00	20	0.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
80	A	7	7	1.00	20	0.350
81	A	5	4	1.00	20	0.200
82	A	6	6	1.00	20	0.300
83	A	4	3	1.00	20	0.150
84	A	4	4	1.00	20	0.200
85	A	4	3	1.00	18	0.167
86	A	8	8	1.00	16	0.500
87	A	5	4	1.00	20	0.200
88	A	9	8	1.00	20	0.400
89	A	9	8	1.00	20	0.400
90	A	7	5	1.00	20	0.250
91	A	8	8	1.00	20	0.400
92	A	6	5	1.00	20	0.250
93	A	5	5	1.00	20	0.250
94	A	5	4	1.00	20	0.200
95	A	5	5	1.00	20	0.250
96	A	5	4	1.00	20	0.200
97	A	5	5	1.00	20	0.250
98	A	5	4	1.00	20	0.200
99	A	9	8	1.00	18	0.444
100	A	6	5	1.00	16	0.312
101	A	9	8	1.00	20	0.400
102	A	7	5	1.00	20	0.250
103	A	9	8	1.00	20	0.400
104	A	3	3	1.00	20	0.150
105	A	5	5	1.00	24	0.208
106	A	5	5	1.00	24	0.208
107	A	5	5	1.00	24	0.208
108	A	8	7	1.00	24	0.292
109	A	8	8	1.00	24	0.333
110	A	6	6	1.00	24	0.250
111	A	6	5	1.00	24	0.208
112	A	6	6	1.00	24	0.250
113	A	4	4	1.00	24	0.167
114	A	2	2	1.00	24	0.083
115	A	2	2	1.00	24	0.083

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
116	A	6	6	1.00	24	0.250
117	A	5	5	1.00	24	0.208
118	A	3	3	1.00	24	0.125
119	A	6	6	1.00	24	0.250
120	A	5	5	1.00	24	0.208
121	A	1	1	1.00	34	0.029
122	A	7	4	1.00	27	0.148
123	A	2	2	1.00	18	0.111
124	A	3	3	1.00	18	0.167
125	A	3	3	1.00	17	0.176
126	A	5	5	1.00	18	0.278
127	A	6	6	1.00	18	0.333
128	A	6	6	1.00	17	0.353
129	A	2	2	1.00	18	0.111
130	A	3	3	1.00	18	0.167
131	A	3	3	1.00	22	0.136
132	A	3	3	1.00	24	0.125
133	A	3	3	1.00	20	0.150
134	A	3	3	1.00	20	0.150
135	A	3	3	1.00	24	0.125
136	A	3	3	1.00	26	0.115
137	A	3	3	1.00	18	0.167
138	A	3	3	1.00	18	0.167
139	A	3	3	1.00	20	0.150
140	A	2	2	1.00	36	0.056

Chapter 3

Listing of integrals

3.1 $\int x^2 \left(ax^2+bx^3+cx^4\right) dx$

Optimal. Leaf size=25

$$\frac{ax^5}{5}+\frac{bx^6}{6}+\frac{cx^7}{7}$$

```
[Out] 1/5*a*x^5+1/6*b*x^6+1/7*c*x^7
```

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {14}

$$\frac{ax^5}{5}+\frac{bx^6}{6}+\frac{cx^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a*x^2 + b*x^3 + c*x^4),x]

[Out] (a*x^5)/5 + (b*x^6)/6 + (c*x^7)/7
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x^2 \left(ax^2+bx^3+cx^4\right) dx &= \int \left(ax^4+bx^5+cx^6\right) dx\\ &= \frac{ax^5}{5}+\frac{bx^6}{6}+\frac{cx^7}{7}\end{aligned}$$

Mathematica [A] time = 0.00, size = 25, normalized size = 1.00

$$\frac{ax^5}{5}+\frac{bx^6}{6}+\frac{cx^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a*x^2 + b*x^3 + c*x^4),x]

[Out] (a*x^5)/5 + (b*x^6)/6 + (c*x^7)/7
```

fricas [A] time = 0.56, size = 19, normalized size = 0.76

$$\frac{1}{7}x^7c + \frac{1}{6}x^6b + \frac{1}{5}x^5a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^3+a*x^2),x, algorithm="fricas")
[Out] 1/7*x^7*c + 1/6*x^6*b + 1/5*x^5*a
```

giac [A] time = 0.54, size = 19, normalized size = 0.76

$$\frac{1}{7}cx^7 + \frac{1}{6}bx^6 + \frac{1}{5}ax^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^3+a*x^2),x, algorithm="giac")
[Out] 1/7*c*x^7 + 1/6*b*x^6 + 1/5*a*x^5
```

maple [A] time = 0.00, size = 20, normalized size = 0.80

$$\frac{1}{7}cx^7 + \frac{1}{6}bx^6 + \frac{1}{5}ax^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^4+b*x^3+a*x^2),x)
[Out] 1/5*a*x^5+1/6*b*x^6+1/7*c*x^7
```

maxima [A] time = 0.44, size = 19, normalized size = 0.76

$$\frac{1}{7}cx^7 + \frac{1}{6}bx^6 + \frac{1}{5}ax^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^3+a*x^2),x, algorithm="maxima")
[Out] 1/7*c*x^7 + 1/6*b*x^6 + 1/5*a*x^5
```

mupad [B] time = 0.03, size = 19, normalized size = 0.76

$$\frac{x^5\left(30\,c\,x^2 + 35\,b\,x + 42\,a\right)}{210}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a*x^2 + b*x^3 + c*x^4),x)
[Out] (x^5*(42*a + 35*b*x + 30*c*x^2))/210
```

sympy [A] time = 0.06, size = 19, normalized size = 0.76

$$\frac{ax^5}{5} + \frac{bx^6}{6} + \frac{cx^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**4+b*x**3+a*x**2),x)
[Out] a*x**5/5 + b*x**6/6 + c*x**7/7
```

3.2 $\int x\left(ax^2+bx^3+cx^4\right)\,dx$

Optimal. Leaf size=25

$$\frac{ax^4}{4}+\frac{bx^5}{5}+\frac{cx^6}{6}$$

```
[Out] 1/4*a*x^4+1/5*b*x^5+1/6*c*x^6
```

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {14}

$$\frac{ax^4}{4}+\frac{bx^5}{5}+\frac{cx^6}{6}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a*x^2 + b*x^3 + c*x^4),x]
```

```
[Out] (a*x^4)/4 + (b*x^5)/5 + (c*x^6)/6
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x\left(ax^2+bx^3+cx^4\right)\,dx &= \int\left(ax^3+bx^4+cx^5\right)\,dx\\ &= \frac{ax^4}{4}+\frac{bx^5}{5}+\frac{cx^6}{6}\end{aligned}$$

Mathematica [A] time = 0.00, size = 25, normalized size = 1.00

$$\frac{ax^4}{4}+\frac{bx^5}{5}+\frac{cx^6}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a*x^2 + b*x^3 + c*x^4),x]
```

```
[Out] (a*x^4)/4 + (b*x^5)/5 + (c*x^6)/6
```

fricas [A] time = 0.47, size = 19, normalized size = 0.76

$$\frac{1}{6}x^6c+\frac{1}{5}x^5b+\frac{1}{4}x^4a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^3+a*x^2),x, algorithm="fricas")
```

```
[Out] 1/6*x^6*c + 1/5*x^5*b + 1/4*x^4*a
```

giac [A] time = 0.45, size = 19, normalized size = 0.76

$$\frac{1}{6}\,cx^6+\frac{1}{5}\,bx^5+\frac{1}{4}\,ax^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^3+a*x^2),x, algorithm="giac")
```

```
[Out] 1/6*c*x^6 + 1/5*b*x^5 + 1/4*a*x^4
```

maple [A] time = 0.00, size = 20, normalized size = 0.80

$$\frac{1}{6}c\,x^{6}+\frac{1}{5}b\,x^{5}+\frac{1}{4}a\,x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^4+b*x^3+a*x^2),x)
```

```
[Out] 1/4*a*x^4+1/5*b*x^5+1/6*c*x^6
```

maxima [A] time = 0.44, size = 19, normalized size = 0.76

$$\frac{1}{6}\,cx^{6}+\frac{1}{5}\,bx^{5}+\frac{1}{4}\,ax^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^3+a*x^2),x, algorithm="maxima")
```

```
[Out] 1/6*c*x^6 + 1/5*b*x^5 + 1/4*a*x^4
```

mupad [B] time = 0.03, size = 19, normalized size = 0.76

$$\frac{x^{4}\,\left(10\,c\,x^{2}+12\,b\,x+15\,a\right)}{60}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a*x^2 + b*x^3 + c*x^4),x)
```

```
[Out] (x^4*(15*a + 12*b*x + 10*c*x^2))/60
```

sympy [A] time = 0.06, size = 19, normalized size = 0.76

$$\frac{ax^{4}}{4}+\frac{bx^{5}}{5}+\frac{cx^{6}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**4+b*x**3+a*x**2),x)
```

```
[Out] a*x**4/4 + b*x**5/5 + c*x**6/6
```

3.3 $\int \left(ax^2+bx^3+cx^4\right) dx$

Optimal. Leaf size=25

$$\frac{ax^3}{3}+\frac{bx^4}{4}+\frac{cx^5}{5}$$

```
[Out] 1/3*a*x^3+1/4*b*x^4+1/5*c*x^5
```

Rubi [A] time = 0.00, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\frac{ax^3}{3}+\frac{bx^4}{4}+\frac{cx^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[a*x^2 + b*x^3 + c*x^4,x]
```

```
[Out] (a*x^3)/3 + (b*x^4)/4 + (c*x^5)/5
```

Rubi steps

$$\int \left(ax^2+bx^3+cx^4\right) dx=\frac{ax^3}{3}+\frac{bx^4}{4}+\frac{cx^5}{5}$$

Mathematica [A] time = 0.00, size = 25, normalized size = 1.00

$$\frac{ax^3}{3}+\frac{bx^4}{4}+\frac{cx^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[a*x^2 + b*x^3 + c*x^4,x]
```

```
[Out] (a*x^3)/3 + (b*x^4)/4 + (c*x^5)/5
```

fricas [A] time = 0.63, size = 19, normalized size = 0.76

$$\frac{1}{5}x^5c+\frac{1}{4}x^4b+\frac{1}{3}x^3a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x^4+b*x^3+a*x^2,x, algorithm="fricas")
```

```
[Out] 1/5*x^5*c + 1/4*x^4*b + 1/3*x^3*a
```

giac [A] time = 0.54, size = 19, normalized size = 0.76

$$\frac{1}{5}\,cx^5+\frac{1}{4}\,bx^4+\frac{1}{3}\,ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x^4+b*x^3+a*x^2,x, algorithm="giac")
```

```
[Out] 1/5*c*x^5 + 1/4*b*x^4 + 1/3*a*x^3
```

maple [A] time = 0.00, size = 20, normalized size = 0.80

$$\frac{1}{5}c\,x^5+\frac{1}{4}b\,x^4+\frac{1}{3}a\,x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(c*x^4+b*x^3+a*x^2,x)
[Out] 1/3*a*x^3+1/4*b*x^4+1/5*c*x^5
```

maxima [A] time = 0.43, size = 19, normalized size = 0.76

$$\frac{1}{5}\,cx^5 + \frac{1}{4}\,bx^4 + \frac{1}{3}\,ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x^4+b*x^3+a*x^2,x, algorithm="maxima")
[Out] 1/5*c*x^5 + 1/4*b*x^4 + 1/3*a*x^3
```

mupad [B] time = 0.03, size = 19, normalized size = 0.76

$$\frac{x^3\,\left(12\,c\,x^2+15\,b\,x+20\,a\right)}{60}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a*x^2 + b*x^3 + c*x^4,x)
[Out] (x^3*(20*a + 15*b*x + 12*c*x^2))/60
```

sympy [A] time = 0.06, size = 19, normalized size = 0.76

$$\frac{ax^3}{3} + \frac{bx^4}{4} + \frac{cx^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x**4+b*x**3+a*x**2,x)
[Out] a*x**3/3 + b*x**4/4 + c*x**5/5
```

3.4 $\int \frac{ax^2+bx^3+cx^4}{x} \, dx$

Optimal. Leaf size=25

$$\frac{ax^2}{2}+\frac{bx^3}{3}+\frac{cx^4}{4}$$

```
[Out] 1/2*a*x^2+1/3*b*x^3+1/4*c*x^4
```

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {14}

$$\frac{ax^2}{2}+\frac{bx^3}{3}+\frac{cx^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3 + c*x^4)/x,x]
```

```
[Out] (a*x^2)/2 + (b*x^3)/3 + (c*x^4)/4
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{ax^2+bx^3+cx^4}{x} \, dx &= \int \left(ax+bx^2+cx^3\right) \, dx\\ &= \frac{ax^2}{2}+\frac{bx^3}{3}+\frac{cx^4}{4}\end{aligned}$$

Mathematica [A] time = 0.00, size = 25, normalized size = 1.00

$$\frac{ax^2}{2}+\frac{bx^3}{3}+\frac{cx^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3 + c*x^4)/x,x]
```

```
[Out] (a*x^2)/2 + (b*x^3)/3 + (c*x^4)/4
```

fricas [A] time = 0.76, size = 19, normalized size = 0.76

$$\frac{1}{4}\,cx^4+\frac{1}{3}\,bx^3+\frac{1}{2}\,ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)/x,x, algorithm="fricas")
```

```
[Out] 1/4*c*x^4 + 1/3*b*x^3 + 1/2*a*x^2
```

giac [A] time = 0.36, size = 19, normalized size = 0.76

$$\frac{1}{4}\,cx^4+\frac{1}{3}\,bx^3+\frac{1}{2}\,ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)/x,x, algorithm="giac")
[Out] 1/4*c*x^4 + 1/3*b*x^3 + 1/2*a*x^2
```

maple [A] time = 0.00, size = 20, normalized size = 0.80

$$\frac{1}{4}c\,x^{4}+\frac{1}{3}b\,x^{3}+\frac{1}{2}a\,x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^3+a*x^2)/x,x)
[Out] 1/2*a*x^2+1/3*b*x^3+1/4*c*x^4
```

maxima [A] time = 0.45, size = 19, normalized size = 0.76

$$\frac{1}{4}\,cx^{4}+\frac{1}{3}\,bx^{3}+\frac{1}{2}\,ax^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)/x,x, algorithm="maxima")
[Out] 1/4*c*x^4 + 1/3*b*x^3 + 1/2*a*x^2
```

mupad [B] time = 0.03, size = 19, normalized size = 0.76

$$\frac{x^{2}\,\left(3\,c\,x^{2}+4\,b\,x+6\,a\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3 + c*x^4)/x,x)
[Out] (x^2*(6*a + 4*b*x + 3*c*x^2))/12
```

sympy [A] time = 0.06, size = 19, normalized size = 0.76

$$\frac{ax^{2}}{2}+\frac{bx^{3}}{3}+\frac{cx^{4}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**3+a*x**2)/x,x)
[Out] a*x**2/2 + b*x**3/3 + c*x**4/4
```

3.5 $\int \frac{ax^2+bx^3+cx^4}{x^2} \, dx$

Optimal. Leaf size=20

$$ax + \frac{bx^2}{2} + \frac{cx^3}{3}$$

```
[Out] a*x+1/2*b*x^2+1/3*c*x^3
```

Rubi [A] time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {14}

$$ax + \frac{bx^2}{2} + \frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3 + c*x^4)/x^2,x]
```

```
[Out] a*x + (b*x^2)/2 + (c*x^3)/3
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}
\int \frac{ax^2 + bx^3 + cx^4}{x^2} \, dx &= \int \left(a + bx + cx^2\right) \, dx \\
&= ax + \frac{bx^2}{2} + \frac{cx^3}{3}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 20, normalized size = 1.00

$$ax + \frac{bx^2}{2} + \frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3 + c*x^4)/x^2,x]
```

```
[Out] a*x + (b*x^2)/2 + (c*x^3)/3
```

fricas [A] time = 0.58, size = 16, normalized size = 0.80

$$\frac{1}{3}cx^3 + \frac{1}{2}bx^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/3*c*x^3 + 1/2*b*x^2 + a*x
```

giac [A] time = 0.49, size = 16, normalized size = 0.80

$$\frac{1}{3}cx^3 + \frac{1}{2}bx^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)/x^2,x, algorithm="giac")
```

```
[Out] 1/3*c*x^3 + 1/2*b*x^2 + a*x
```

maple [A] time = 0.00, size = 17, normalized size = 0.85

$$\frac{1}{3}c\,x^3+\frac{1}{2}b\,x^2+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^3+a*x^2)/x^2,x)
```

```
[Out] a*x+1/2*b*x^2+1/3*c*x^3
```

maxima [A] time = 0.43, size = 16, normalized size = 0.80

$$\frac{1}{3}\,cx^3+\frac{1}{2}\,bx^2+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)/x^2,x, algorithm="maxima")
```

```
[Out] 1/3*c*x^3 + 1/2*b*x^2 + a*x
```

mupad [B] time = 0.03, size = 16, normalized size = 0.80

$$\frac{c\,x^3}{3}+\frac{b\,x^2}{2}+a\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3 + c*x^4)/x^2,x)
```

```
[Out] a*x + (b*x^2)/2 + (c*x^3)/3
```

sympy [A] time = 0.06, size = 15, normalized size = 0.75

$$ax+\frac{bx^2}{2}+\frac{cx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**3+a*x**2)/x**2,x)
```

```
[Out] a*x + b*x**2/2 + c*x**3/3
```

3.6 $\int x^2 \left(ax^2+bx^3+cx^4\right)^2 \, dx$

Optimal. Leaf size=54

$$\frac{a^2x^7}{7}+\frac{1}{9}x^9\left(2ac+b^2\right)+\frac{1}{4}abx^8+\frac{1}{5}bcx^{10}+\frac{c^2x^{11}}{11}$$

```
[Out] 1/7*a^2*x^7+1/4*a*b*x^8+1/9*(2*a*c+b^2)*x^9+1/5*b*c*x^10+1/11*c^2*x^11
```

Rubi [A] time = 0.05, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1585, 698}

$$\frac{a^2x^7}{7}+\frac{1}{9}x^9\left(2ac+b^2\right)+\frac{1}{4}abx^8+\frac{1}{5}bcx^{10}+\frac{c^2x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] (a^2*x^7)/7 + (a*b*x^8)/4 + ((b^2 + 2*a*c)*x^9)/9 + (b*c*x^10)/5 + (c^2*x^1
1)/11
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\begin{aligned}\int x^2\left(ax^2+bx^3+cx^4\right)^2\,dx &= \int x^6\left(a+bx+cx^2\right)^2\,dx\\ &= \int\left(a^2x^6+2abx^7+\left(b^2+2ac\right)x^8+2bcx^9+c^2x^{10}\right)dx\\ &= \frac{a^2x^7}{7}+\frac{1}{4}abx^8+\frac{1}{9}\left(b^2+2ac\right)x^9+\frac{1}{5}bcx^{10}+\frac{c^2x^{11}}{11}\end{aligned}$$

Mathematica [A] time = 0.01, size = 54, normalized size = 1.00

$$\frac{a^2x^7}{7}+\frac{1}{9}x^9\left(2ac+b^2\right)+\frac{1}{4}abx^8+\frac{1}{5}bcx^{10}+\frac{c^2x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] (a^2*x^7)/7 + (a*b*x^8)/4 + ((b^2 + 2*a*c)*x^9)/9 + (b*c*x^10)/5 + (c^2*x^1
1)/11
```

fricas [A] time = 0.43, size = 46, normalized size = 0.85

$$\frac{1}{11}x^{11}c^2 + \frac{1}{5}x^{10}cb + \frac{1}{9}x^9b^2 + \frac{2}{9}x^9ca + \frac{1}{4}x^8ba + \frac{1}{7}x^7a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/11*x^11*c^2 + 1/5*x^10*c*b + 1/9*x^9*b^2 + 2/9*x^9*c*a + 1/4*x^8*b*a + 1/
7*x^7*a^2
```

giac [A] time = 0.58, size = 46, normalized size = 0.85

$$\frac{1}{11}c^2x^{11} + \frac{1}{5}bcx^{10} + \frac{1}{9}b^2x^9 + \frac{2}{9}acx^9 + \frac{1}{4}abx^8 + \frac{1}{7}a^2x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/11*c^2*x^11 + 1/5*b*c*x^10 + 1/9*b^2*x^9 + 2/9*a*c*x^9 + 1/4*a*b*x^8 + 1/
7*a^2*x^7
```

maple [A] time = 0.00, size = 45, normalized size = 0.83

$$\frac{c^2x^{11}}{11} + \frac{bc\,x^{10}}{5} + \frac{ab\,x^8}{4} + \frac{a^2x^7}{7} + \frac{\left(2ac + b^2\right)x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^4+b*x^3+a*x^2)^2,x)
```

```
[Out] 1/7*a^2*x^7+1/4*a*b*x^8+1/9*(2*a*c+b^2)*x^9+1/5*b*c*x^10+1/11*c^2*x^11
```

maxima [A] time = 0.43, size = 44, normalized size = 0.81

$$\frac{1}{11}c^2x^{11} + \frac{1}{5}bcx^{10} + \frac{1}{4}abx^8 + \frac{1}{9}\left(b^2 + 2\,ac\right)x^9 + \frac{1}{7}a^2x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/11*c^2*x^11 + 1/5*b*c*x^10 + 1/4*a*b*x^8 + 1/9*(b^2 + 2*a*c)*x^9 + 1/7*a^
2*x^7
```

mupad [B] time = 0.03, size = 45, normalized size = 0.83

$$x^9\left(\frac{b^2}{9} + \frac{2\,a\,c}{9}\right) + \frac{a^2\,x^7}{7} + \frac{c^2\,x^{11}}{11} + \frac{a\,b\,x^8}{4} + \frac{b\,c\,x^{10}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a*x^2 + b*x^3 + c*x^4)^2,x)
```

```
[Out] x^9*((2*a*c)/9 + b^2/9) + (a^2*x^7)/7 + (c^2*x^11)/11 + (a*b*x^8)/4 + (b*c*
x^10)/5
```

sympy [A] time = 0.08, size = 48, normalized size = 0.89

$$\frac{a^2x^7}{7} + \frac{abx^8}{4} + \frac{bcx^{10}}{5} + \frac{c^2x^{11}}{11} + x^9\left(\frac{2ac}{9} + \frac{b^2}{9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**4+b*x**3+a*x**2)**2,x)
```

```
[Out] a**2*x**7/7 + a*b*x**8/4 + b*c*x**10/5 + c**2*x**11/11 + x**9*(2*a*c/9 + b*
*2/9)
```

3.7 $\int x\left(ax^2+bx^3+cx^4\right)^2\,dx$

Optimal. Leaf size=54

$$\frac{a^2x^6}{6}+\frac{1}{8}x^8\left(2ac+b^2\right)+\frac{2}{7}abx^7+\frac{2}{9}bcx^9+\frac{c^2x^{10}}{10}$$

```
[Out] 1/6*a^2*x^6+2/7*a*b*x^7+1/8*(2*a*c+b^2)*x^8+2/9*b*c*x^9+1/10*c^2*x^10
```

Rubi [A] time = 0.03, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1585, 698}

$$\frac{a^2x^6}{6}+\frac{1}{8}x^8\left(2ac+b^2\right)+\frac{2}{7}abx^7+\frac{2}{9}bcx^9+\frac{c^2x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] (a^2*x^6)/6 + (2*a*b*x^7)/7 + ((b^2 + 2*a*c)*x^8)/8 + (2*b*c*x^9)/9 + (c^2*
x^10)/10
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\begin{aligned}\int x\left(ax^2+bx^3+cx^4\right)^2\,dx&=\int x^5\left(a+bx+cx^2\right)^2\,dx\\&=\int\left(a^2x^5+2abx^6+\left(b^2+2ac\right)x^7+2bcx^8+c^2x^9\right)dx\\&=\frac{a^2x^6}{6}+\frac{2}{7}abx^7+\frac{1}{8}\left(b^2+2ac\right)x^8+\frac{2}{9}bcx^9+\frac{c^2x^{10}}{10}\end{aligned}$$

Mathematica [A] time = 0.01, size = 54, normalized size = 1.00

$$\frac{a^2x^6}{6}+\frac{1}{8}x^8\left(2ac+b^2\right)+\frac{2}{7}abx^7+\frac{2}{9}bcx^9+\frac{c^2x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] (a^2*x^6)/6 + (2*a*b*x^7)/7 + ((b^2 + 2*a*c)*x^8)/8 + (2*b*c*x^9)/9 + (c^2*
x^10)/10
```

fricas [A] time = 0.59, size = 46, normalized size = 0.85

$$\frac{1}{10}x^{10}c^2 + \frac{2}{9}x^9cb + \frac{1}{8}x^8b^2 + \frac{1}{4}x^8ca + \frac{2}{7}x^7ba + \frac{1}{6}x^6a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/10*x^10*c^2 + 2/9*x^9*c*b + 1/8*x^8*b^2 + 1/4*x^8*c*a + 2/7*x^7*b*a + 1/6
*x^6*a^2
```

giac [A] time = 0.48, size = 46, normalized size = 0.85

$$\frac{1}{10}c^2x^{10} + \frac{2}{9}bcx^9 + \frac{1}{8}b^2x^8 + \frac{1}{4}acx^8 + \frac{2}{7}abx^7 + \frac{1}{6}a^2x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/10*c^2*x^10 + 2/9*b*c*x^9 + 1/8*b^2*x^8 + 1/4*a*c*x^8 + 2/7*a*b*x^7 + 1/6
*a^2*x^6
```

maple [A] time = 0.00, size = 45, normalized size = 0.83

$$\frac{c^2x^{10}}{10} + \frac{2bc\,x^9}{9} + \frac{2ab\,x^7}{7} + \frac{a^2x^6}{6} + \frac{\left(2ac + b^2\right)x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^4+b*x^3+a*x^2)^2,x)
```

```
[Out] 1/6*a^2*x^6+2/7*a*b*x^7+1/8*(2*a*c+b^2)*x^8+2/9*b*c*x^9+1/10*c^2*x^10
```

maxima [A] time = 0.44, size = 44, normalized size = 0.81

$$\frac{1}{10}c^2x^{10} + \frac{2}{9}bcx^9 + \frac{2}{7}abx^7 + \frac{1}{8}\left(b^2 + 2\,ac\right)x^8 + \frac{1}{6}a^2x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/10*c^2*x^10 + 2/9*b*c*x^9 + 2/7*a*b*x^7 + 1/8*(b^2 + 2*a*c)*x^8 + 1/6*a^2
*x^6
```

mupad [B] time = 0.02, size = 45, normalized size = 0.83

$$x^8\left(\frac{b^2}{8} + \frac{a\,c}{4}\right) + \frac{a^2\,x^6}{6} + \frac{c^2\,x^{10}}{10} + \frac{2\,a\,b\,x^7}{7} + \frac{2\,b\,c\,x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a*x^2 + b*x^3 + c*x^4)^2,x)
```

```
[Out] x^8*((a*c)/4 + b^2/8) + (a^2*x^6)/6 + (c^2*x^10)/10 + (2*a*b*x^7)/7 + (2*b*
c*x^9)/9
```

sympy [A] time = 0.07, size = 49, normalized size = 0.91

$$\frac{a^2x^6}{6} + \frac{2abx^7}{7} + \frac{2bcx^9}{9} + \frac{c^2x^{10}}{10} + x^8\left(\frac{ac}{4} + \frac{b^2}{8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**4+b*x**3+a*x**2)**2,x)
```

```
[Out] a**2*x**6/6 + 2*a*b*x**7/7 + 2*b*c*x**9/9 + c**2*x**10/10 + x**8*(a*c/4 + b
**2/8)
```

3.8 $\int \left(ax^2+bx^3+cx^4\right)^2 \, dx$

Optimal. Leaf size=54

$$\frac{a^2x^5}{5}+\frac{1}{7}x^7\left(2ac+b^2\right)+\frac{1}{3}abx^6+\frac{1}{4}bcx^8+\frac{c^2x^9}{9}$$

```
[Out] 1/5*a^2*x^5+1/3*a*b*x^6+1/7*(2*a*c+b^2)*x^7+1/4*b*c*x^8+1/9*c^2*x^9
```

Rubi [A] time = 0.03, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1594, 698}

$$\frac{a^2x^5}{5}+\frac{1}{7}x^7\left(2ac+b^2\right)+\frac{1}{3}abx^6+\frac{1}{4}bcx^8+\frac{c^2x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] (a^2*x^5)/5 + (a*b*x^6)/3 + ((b^2 + 2*a*c)*x^7)/7 + (b*c*x^8)/4 + (c^2*x^9)
/9
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rubi steps

$$\begin{aligned}\int \left(ax^2+bx^3+cx^4\right)^2 \, dx &= \int x^4\left(a+bx+cx^2\right)^2 \, dx\\ &= \int \left(a^2x^4+2abx^5+\left(b^2+2ac\right)x^6+2bcx^7+c^2x^8\right) dx\\ &= \frac{a^2x^5}{5}+\frac{1}{3}abx^6+\frac{1}{7}\left(b^2+2ac\right)x^7+\frac{1}{4}bcx^8+\frac{c^2x^9}{9}\end{aligned}$$

Mathematica [A] time = 0.01, size = 54, normalized size = 1.00

$$\frac{a^2x^5}{5}+\frac{1}{7}x^7\left(2ac+b^2\right)+\frac{1}{3}abx^6+\frac{1}{4}bcx^8+\frac{c^2x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] (a^2*x^5)/5 + (a*b*x^6)/3 + ((b^2 + 2*a*c)*x^7)/7 + (b*c*x^8)/4 + (c^2*x^9)
/9
```

fricas [A] time = 0.48, size = 46, normalized size = 0.85

$$\frac{1}{9}x^9c^2 + \frac{1}{4}x^8cb + \frac{1}{7}x^7b^2 + \frac{2}{7}x^7ca + \frac{1}{3}x^6ba + \frac{1}{5}x^5a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/9*x^9*c^2 + 1/4*x^8*c*b + 1/7*x^7*b^2 + 2/7*x^7*c*a + 1/3*x^6*b*a + 1/5*x
^5*a^2
```

giac [A] time = 0.40, size = 46, normalized size = 0.85

$$\frac{1}{9}c^2x^9 + \frac{1}{4}bcx^8 + \frac{1}{7}b^2x^7 + \frac{2}{7}acx^7 + \frac{1}{3}abx^6 + \frac{1}{5}a^2x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/9*c^2*x^9 + 1/4*b*c*x^8 + 1/7*b^2*x^7 + 2/7*a*c*x^7 + 1/3*a*b*x^6 + 1/5*a
^2*x^5
```

maple [A] time = 0.00, size = 45, normalized size = 0.83

$$\frac{c^2x^9}{9} + \frac{bc\,x^8}{4} + \frac{ab\,x^6}{3} + \frac{a^2x^5}{5} + \frac{\left(2ac + b^2\right)x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^3+a*x^2)^2,x)
```

```
[Out] 1/5*a^2*x^5+1/3*a*b*x^6+1/7*(2*a*c+b^2)*x^7+1/4*b*c*x^8+1/9*c^2*x^9
```

maxima [A] time = 0.45, size = 48, normalized size = 0.89

$$\frac{1}{9}c^2x^9 + \frac{1}{4}bcx^8 + \frac{1}{7}b^2x^7 + \frac{1}{5}a^2x^5 + \frac{1}{21}\left(6\,cx^7 + 7\,bx^6\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/9*c^2*x^9 + 1/4*b*c*x^8 + 1/7*b^2*x^7 + 1/5*a^2*x^5 + 1/21*(6*c*x^7 + 7*b
*x^6)*a
```

mupad [B] time = 0.02, size = 45, normalized size = 0.83

$$x^7\left(\frac{b^2}{7} + \frac{2\,a\,c}{7}\right) + \frac{a^2\,x^5}{5} + \frac{c^2\,x^9}{9} + \frac{a\,b\,x^6}{3} + \frac{b\,c\,x^8}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3 + c*x^4)^2,x)
```

```
[Out] x^7*((2*a*c)/7 + b^2/7) + (a^2*x^5)/5 + (c^2*x^9)/9 + (a*b*x^6)/3 + (b*c*x^
8)/4
```

sympy [A] time = 0.07, size = 48, normalized size = 0.89

$$\frac{a^2x^5}{5} + \frac{abx^6}{3} + \frac{bcx^8}{4} + \frac{c^2x^9}{9} + x^7\left(\frac{2ac}{7} + \frac{b^2}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**3+a*x**2)**2,x)
```

```
[Out] a**2*x**5/5 + a*b*x**6/3 + b*c*x**8/4 + c**2*x**9/9 + x**7*(2*a*c/7 + b**2/
7)
```

3.9 $\int \frac{\left(ax^2+bx^3+cx^4\right)^2}{x} \, dx$

<u>**Optimal**</u>. Leaf size=54

$$\frac{a^2x^4}{4}+\frac{1}{6}x^6\left(2ac+b^2\right)+\frac{2}{5}abx^5+\frac{2}{7}bcx^7+\frac{c^2x^8}{8}$$

```
[Out] 1/4*a^2*x^4+2/5*a*b*x^5+1/6*(2*a*c+b^2)*x^6+2/7*b*c*x^7+1/8*c^2*x^8
```

<u>**Rubi**</u> [A] time = 0.03, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1585, 698}

$$\frac{a^2x^4}{4}+\frac{1}{6}x^6\left(2ac+b^2\right)+\frac{2}{5}abx^5+\frac{2}{7}bcx^7+\frac{c^2x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3 + c*x^4)^2/x,x]
```

```
[Out] (a^2*x^4)/4 + (2*a*b*x^5)/5 + ((b^2 + 2*a*c)*x^6)/6 + (2*b*c*x^7)/7 + (c^2*
x^8)/8
```

<u>Rule 698</u>

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

<u>Rule 1585</u>

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

<u>Rubi steps</u>

$$\begin{aligned}\int \frac{\left(ax^2+bx^3+cx^4\right)^2}{x}\,dx &= \int x^3\left(a+bx+cx^2\right)^2\,dx\\ &= \int \left(a^2x^3+2abx^4+\left(b^2+2ac\right)x^5+2bcx^6+c^2x^7\right)dx\\ &= \frac{a^2x^4}{4}+\frac{2}{5}abx^5+\frac{1}{6}\left(b^2+2ac\right)x^6+\frac{2}{7}bcx^7+\frac{c^2x^8}{8}\end{aligned}$$

<u>**Mathematica**</u> [A] time = 0.01, size = 54, normalized size = 1.00

$$\frac{a^2x^4}{4}+\frac{1}{6}x^6\left(2ac+b^2\right)+\frac{2}{5}abx^5+\frac{2}{7}bcx^7+\frac{c^2x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3 + c*x^4)^2/x,x]
```

```
[Out] (a^2*x^4)/4 + (2*a*b*x^5)/5 + ((b^2 + 2*a*c)*x^6)/6 + (2*b*c*x^7)/7 + (c^2*
x^8)/8
```

fricas [A] time = 0.67, size = 44, normalized size = 0.81

$$\frac{1}{8}\,c^2x^8 + \frac{2}{7}\,bcx^7 + \frac{2}{5}\,abx^5 + \frac{1}{6}\left(b^2 + 2\,ac\right)x^6 + \frac{1}{4}\,a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^2/x,x, algorithm="fricas")
```

```
[Out] 1/8*c^2*x^8 + 2/7*b*c*x^7 + 2/5*a*b*x^5 + 1/6*(b^2 + 2*a*c)*x^6 + 1/4*a^2*x
^4
```

giac [A] time = 0.49, size = 46, normalized size = 0.85

$$\frac{1}{8}\,c^2x^8 + \frac{2}{7}\,bcx^7 + \frac{1}{6}\,b^2x^6 + \frac{1}{3}\,acx^6 + \frac{2}{5}\,abx^5 + \frac{1}{4}\,a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^2/x,x, algorithm="giac")
```

```
[Out] 1/8*c^2*x^8 + 2/7*b*c*x^7 + 1/6*b^2*x^6 + 1/3*a*c*x^6 + 2/5*a*b*x^5 + 1/4*a
^2*x^4
```

maple [A] time = 0.00, size = 45, normalized size = 0.83

$$\frac{c^2x^8}{8} + \frac{2bc\,x^7}{7} + \frac{2ab\,x^5}{5} + \frac{a^2x^4}{4} + \frac{\left(2ac + b^2\right)x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^3+a*x^2)^2/x,x)
```

```
[Out] 1/4*a^2*x^4+2/5*a*b*x^5+1/6*(2*a*c+b^2)*x^6+2/7*b*c*x^7+1/8*c^2*x^8
```

maxima [A] time = 0.43, size = 44, normalized size = 0.81

$$\frac{1}{8}\,c^2x^8 + \frac{2}{7}\,bcx^7 + \frac{2}{5}\,abx^5 + \frac{1}{6}\left(b^2 + 2\,ac\right)x^6 + \frac{1}{4}\,a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^2/x,x, algorithm="maxima")
```

```
[Out] 1/8*c^2*x^8 + 2/7*b*c*x^7 + 2/5*a*b*x^5 + 1/6*(b^2 + 2*a*c)*x^6 + 1/4*a^2*x
^4
```

mupad [B] time = 0.02, size = 45, normalized size = 0.83

$$x^6\left(\frac{b^2}{6} + \frac{a\,c}{3}\right) + \frac{a^2\,x^4}{4} + \frac{c^2\,x^8}{8} + \frac{2\,a\,b\,x^5}{5} + \frac{2\,b\,c\,x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3 + c*x^4)^2/x,x)
```

```
[Out] x^6*((a*c)/3 + b^2/6) + (a^2*x^4)/4 + (c^2*x^8)/8 + (2*a*b*x^5)/5 + (2*b*c*
x^7)/7
```

sympy [A] time = 0.08, size = 49, normalized size = 0.91

$$\frac{a^2x^4}{4} + \frac{2abx^5}{5} + \frac{2bcx^7}{7} + \frac{c^2x^8}{8} + x^6\left(\frac{ac}{3} + \frac{b^2}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**3+a*x**2)**2/x,x)
```

```
[Out] a**2*x**4/4 + 2*a*b*x**5/5 + 2*b*c*x**7/7 + c**2*x**8/8 + x**6*(a*c/3 + b**
2/6)
```

3.10 $\int \frac{(ax^2+bx^3+cx^4)^2}{x^2}\,dx$

Optimal. Leaf size=54

$$\frac{a^2x^3}{3}+\frac{1}{5}x^5\left(2ac+b^2\right)+\frac{1}{2}abx^4+\frac{1}{3}bcx^6+\frac{c^2x^7}{7}$$

```
[Out] 1/3*a^2*x^3+1/2*a*b*x^4+1/5*(2*a*c+b^2)*x^5+1/3*b*c*x^6+1/7*c^2*x^7
```

Rubi [A] time = 0.03, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1585, 698}

$$\frac{a^2x^3}{3}+\frac{1}{5}x^5\left(2ac+b^2\right)+\frac{1}{2}abx^4+\frac{1}{3}bcx^6+\frac{c^2x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3 + c*x^4)^2/x^2,x]
```

```
[Out] (a^2*x^3)/3 + (a*b*x^4)/2 + ((b^2 + 2*a*c)*x^5)/5 + (b*c*x^6)/3 + (c^2*x^7)
/7
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(ax^2+bx^3+cx^4\right)^2}{x^2}\,dx &= \int x^2\left(a+bx+cx^2\right)^2\,dx\\ &= \int \left(a^2x^2+2abx^3+\left(b^2+2ac\right)x^4+2bcx^5+c^2x^6\right)dx\\ &= \frac{a^2x^3}{3}+\frac{1}{2}abx^4+\frac{1}{5}\left(b^2+2ac\right)x^5+\frac{1}{3}bcx^6+\frac{c^2x^7}{7}\end{aligned}$$

Mathematica [A] time = 0.01, size = 54, normalized size = 1.00

$$\frac{a^2x^3}{3}+\frac{1}{5}x^5\left(2ac+b^2\right)+\frac{1}{2}abx^4+\frac{1}{3}bcx^6+\frac{c^2x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3 + c*x^4)^2/x^2,x]
```

```
[Out] (a^2*x^3)/3 + (a*b*x^4)/2 + ((b^2 + 2*a*c)*x^5)/5 + (b*c*x^6)/3 + (c^2*x^7)
/7
```

fricas [A] time = 0.58, size = 44, normalized size = 0.81

$$\frac{1}{7}c^2x^7 + \frac{1}{3}bcx^6 + \frac{1}{2}abx^4 + \frac{1}{5}\left(b^2 + 2\,ac\right)x^5 + \frac{1}{3}a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^2/x^2,x, algorithm="fricas")
```

```
[Out] 1/7*c^2*x^7 + 1/3*b*c*x^6 + 1/2*a*b*x^4 + 1/5*(b^2 + 2*a*c)*x^5 + 1/3*a^2*x
^3
```

giac [A] time = 0.48, size = 46, normalized size = 0.85

$$\frac{1}{7}c^2x^7 + \frac{1}{3}bcx^6 + \frac{1}{5}b^2x^5 + \frac{2}{5}acx^5 + \frac{1}{2}abx^4 + \frac{1}{3}a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^2/x^2,x, algorithm="giac")
```

```
[Out] 1/7*c^2*x^7 + 1/3*b*c*x^6 + 1/5*b^2*x^5 + 2/5*a*c*x^5 + 1/2*a*b*x^4 + 1/3*a
^2*x^3
```

maple [A] time = 0.00, size = 45, normalized size = 0.83

$$\frac{c^2x^7}{7} + \frac{bc\,x^6}{3} + \frac{ab\,x^4}{2} + \frac{a^2x^3}{3} + \frac{\left(2ac + b^2\right)x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^3+a*x^2)^2/x^2,x)
```

```
[Out] 1/3*a^2*x^3+1/2*a*b*x^4+1/5*(2*a*c+b^2)*x^5+1/3*b*c*x^6+1/7*c^2*x^7
```

maxima [A] time = 0.42, size = 44, normalized size = 0.81

$$\frac{1}{7}c^2x^7 + \frac{1}{3}bcx^6 + \frac{1}{2}abx^4 + \frac{1}{5}\left(b^2 + 2\,ac\right)x^5 + \frac{1}{3}a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^2/x^2,x, algorithm="maxima")
```

```
[Out] 1/7*c^2*x^7 + 1/3*b*c*x^6 + 1/2*a*b*x^4 + 1/5*(b^2 + 2*a*c)*x^5 + 1/3*a^2*x
^3
```

mupad [B] time = 0.02, size = 45, normalized size = 0.83

$$x^5\left(\frac{b^2}{5} + \frac{2\,a\,c}{5}\right) + \frac{a^2\,x^3}{3} + \frac{c^2\,x^7}{7} + \frac{a\,b\,x^4}{2} + \frac{b\,c\,x^6}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3 + c*x^4)^2/x^2,x)
```

```
[Out] x^5*((2*a*c)/5 + b^2/5) + (a^2*x^3)/3 + (c^2*x^7)/7 + (a*b*x^4)/2 + (b*c*x^
6)/3
```

sympy [A] time = 0.08, size = 48, normalized size = 0.89

$$\frac{a^2x^3}{3} + \frac{abx^4}{2} + \frac{bcx^6}{3} + \frac{c^2x^7}{7} + x^5\left(\frac{2ac}{5} + \frac{b^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**3+a*x**2)**2/x**2,x)
```

```
[Out] a**2*x**3/3 + a*b*x**4/2 + b*c*x**6/3 + c**2*x**7/7 + x**5*(2*a*c/5 + b**2/
5)
```

3.11 $\int \frac{x^5}{ax^2+bx^3+cx^4}\,dx$

Optimal. Leaf size=89

$$\frac{(b^2-ac)\log(a+bx+cx^2)}{2c^3}+\frac{b(b^2-3ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}}-\frac{bx}{c^2}+\frac{x^2}{2c}$$

```
[Out] -b*x/c^2+1/2*x^2/c+1/2*(-a*c+b^2)*ln(c*x^2+b*x+a)/c^3+b*(-3*a*c+b^2)*arctan
h((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c^3/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1585, 701, 634, 618, 206, 628}

$$\frac{(b^2-ac)\log(a+bx+cx^2)}{2c^3}+\frac{b(b^2-3ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}}-\frac{bx}{c^2}+\frac{x^2}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a*x^2 + b*x^3 + c*x^4),x]
```

```
[Out] -((b*x)/c^2) + x^2/(2*c) + (b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 -
4*a*c]])/(c^3*Sqrt[b^2 - 4*a*c]) + ((b^2 - a*c)*Log[a + b*x + c*x^2])/(2*c^
3)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 701

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Int[PolynomialDivide[(d + e*x)^m, a + b*x + c*x^2, x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 NeQ[2*c*d - b*e, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5}{ax^2+bx^3+cx^4}\,dx &= \int \frac{x^3}{a+bx+cx^2}\,dx \\
&= \int \left(-\frac{b}{c^2}+\frac{x}{c}+\frac{ab+\left(b^2-ac\right)x}{c^2\left(a+bx+cx^2\right)}\right)dx \\
&= -\frac{bx}{c^2}+\frac{x^2}{2c}+\frac{\int \frac{ab+\left(b^2-ac\right)x}{a+bx+cx^2}\,dx}{c^2} \\
&= -\frac{bx}{c^2}+\frac{x^2}{2c}-\frac{\left(b\left(b^2-3ac\right)\right)\int \frac{1}{a+bx+cx^2}\,dx}{2c^3}+\frac{\left(b^2-ac\right)\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2c^3} \\
&= -\frac{bx}{c^2}+\frac{x^2}{2c}+\frac{\left(b^2-ac\right)\log\left(a+bx+cx^2\right)}{2c^3}+\frac{\left(b\left(b^2-3ac\right)\right)\operatorname{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b\ \cdots\right.}{c^3} \\
&= -\frac{bx}{c^2}+\frac{x^2}{2c}+\frac{b\left(b^2-3ac\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}}+\frac{\left(b^2-ac\right)\log\left(a+bx+cx^2\right)}{2c^3}
\end{aligned}
$$

Mathematica [A] time = 0.11, size = 84, normalized size = 0.94

$$
\frac{\left(b^2-ac\right)\log(a+x(b+cx))-\frac{2b\left(b^2-3ac\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+cx(cx-2b)}{2c^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a*x^2 + b*x^3 + c*x^4),x]
```

```
[Out] (c*x*(-2*b + c*x) - (2*b*(b^2 - 3*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c
]])/Sqrt[-b^2 + 4*a*c] + (b^2 - a*c)*Log[a + x*(b + c*x)])/(2*c^3)
```

fricas [A] time = 0.77, size = 297, normalized size = 3.34

$$
\left[\frac{\left(b^2c^2-4\,ac^3\right)x^2-\left(b^3-3\,abc\right)\sqrt{b^2-4\,ac}\,\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}\,(2\,cx+b)}{cx^2+bx+a}\right)-2\left(b^3c-4\,abc^2\right)x+\left(b^4-5\ \cdots\right.}{2\left(b^2c^3-4\,ac^4\right)}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^3+a*x^2),x, algorithm="fricas")
```

```
[Out] [1/2*((b^2*c^2 - 4*a*c^3)*x^2 - (b^3 - 3*a*b*c)*sqrt(b^2 - 4*a*c)*log((2*c^
2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x
 + a)) - 2*(b^3*c - 4*a*b*c^2)*x + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*log(c*x^2
+ b*x + a))/(b^2*c^3 - 4*a*c^4), 1/2*((b^2*c^2 - 4*a*c^3)*x^2 + 2*(b^3 - 3*
a*b*c)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a
*c)) - 2*(b^3*c - 4*a*b*c^2)*x + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*log(c*x^2 +
b*x + a))/(b^2*c^3 - 4*a*c^4)]
```

giac [A] time = 0.45, size = 86, normalized size = 0.97

$$\frac{cx^2 - 2\,bx}{2\,c^2} + \frac{\left(b^2 - ac\right)\log\left(cx^2 + bx + a\right)}{2\,c^3} - \frac{\left(b^3 - 3\,abc\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2 + 4\,ac}\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^3+a*x^2),x, algorithm="giac")
```

```
[Out] 1/2*(c*x^2 - 2*b*x)/c^2 + 1/2*(b^2 - a*c)*log(c*x^2 + b*x + a)/c^3 - (b^3 -
 3*a*b*c)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^3)
```

maple [A] time = 0.00, size = 132, normalized size = 1.48

$$\frac{3ab\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\;c^2} - \frac{b^3\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\;c^3} + \frac{x^2}{2c} - \frac{a\ln\left(c\,x^2+bx+a\right)}{2c^2} + \frac{b^2\ln\left(c\,x^2+bx+a\right)}{2c^3} - \frac{bx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(c*x^4+b*x^3+a*x^2),x)
```

```
[Out] 1/2/c*x^2-b/c^2*x-1/2/c^2*ln(c*x^2+b*x+a)*a+1/2/c^3*ln(c*x^2+b*x+a)*b^2+3/c
^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b-1/c^3/(4*a*c-b
^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^3
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^3+a*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 0.14, size = 112, normalized size = 1.26

$$\frac{x^2}{2\,c} - \frac{\ln\left(c\,x^2 + b\,x + a\right)\left(4\,a^2\,c^2 - 5\,a\,b^2\,c + b^4\right)}{2\left(4\,a\,c^4 - b^2\,c^3\right)} - \frac{b\,x}{c^2} + \frac{b\,\mathrm{atan}\left(\frac{b+2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)\left(3\,a\,c - b^2\right)}{c^3\,\sqrt{4\,a\,c - b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a*x^2 + b*x^3 + c*x^4),x)
```

```
[Out] x^2/(2*c) - (log(a + b*x + c*x^2)*(b^4 + 4*a^2*c^2 - 5*a*b^2*c))/(2*(4*a*c^
4 - b^2*c^3)) - (b*x)/c^2 + (b*atan((b + 2*c*x)/(4*a*c - b^2)^(1/2))*(3*a*c
 - b^2))/(c^3*(4*a*c - b^2)^(1/2))
```

sympy [B] time = 0.84, size = 381, normalized size = 4.28

$$-\frac{bx}{c^2} + \left(-\frac{b\sqrt{-4ac+b^2}\left(3ac-b^2\right)}{2c^3\left(4ac-b^2\right)} - \frac{ac-b^2}{2c^3}\right)\log\left(x + \frac{2a^2c - ab^2 + 4ac^3\left(-\frac{b\sqrt{-4ac+b^2}\left(3ac-b^2\right)}{2c^3\left(4ac-b^2\right)} - \frac{ac-b^2}{2c^3}\right) - b^2c^2\left(-\frac{b\sqrt{-}}{}\right.}{3abc - b^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(c*x**4+b*x**3+a*x**2),x)
```

```
[Out] -b*x/c**2 + (-b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(2*c**3*(4*a*c - b**2))
- (a*c - b**2)/(2*c**3))*log(x + (2*a**2*c - a*b**2 + 4*a*c**3*(-b*sqrt(-4*
a*c + b**2)*(3*a*c - b**2)/(2*c**3*(4*a*c - b**2)) - (a*c - b**2)/(2*c**3))
 - b**2*c**2*(-b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(2*c**3*(4*a*c - b**2))
 - (a*c - b**2)/(2*c**3)))/(3*a*b*c - b**3)) + (b*sqrt(-4*a*c + b**2)*(3*a*
c - b**2)/(2*c**3*(4*a*c - b**2)) - (a*c - b**2)/(2*c**3))*log(x + (2*a**2*
c - a*b**2 + 4*a*c**3*(b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(2*c**3*(4*a*c
- b**2)) - (a*c - b**2)/(2*c**3)) - b**2*c**2*(b*sqrt(-4*a*c + b**2)*(3*a*c
 - b**2)/(2*c**3*(4*a*c - b**2)) - (a*c - b**2)/(2*c**3)))/(3*a*b*c - b**3)
) + x**2/(2*c)
```

3.12 $\int \frac{x^4}{ax^2+bx^3+cx^4} dx$

Optimal. Leaf size=70

$$-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx+cx^2\right)}{2c^2}+\frac{x}{c}$$

```
[Out] x/c-1/2*b*ln(c*x^2+b*x+a)/c^2-(-2*a*c+b^2)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(
1/2))/c^2/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1585, 703, 634, 618, 206, 628}

$$-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx+cx^2\right)}{2c^2}+\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a*x^2 + b*x^3 + c*x^4),x]
```

```
[Out] x/c - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^2*Sqrt[b^2
- 4*a*c]) - (b*Log[a + b*x + c*x^2])/(2*c^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 703

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^
(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{ax^2+bx^3+cx^4}\,dx &= \int \frac{x^2}{a+bx+cx^2}\,dx\\ &= \frac{x}{c}+\frac{\int \frac{-a-bx}{a+bx+cx^2}\,dx}{c}\\ &= \frac{x}{c}-\frac{b\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2c^2}+\frac{\left(b^2-2ac\right)\int \frac{1}{a+bx+cx^2}\,dx}{2c^2}\\ &= \frac{x}{c}-\frac{b\log\left(a+bx+cx^2\right)}{2c^2}-\frac{\left(b^2-2ac\right)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx\right)}{c^2}\\ &= \frac{x}{c}-\frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx+cx^2\right)}{2c^2}\end{aligned}$$

Mathematica [A] time = 0.06, size = 73, normalized size = 1.04

$$\frac{\left(b^2-2ac\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{c^2\sqrt{4ac-b^2}}-\frac{b\log\left(a+bx+cx^2\right)}{2c^2}+\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a*x^2 + b*x^3 + c*x^4),x]
```

```
[Out] x/c + ((b^2 - 2*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(c^2*Sqrt[-b^2
 + 4*a*c]) - (b*Log[a + b*x + c*x^2])/(2*c^2)
```

fricas [A] time = 0.88, size = 235, normalized size = 3.36

$$\left[-\frac{\left(b^2-2\,ac\right)\sqrt{b^2-4\,ac}\,\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}\,(2\,cx+b)}{cx^2+bx+a}\right)-2\left(b^2c-4\,ac^2\right)x+\left(b^3-4\,abc\right)\log\left(cx^2+bx\right.}{2\left(b^2c^2-4\,ac^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^3+a*x^2),x, algorithm="fricas")
```

```
[Out] [-1/2*((b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a
*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - 2*(b^2*c - 4*a*c^2
)*x + (b^3 - 4*a*b*c)*log(c*x^2 + b*x + a))/(b^2*c^2 - 4*a*c^3), -1/2*(2*(b
^2 - 2*a*c)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2
- 4*a*c)) - 2*(b^2*c - 4*a*c^2)*x + (b^3 - 4*a*b*c)*log(c*x^2 + b*x + a))/(
b^2*c^2 - 4*a*c^3)]
```

giac [A] time = 0.55, size = 67, normalized size = 0.96

$$\frac{x}{c}-\frac{b\log\left(cx^2+bx+a\right)}{2\,c^2}+\frac{\left(b^2-2\,ac\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^3+a*x^2),x, algorithm="giac")
```

```
[Out] x/c - 1/2*b*log(c*x^2 + b*x + a)/c^2 + (b^2 - 2*a*c)*arctan((2*c*x + b)/sqr
t(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^2)
```

maple [A] time = 0.00, size = 101, normalized size = 1.44

$$-\frac{2a\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\,c}+\frac{b^2\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\,c^2}-\frac{b\ln\left(cx^2+bx+a\right)}{2c^2}+\frac{x}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^4+b*x^3+a*x^2),x)
```

```
[Out] 1/c*x-1/2*b/c^2*ln(c*x^2+b*x+a)-2/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a
*c-b^2)^(1/2))*a+1/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)
)*b^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^3+a*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 2.03, size = 172, normalized size = 2.46

$$\frac{x}{c}+\frac{b^3\ln\left(c\,x^2+b\,x+a\right)}{2\,\left(4\,a\,c^3-b^2\,c^2\right)}-\frac{2\,a\,\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)}{c\,\sqrt{4\,a\,c-b^2}}+\frac{b^2\,\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)}{c^2\,\sqrt{4\,a\,c-b^2}}-\frac{2\,a\,b\,c\,\ln\left(c\,x^2+b\,x+a\right)}{4\,a\,c^3-b^2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a*x^2 + b*x^3 + c*x^4),x)
```

```
[Out] x/c + (b^3*log(a + b*x + c*x^2))/(2*(4*a*c^3 - b^2*c^2)) - (2*a*atan(b/(4*a
*c - b^2)^(1/2) + (2*c*x)/(4*a*c - b^2)^(1/2)))/(c*(4*a*c - b^2)^(1/2)) + (
b^2*atan(b/(4*a*c - b^2)^(1/2) + (2*c*x)/(4*a*c - b^2)^(1/2)))/(c^2*(4*a*c
- b^2)^(1/2)) - (2*a*b*c*log(a + b*x + c*x^2))/(4*a*c^3 - b^2*c^2)
```

sympy [B] time = 0.62, size = 306, normalized size = 4.37

$$\left(-\frac{b}{2c^2}-\frac{\sqrt{-4ac+b^2}\,\left(2ac-b^2\right)}{2c^2\left(4ac-b^2\right)}\right)\log\left(x+\frac{-ab-4ac^2\left(-\frac{b}{2c^2}-\frac{\sqrt{-4ac+b^2}\left(2ac-b^2\right)}{2c^2\left(4ac-b^2\right)}\right)+b^2c\left(-\frac{b}{2c^2}-\frac{\sqrt{-4ac+b^2}\left(2ac-b^2\right)}{2c^2\left(4ac-b^2\right)}\right)}{2ac-b^2}\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**4+b*x**3+a*x**2),x)
```

```
[Out] (-b/(2*c**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*c**2*(4*a*c - b**2)))*
log(x + (-a*b - 4*a*c**2*(-b/(2*c**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/
(2*c**2*(4*a*c - b**2))) + b**2*c*(-b/(2*c**2) - sqrt(-4*a*c + b**2)*(2*a*c
```

```
 - b**2)/(2*c**2*(4*a*c - b**2))))/(2*a*c - b**2)) + (-b/(2*c**2) + sqrt(-4
*a*c + b**2)*(2*a*c - b**2)/(2*c**2*(4*a*c - b**2)))*log(x + (-a*b - 4*a*c*
*2*(-b/(2*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*c**2*(4*a*c - b**2)
)) + b**2*c*(-b/(2*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*c**2*(4*a*
c - b**2))))/(2*a*c - b**2)) + x/c
```

3.13 $\int \frac{x^3}{ax^2+bx^3+cx^4}\,dx$

Optimal. Leaf size=56

$$\frac{b\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}}+\frac{\log\left(a+bx+cx^2\right)}{2c}$$

```
[Out] 1/2*ln(c*x^2+b*x+a)/c+b*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c/(-4*a*c+b^2
)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1585, 634, 618, 206, 628}

$$\frac{b\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}}+\frac{\log\left(a+bx+cx^2\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a*x^2 + b*x^3 + c*x^4),x]
```

```
[Out] (b*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c*Sqrt[b^2 - 4*a*c]) + Log[a +
b*x + c*x^2]/(2*c)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\int \frac{x^3}{ax^2+bx^3+cx^4}\,dx = \int \frac{x}{a+bx+cx^2}\,dx$$
$$= \frac{\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2c} - \frac{b\int \frac{1}{a+bx+cx^2}\,dx}{2c}$$
$$= \frac{\log\left(a+bx+cx^2\right)}{2c} + \frac{b\,\mathrm{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx\right)}{c}$$
$$= \frac{b\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}} + \frac{\log\left(a+bx+cx^2\right)}{2c}$$

Mathematica [A] time = 0.03, size = 57, normalized size = 1.02

$$\frac{\log(a+x(b+cx)) - \frac{2b\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a*x^2 + b*x^3 + c*x^4),x]
```

```
[Out] ((-2*b*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + Log[a +
 x*(b + c*x)])/(2*c)
```

fricas [A] time = 0.93, size = 185, normalized size = 3.30

$$\left[\frac{\sqrt{b^2-4\,ac}\,b\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}\,(2\,cx+b)}{cx^2+bx+a}\right) + \left(b^2-4\,ac\right)\log\left(cx^2+bx+a\right)}{2\left(b^2c-4\,ac^2\right)}, \frac{2\sqrt{-b^2+4\,ac}\,b\arctan\left(\cdot\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^3+a*x^2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2
 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + (b^2 - 4*a*c)*log(c*x^2 + b*x +
 a))/(b^2*c - 4*a*c^2), 1/2*(2*sqrt(-b^2 + 4*a*c)*b*arctan(-sqrt(-b^2 + 4*a
*c)*(2*c*x + b)/(b^2 - 4*a*c)) + (b^2 - 4*a*c)*log(c*x^2 + b*x + a))/(b^2*c
 - 4*a*c^2)]
```

giac [A] time = 0.42, size = 55, normalized size = 0.98

$$-\frac{b\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}\,c} + \frac{\log\left(cx^2+bx+a\right)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^3+a*x^2),x, algorithm="giac")
```

```
[Out] -b*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c) + 1/2*log(
c*x^2 + b*x + a)/c
```

maple [A] time = 0.00, size = 56, normalized size = 1.00

$$-\frac{b\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\,c} + \frac{\ln\left(c\,x^2+bx+a\right)}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^4+b*x^3+a*x^2),x)
```

```
[Out] 1/2/c*ln(c*x^2+b*x+a)-b/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1
/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^3+a*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 0.13, size = 112, normalized size = 2.00

$$\frac{2\,a\,c\,\ln\left(c\,x^2+b\,x+a\right)}{4\,a\,c^2-b^2\,c}-\frac{b\,\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)}{c\,\sqrt{4\,a\,c-b^2}}-\frac{b^2\,\ln\left(c\,x^2+b\,x+a\right)}{2\,\left(4\,a\,c^2-b^2\,c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*x^2 + b*x^3 + c*x^4),x)
```

```
[Out] (2*a*c*log(a + b*x + c*x^2))/(4*a*c^2 - b^2*c) - (b*atan(b/(4*a*c - b^2)^(1
/2) + (2*c*x)/(4*a*c - b^2)^(1/2)))/(c*(4*a*c - b^2)^(1/2)) - (b^2*log(a +
b*x + c*x^2))/(2*(4*a*c^2 - b^2*c))
```

sympy [B] time = 0.33, size = 216, normalized size = 3.86

$$\left(-\frac{b\sqrt{-4ac+b^2}}{2c\left(4ac-b^2\right)}+\frac{1}{2c}\right)\log\left(x+\frac{-4ac\left(-\frac{b\sqrt{-4ac+b^2}}{2c\left(4ac-b^2\right)}+\frac{1}{2c}\right)+2a+b^2\left(-\frac{b\sqrt{-4ac+b^2}}{2c\left(4ac-b^2\right)}+\frac{1}{2c}\right)}{b}\right)+\left(\frac{b\sqrt{-4ac+b^2}}{2c\left(4ac-b^2\right)}+\frac{1}{2c}\right)\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**4+b*x**3+a*x**2),x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)/(2*c*(4*a*c - b**2)) + 1/(2*c))*log(x + (-4*a*c*(-b
*sqrt(-4*a*c + b**2)/(2*c*(4*a*c - b**2)) + 1/(2*c)) + 2*a + b**2*(-b*sqrt(
-4*a*c + b**2)/(2*c*(4*a*c - b**2)) + 1/(2*c)))/b) + (b*sqrt(-4*a*c + b**2)
/(2*c*(4*a*c - b**2)) + 1/(2*c))*log(x + (-4*a*c*(b*sqrt(-4*a*c + b**2)/(2*
c*(4*a*c - b**2)) + 1/(2*c)) + 2*a + b**2*(b*sqrt(-4*a*c + b**2)/(2*c*(4*a*
c - b**2)) + 1/(2*c)))/b)
```

3.14 $\int \frac{x^2}{ax^2+bx^3+cx^4} \, dx$

Optimal. Leaf size=34

$$-\frac{2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

```
[Out] -2*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1585, 618, 206}

$$-\frac{2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a*x^2 + b*x^3 + c*x^4),x]
```

```
[Out] (-2*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 - 4*a*c]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{ax^2+bx^3+cx^4} \, dx &= \int \frac{1}{a+bx+cx^2} \, dx \\
&= -\left(2\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2} \, dx, x, b+2cx\right)\right) \\
&= -\frac{2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 38, normalized size = 1.12

$$\frac{2\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a*x^2 + b*x^3 + c*x^4),x]
```

```
[Out] (2*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c]
```

fricas [A] time = 0.75, size = 120, normalized size = 3.53

$$\left[\frac{\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}\,(2\,cx+b)}{cx^2+bx+a}\right)}{\sqrt{b^2-4\,ac}}, -\frac{2\,\sqrt{-b^2+4\,ac}\,\arctan\left(-\frac{\sqrt{-b^2+4\,ac}\,(2\,cx+b)}{b^2-4\,ac}\right)}{b^2-4\,ac}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^3+a*x^2),x, algorithm="fricas")
```

```
[Out] [log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c
*x^2 + b*x + a))/sqrt(b^2 - 4*a*c), -2*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2
 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c))/(b^2 - 4*a*c)]
```

giac [A] time = 0.50, size = 34, normalized size = 1.00

$$\frac{2\,\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^3+a*x^2),x, algorithm="giac")
```

```
[Out] 2*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/sqrt(-b^2 + 4*a*c)
```

maple [A] time = 0.00, size = 35, normalized size = 1.03

$$\frac{2\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^4+b*x^3+a*x^2),x)
```

```
[Out] 2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^3+a*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 0.03, size = 46, normalized size = 1.35

$$\frac{2\,\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)}{\sqrt{4\,a\,c-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x^2 + b*x^3 + c*x^4),x)
```

```
[Out] (2*atan(b/(4*a*c - b^2)^(1/2) + (2*c*x)/(4*a*c - b^2)^(1/2)))/(4*a*c - b^2)
^(1/2)
```

sympy [B] time = 0.22, size = 124, normalized size = 3.65

$$-\sqrt{-\frac{1}{4ac-b^2}}\log\left(x+\frac{-4ac\sqrt{-\frac{1}{4ac-b^2}}+b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2c}\right)+\sqrt{-\frac{1}{4ac-b^2}}\log\left(x+\frac{4ac\sqrt{-\frac{1}{4ac-b^2}}-b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**4+b*x**3+a*x**2),x)
```

```
[Out] -sqrt(-1/(4*a*c - b**2))*log(x + (-4*a*c*sqrt(-1/(4*a*c - b**2)) + b**2*sqr
t(-1/(4*a*c - b**2)) + b)/(2*c)) + sqrt(-1/(4*a*c - b**2))*log(x + (4*a*c*s
qrt(-1/(4*a*c - b**2)) - b**2*sqrt(-1/(4*a*c - b**2)) + b)/(2*c))
```

3.15 $\int \frac{x}{ax^2+bx^3+cx^4}\,dx$

Optimal. Leaf size=62

$$\frac{b\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}}-\frac{\log\left(a+bx+cx^2\right)}{2a}+\frac{\log(x)}{a}$$

```
[Out] ln(x)/a-1/2*ln(c*x^2+b*x+a)/a+b*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/a/(-4
*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1585, 705, 29, 634, 618, 206, 628}

$$\frac{b\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}}-\frac{\log\left(a+bx+cx^2\right)}{2a}+\frac{\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a*x^2 + b*x^3 + c*x^4),x]
```

```
[Out] (b*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a*Sqrt[b^2 - 4*a*c]) + Log[x]/a
 - Log[a + b*x + c*x^2]/(2*a)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
```

```
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x}{ax^2+bx^3+cx^4}\,dx &= \int \frac{1}{x\left(a+bx+cx^2\right)}\,dx \\
&= \frac{\int \frac{1}{x}\,dx}{a}+\frac{\int \frac{-b-cx}{a+bx+cx^2}\,dx}{a} \\
&= \frac{\log(x)}{a}-\frac{\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2a}-\frac{b\int \frac{1}{a+bx+cx^2}\,dx}{2a} \\
&= \frac{\log(x)}{a}-\frac{\log\left(a+bx+cx^2\right)}{2a}+\frac{b\,\mathrm{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx\right)}{a} \\
&= \frac{b\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}}+\frac{\log(x)}{a}-\frac{\log\left(a+bx+cx^2\right)}{2a}
\end{aligned}
$$

Mathematica [A] time = 0.07, size = 61, normalized size = 0.98

$$-\frac{\frac{2b\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+\log(a+x(b+cx))-2\log(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a*x^2 + b*x^3 + c*x^4),x]
```

```
[Out] -1/2*((2*b*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] - 2*L
og[x] + Log[a + x*(b + c*x)])/a
```

fricas [A] time = 0.95, size = 211, normalized size = 3.40

$$\left[\frac{\sqrt{b^2-4\,ac}\,b\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}\,(2\,cx+b)}{cx^2+bx+a}\right)-\left(b^2-4\,ac\right)\log\left(cx^2+bx+a\right)+2\left(b^2-4\,ac\right)\log(x)}{2\left(ab^2-4\,a^2c\right)},\ 2\,\sqrt{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^3+a*x^2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2
 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - (b^2 - 4*a*c)*log(c*x^2 + b*x +
 a) + 2*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2*c), 1/2*(2*sqrt(-b^2 + 4*a*c)*
b*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - (b^2 - 4*a*c)*log
(c*x^2 + b*x + a) + 2*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2*c)]
```

giac [A] time = 0.50, size = 62, normalized size = 1.00

$$-\frac{b\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}\,a}-\frac{\log\left(cx^2+bx+a\right)}{2\,a}+\frac{\log\left(|x|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^3+a*x^2),x, algorithm="giac")
```

```
[Out] -b*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a) - 1/2*log(
c*x^2 + b*x + a)/a + log(abs(x))/a
```

maple [A] time = 0.01, size = 62, normalized size = 1.00

$$-\frac{b \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\, a}+\frac{\ln(x)}{a}-\frac{\ln\left(c x^2+bx+a\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^4+b*x^3+a*x^2),x)
```

```
[Out] 1/a*ln(x)-1/2/a*ln(c*x^2+b*x+a)-1/a*b/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4
*a*c-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^3+a*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 2.30, size = 213, normalized size = 3.44

$$\frac{\ln(x)}{a}-\ln\left(b\,c-\left(x\,\left(6\,a\,c^2-2\,b^2\,c\right)-a\,b\,c\right)\,\left(\frac{1}{2\,a}-\frac{b\,\sqrt{b^2-4\,a\,c}}{2\,\left(a\,b^2-4\,a^2\,c\right)}\right)+3\,c^2\,x\right)\,\left(\frac{1}{2\,a}-\frac{b\,\sqrt{b^2-4\,a\,c}}{2\,\left(a\,b^2-4\,a^2\,c\right)}\right)-\ln\left(\left(x\,\left(6\,a\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x^2 + b*x^3 + c*x^4),x)
```

```
[Out] log(x)/a - log(b*c - (x*(6*a*c^2 - 2*b^2*c) - a*b*c)*(1/(2*a) - (b*(b^2 - 4
*a*c)^(1/2))/(2*(a*b^2 - 4*a^2*c))) + 3*c^2*x)*(1/(2*a) - (b*(b^2 - 4*a*c)^
(1/2))/(2*(a*b^2 - 4*a^2*c))) - log((x*(6*a*c^2 - 2*b^2*c) - a*b*c)*(1/(2*a
) + (b*(b^2 - 4*a*c)^(1/2))/(2*(a*b^2 - 4*a^2*c))) - b*c - 3*c^2*x)*(1/(2*a
) + (b*(b^2 - 4*a*c)^(1/2))/(2*(a*b^2 - 4*a^2*c)))
```

sympy [B] time = 4.36, size = 564, normalized size = 9.10

$$\left(-\frac{b\sqrt{-4ac+b^2}}{2a\left(4ac-b^2\right)}-\frac{1}{2a}\right)\log\left(x+\frac{24a^4c^2\left(-\frac{b\sqrt{-4ac+b^2}}{2a\left(4ac-b^2\right)}-\frac{1}{2a}\right)^2-14a^3b^2c\left(-\frac{b\sqrt{-4ac+b^2}}{2a\left(4ac-b^2\right)}-\frac{1}{2a}\right)^2-12a^3c^2\left(-\frac{b\sqrt{-4ac+b^2}}{2a\left(4ac-b^2\right)}-\right.}{9ab\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**4+b*x**3+a*x**2),x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a))*log(x + (24*a**4*c*
*2*(-b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a))**2 - 14*a**3*b**
2*c*(-b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a))**2 - 12*a**3*c*
```

```
*2*(-b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a)) + 2*a**2*b**4*(-
b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a))**2 + 3*a**2*b**2*c*(-
b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a)) - 12*a**2*c**2 + 11*a
*b**2*c - 2*b**4)/(9*a*b*c**2 - 2*b**3*c)) + (b*sqrt(-4*a*c + b**2)/(2*a*(4
*a*c - b**2)) - 1/(2*a))*log(x + (24*a**4*c**2*(b*sqrt(-4*a*c + b**2)/(2*a*
(4*a*c - b**2)) - 1/(2*a))**2 - 14*a**3*b**2*c*(b*sqrt(-4*a*c + b**2)/(2*a*
(4*a*c - b**2)) - 1/(2*a))**2 - 12*a**3*c**2*(b*sqrt(-4*a*c + b**2)/(2*a*(4
*a*c - b**2)) - 1/(2*a)) + 2*a**2*b**4*(b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c -
 b**2)) - 1/(2*a))**2 + 3*a**2*b**2*c*(b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c -
b**2)) - 1/(2*a)) - 12*a**2*c**2 + 11*a*b**2*c - 2*b**4)/(9*a*b*c**2 - 2*b*
*3*c)) + log(x)/a
```

3.16 $\int \frac{1}{ax^2+bx^3+cx^4}\,dx$

Optimal. Leaf size=81

$$-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}}+\frac{b\log\left(a+bx+cx^2\right)}{2a^2}-\frac{b\log(x)}{a^2}-\frac{1}{ax}$$

```
[Out] -1/a/x-b*ln(x)/a^2+1/2*b*ln(c*x^2+b*x+a)/a^2-(-2*a*c+b^2)*arctanh((2*c*x+b)
/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1594, 709, 800, 634, 618, 206, 628}

$$-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}}+\frac{b\log\left(a+bx+cx^2\right)}{2a^2}-\frac{b\log(x)}{a^2}-\frac{1}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3 + c*x^4)^(-1),x]
```

```
[Out] -(1/(a*x)) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^2*Sq
rt[b^2 - 4*a*c]) - (b*Log[x])/a^2 + (b*Log[a + b*x + c*x^2])/(2*a^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{ax^2+bx^3+cx^4}\,dx &= \int \frac{1}{x^2\left(a+bx+cx^2\right)}\,dx \\
&= -\frac{1}{ax}+\frac{\int \frac{-b-cx}{x\left(a+bx+cx^2\right)}\,dx}{a} \\
&= -\frac{1}{ax}+\frac{\int \left(-\frac{b}{ax}+\frac{b^2-ac+bcx}{a\left(a+bx+cx^2\right)}\right)dx}{a} \\
&= -\frac{1}{ax}-\frac{b\log(x)}{a^2}+\frac{\int \frac{b^2-ac+bcx}{a+bx+cx^2}\,dx}{a^2} \\
&= -\frac{1}{ax}-\frac{b\log(x)}{a^2}+\frac{b\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2a^2}+\frac{\left(b^2-2ac\right)\int \frac{1}{a+bx+cx^2}\,dx}{2a^2} \\
&= -\frac{1}{ax}-\frac{b\log(x)}{a^2}+\frac{b\log\left(a+bx+cx^2\right)}{2a^2}-\frac{\left(b^2-2ac\right)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx\right)}{a^2} \\
&= -\frac{1}{ax}-\frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}}-\frac{b\log(x)}{a^2}+\frac{b\log\left(a+bx+cx^2\right)}{2a^2}
\end{aligned}
$$

Mathematica [A] time = 0.08, size = 77, normalized size = 0.95

$$
\frac{\frac{2\left(b^2-2ac\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+b\log(a+x(b+cx))-\frac{2a}{x}-2b\log(x)}{2a^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3 + c*x^4)^(-1),x]
```

```
[Out] ((-2*a)/x + (2*(b^2 - 2*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-
b^2 + 4*a*c] - 2*b*Log[x] + b*Log[a + x*(b + c*x)])/(2*a^2)
```

fricas [A] time = 0.92, size = 269, normalized size = 3.32

$$
\left[-\frac{\left(b^2-2\,ac\right)\sqrt{b^2-4\,ac}\,x\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}\,(2\,cx+b)}{cx^2+bx+a}\right)+2\,ab^2-8\,a^2c-\left(b^3-4\,abc\right)x\log\left(cx^2+bx+\right.}{2\left(a^2b^2-4\,a^3c\right)x}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^3+a*x^2),x, algorithm="fricas")
```

```
[Out] [-1/2*((b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*x*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2
*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 2*a*b^2 - 8*a^2*
c - (b^3 - 4*a*b*c)*x*log(c*x^2 + b*x + a) + 2*(b^3 - 4*a*b*c)*x*log(x))/((
a^2*b^2 - 4*a^3*c)*x), -1/2*(2*(b^2 - 2*a*c)*sqrt(-b^2 + 4*a*c)*x*arctan(-s
qrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*a*b^2 - 8*a^2*c - (b^3 - 4
*a*b*c)*x*log(c*x^2 + b*x + a) + 2*(b^3 - 4*a*b*c)*x*log(x))/((a^2*b^2 - 4*
a^3*c)*x)]
```

giac [A] time = 0.33, size = 79, normalized size = 0.98

$$\frac{b\log\left(cx^2+bx+a\right)}{2\,a^2}-\frac{b\log\left(|x|\right)}{a^2}+\frac{\left(b^2-2\,ac\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}\,a^2}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^3+a*x^2),x, algorithm="giac")
```

```
[Out] 1/2*b*log(c*x^2 + b*x + a)/a^2 - b*log(abs(x))/a^2 + (b^2 - 2*a*c)*arctan((
2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a^2) - 1/(a*x)
```

maple [A] time = 0.01, size = 112, normalized size = 1.38

$$-\frac{2c\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\;a}+\frac{b^2\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\;a^2}-\frac{b\ln(x)}{a^2}+\frac{b\ln\left(c\,x^2+bx+a\right)}{2a^2}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+b*x^3+a*x^2),x)
```

```
[Out] -1/a/x-1/a^2*b*ln(x)+1/2/a^2*b*ln(c*x^2+b*x+a)-2/a/(4*a*c-b^2)^(1/2)*arctan
((2*c*x+b)/(4*a*c-b^2)^(1/2))*c+1/a^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4
*a*c-b^2)^(1/2))*b^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^3+a*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 2.50, size = 339, normalized size = 4.19

$$\frac{\ln\left(2\,a\,b^3+2\,b^4\,x-2\,a\,b^2\,\sqrt{b^2-4\,a\,c}+a^2\,c\,\sqrt{b^2-4\,a\,c}-2\,b^3\,x\,\sqrt{b^2-4\,a\,c}+2\,a^2\,c^2\,x-7\,a^2\,b\,c-8\,a\,b^2\,c\,x+4\,a\right.}{4\,a^3\,c-a^2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^2 + b*x^3 + c*x^4),x)
```

```
[Out] (log(2*a*b^3 + 2*b^4*x - 2*a*b^2*(b^2 - 4*a*c)^(1/2) + a^2*c*(b^2 - 4*a*c)^
(1/2) - 2*b^3*x*(b^2 - 4*a*c)^(1/2) + 2*a^2*c^2*x - 7*a^2*b*c - 8*a*b^2*c*x
 + 4*a*b*c*x*(b^2 - 4*a*c)^(1/2))*(a*(2*b*c - c*(b^2 - 4*a*c)^(1/2)) - b^3/
```

```
2 + (b^2*(b^2 - 4*a*c)^(1/2))/2))/(4*a^3*c - a^2*b^2) - 1/(a*x) - (log(2*a*
b^3 + 2*b^4*x + 2*a*b^2*(b^2 - 4*a*c)^(1/2) - a^2*c*(b^2 - 4*a*c)^(1/2) + 2
*b^3*x*(b^2 - 4*a*c)^(1/2) + 2*a^2*c^2*x - 7*a^2*b*c - 8*a*b^2*c*x - 4*a*b*
c*x*(b^2 - 4*a*c)^(1/2))*(b^3/2 - a*(2*b*c + c*(b^2 - 4*a*c)^(1/2)) + (b^2*
(b^2 - 4*a*c)^(1/2))/2))/(4*a^3*c - a^2*b^2) - (b*log(x))/a^2
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+b*x**3+a*x**2),x)
[Out] Timed out
```

3.17 $\int \frac{1}{x(ax^2+bx^3+cx^4)}\,dx$

Optimal. Leaf size=104

$$-\frac{(b^2-ac)\log(a+bx+cx^2)}{2a^3}+\frac{\log(x)(b^2-ac)}{a^3}+\frac{b(b^2-3ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3\sqrt{b^2-4ac}}+\frac{b}{a^2x}-\frac{1}{2ax^2}$$

```
[Out] -1/2/a/x^2+b/a^2/x+(-a*c+b^2)*ln(x)/a^3-1/2*(-a*c+b^2)*ln(c*x^2+b*x+a)/a^3+
b*(-3*a*c+b^2)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/a^3/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {1585, 709, 800, 634, 618, 206, 628}

$$-\frac{(b^2-ac)\log(a+bx+cx^2)}{2a^3}+\frac{\log(x)(b^2-ac)}{a^3}+\frac{b(b^2-3ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3\sqrt{b^2-4ac}}+\frac{b}{a^2x}-\frac{1}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a*x^2 + b*x^3 + c*x^4)),x]
```

```
[Out] -1/(2*a*x^2) + b/(a^2*x) + (b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 -
4*a*c]])/(a^3*Sqrt[b^2 - 4*a*c]) + ((b^2 - a*c)*Log[x])/a^3 - ((b^2 - a*c)*
Log[a + b*x + c*x^2])/(2*a^3)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
```

```
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(ax^2+bx^3+cx^4\right)}\,dx &= \int \frac{1}{x^3\left(a+bx+cx^2\right)}\,dx\\
&= -\frac{1}{2ax^2}+\frac{\int \frac{-b-cx}{x^2\left(a+bx+cx^2\right)}\,dx}{a}\\
&= -\frac{1}{2ax^2}+\frac{\int \left(-\frac{b}{ax^2}+\frac{b^2-ac}{a^2x}+\frac{-b\left(b^2-2ac\right)-c\left(b^2-ac\right)x}{a^2\left(a+bx+cx^2\right)}\right)dx}{a}\\
&= -\frac{1}{2ax^2}+\frac{b}{a^2x}+\frac{\left(b^2-ac\right)\log(x)}{a^3}+\frac{\int \frac{-b\left(b^2-2ac\right)-c\left(b^2-ac\right)x}{a+bx+cx^2}\,dx}{a^3}\\
&= -\frac{1}{2ax^2}+\frac{b}{a^2x}+\frac{\left(b^2-ac\right)\log(x)}{a^3}-\frac{\left(b\left(b^2-3ac\right)\right)\int \frac{1}{a+bx+cx^2}\,dx}{2a^3}-\frac{\left(b^2-ac\right)\int \frac{b+}{a+bx}}{2a^3}\\
&= -\frac{1}{2ax^2}+\frac{b}{a^2x}+\frac{\left(b^2-ac\right)\log(x)}{a^3}-\frac{\left(b^2-ac\right)\log\left(a+bx+cx^2\right)}{2a^3}+\frac{\left(b\left(b^2-3ac\right)\right)S}{}\\
&= -\frac{1}{2ax^2}+\frac{b}{a^2x}+\frac{b\left(b^2-3ac\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3\sqrt{b^2-4ac}}+\frac{\left(b^2-ac\right)\log(x)}{a^3}-\frac{\left(b^2-ac\right)\log}{2}
\end{aligned}$$

Mathematica [A] time = 0.14, size = 102, normalized size = 0.98

$$\frac{-\frac{a^2}{x^2}+2\log(x)\left(b^2-ac\right)+\left(ac-b^2\right)\log(a+x(b+cx))-\frac{2b\left(b^2-3ac\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+\frac{2ab}{x}}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a*x^2 + b*x^3 + c*x^4)),x]
```

```
[Out] (-(a^2/x^2) + (2*a*b)/x - (2*b*(b^2 - 3*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 +
 4*a*c]])/Sqrt[-b^2 + 4*a*c] + 2*(b^2 - a*c)*Log[x] + (-b^2 + a*c)*Log[a +
x*(b + c*x)])/(2*a^3)
```

fricas [A] time = 0.98, size = 358, normalized size = 3.44

$$\left[-\frac{\left(b^3-3\,abc\right)\sqrt{b^2-4\,ac}\,x^2\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}\,(2\,cx+b)}{cx^2+bx+a}\right)+a^2b^2-4\,a^3c+\left(b^4-5\,ab^2c+4\,a^2c^2\right)x^2\log}{2\left(a^3b^2-4\,a^4c\right)x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^3+a*x^2),x, algorithm="fricas")
```

```
[Out] [-1/2*((b^3 - 3*a*b*c)*sqrt(b^2 - 4*a*c)*x^2*log((2*c^2*x^2 + 2*b*c*x + b^2
 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + a^2*b^2 - 4*
a^3*c + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*x^2*log(c*x^2 + b*x + a) - 2*(b^4 - 5
*a*b^2*c + 4*a^2*c^2)*x^2*log(x) - 2*(a*b^3 - 4*a^2*b*c)*x)/((a^3*b^2 - 4*a
^4*c)*x^2), 1/2*(2*(b^3 - 3*a*b*c)*sqrt(-b^2 + 4*a*c)*x^2*arctan(-sqrt(-b^2
 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - a^2*b^2 + 4*a^3*c - (b^4 - 5*a*b^2*c
 + 4*a^2*c^2)*x^2*log(c*x^2 + b*x + a) + 2*(b^4 - 5*a*b^2*c + 4*a^2*c^2)*x^
2*log(x) + 2*(a*b^3 - 4*a^2*b*c)*x)/((a^3*b^2 - 4*a^4*c)*x^2)]
```

giac [A] time = 0.51, size = 105, normalized size = 1.01

$$-\frac{\left(b^2-ac\right)\log\left(cx^2+bx+a\right)}{2\,a^3}+\frac{\left(b^2-ac\right)\log\left(|x|\right)}{a^3}-\frac{\left(b^3-3\,abc\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}\,a^3}+\frac{2\,abx-a^2}{2\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^3+a*x^2),x, algorithm="giac")
```

```
[Out] -1/2*(b^2 - a*c)*log(c*x^2 + b*x + a)/a^3 + (b^2 - a*c)*log(abs(x))/a^3 - (
b^3 - 3*a*b*c)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a
^3) + 1/2*(2*a*b*x - a^2)/(a^3*x^2)
```

maple [A] time = 0.01, size = 150, normalized size = 1.44

$$\frac{3bc\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\;a^2}-\frac{b^3\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\;a^3}-\frac{c\ln(x)}{a^2}+\frac{c\ln\left(c\,x^2+bx+a\right)}{2a^2}+\frac{b^2\ln(x)}{a^3}-\frac{b^2\ln\left(c\,x^2+bx+a\right)}{2a^3}+\frac{b}{a^2x}-2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^4+b*x^3+a*x^2),x)
```

```
[Out] -1/2/a/x^2-1/a^2*ln(x)*c+1/a^3*ln(x)*b^2+1/a^2*b/x+1/2/a^2*c*ln(c*x^2+b*x+a
)-1/2/a^3*ln(c*x^2+b*x+a)*b^2+3/a^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a
*c-b^2)^(1/2))*b*c-1/a^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/
2))*b^3
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^3+a*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 0.59, size = 447, normalized size = 4.30

$$\frac{\ln\left(2\,a\,b^4+2\,b^5\,x+6\,a^3\,c^2+2\,a\,b^3\,\sqrt{b^2-4\,a\,c}+2\,b^4\,x\,\sqrt{b^2-4\,a\,c}-9\,a^2\,b^2\,c-10\,a\,b^3\,c\,x-3\,a^2\,b\,c\,\sqrt{b^2-4\,a\,c}+\right.}{4\,a^4\,c-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a*x^2 + b*x^3 + c*x^4)),x)
```

```
[Out] (log(2*a*b^4 + 2*b^5*x + 6*a^3*c^2 + 2*a*b^3*(b^2 - 4*a*c)^(1/2) + 2*b^4*x*
(b^2 - 4*a*c)^(1/2) - 9*a^2*b^2*c - 10*a*b^3*c*x - 3*a^2*b*c*(b^2 - 4*a*c)^
(1/2) + 9*a^2*b*c^2*x + 3*a^2*c^2*x*(b^2 - 4*a*c)^(1/2) - 6*a*b^2*c*x*(b^2
- 4*a*c)^(1/2))*(b^4/2 - a*((5*b^2*c)/2 + (3*b*c*(b^2 - 4*a*c)^(1/2))/2) +
(b^3*(b^2 - 4*a*c)^(1/2))/2 + 2*a^2*c^2))/(4*a^4*c - a^3*b^2) - (log(2*a*b^
4 + 2*b^5*x + 6*a^3*c^2 - 2*a*b^3*(b^2 - 4*a*c)^(1/2) - 2*b^4*x*(b^2 - 4*a*
c)^(1/2) - 9*a^2*b^2*c - 10*a*b^3*c*x + 3*a^2*b*c*(b^2 - 4*a*c)^(1/2) + 9*a
^2*b*c^2*x - 3*a^2*c^2*x*(b^2 - 4*a*c)^(1/2) + 6*a*b^2*c*x*(b^2 - 4*a*c)^(1
/2))*(a*((5*b^2*c)/2 - (3*b*c*(b^2 - 4*a*c)^(1/2))/2) - b^4/2 + (b^3*(b^2 -
 4*a*c)^(1/2))/2 - 2*a^2*c^2))/(4*a^4*c - a^3*b^2) - (1/(2*a) - (b*x)/a^2)/
x^2 - (log(x)*(a*c - b^2))/a^3
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**4+b*x**3+a*x**2),x)
```

```
[Out] Timed out
```

3.18 $\int \frac{1}{x^2\left(ax^2+bx^3+cx^4\right)}\,dx$

Optimal. Leaf size=137

$$\frac{b\left(b^2-2ac\right)\log\left(a+bx+cx^2\right)}{2a^4}-\frac{b\log(x)\left(b^2-2ac\right)}{a^4}-\frac{b^2-ac}{a^3x}+\frac{b}{2a^2x^2}-\frac{\left(2a^2c^2-4ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^4\sqrt{b^2-4ac}}-\frac{1}{3a}$$

```
[Out] -1/3/a/x^3+1/2*b/a^2/x^2+(a*c-b^2)/a^3/x-b*(-2*a*c+b^2)*ln(x)/a^4+1/2*b*(-2
*a*c+b^2)*ln(c*x^2+b*x+a)/a^4-(2*a^2*c^2-4*a*b^2*c+b^4)*arctanh((2*c*x+b)/(
-4*a*c+b^2)^(1/2))/a^4/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {1585, 709, 800, 634, 618, 206, 628}

$$-\frac{\left(2a^2c^2-4ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^4\sqrt{b^2-4ac}}+\frac{b\left(b^2-2ac\right)\log\left(a+bx+cx^2\right)}{2a^4}-\frac{b^2-ac}{a^3x}-\frac{b\log(x)\left(b^2-2ac\right)}{a^4}+\frac{b}{2a^2x^2}-$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a*x^2 + b*x^3 + c*x^4)),x]
```

```
[Out] -1/(3*a*x^3) + b/(2*a^2*x^2) - (b^2 - a*c)/(a^3*x) - ((b^4 - 4*a*b^2*c + 2*
a^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^4*Sqrt[b^2 - 4*a*c]) -
(b*(b^2 - 2*a*c)*Log[x])/a^4 + (b*(b^2 - 2*a*c)*Log[a + b*x + c*x^2])/(2*a^
4)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
```

```
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\left(ax^2+bx^3+cx^4\right)}dx &= \int \frac{1}{x^4\left(a+bx+cx^2\right)}dx\\
&= -\frac{1}{3ax^3}+\frac{\int \frac{-b-cx}{x^3\left(a+bx+cx^2\right)}dx}{a}\\
&= -\frac{1}{3ax^3}+\frac{\int \left(-\frac{b}{ax^3}+\frac{b^2-ac}{a^2x^2}+\frac{-b^3+2abc}{a^3x}+\frac{b^4-3ab^2c+a^2c^2+bc\left(b^2-2ac\right)x}{a^3\left(a+bx+cx^2\right)}\right)dx}{a}\\
&= -\frac{1}{3ax^3}+\frac{b}{2a^2x^2}-\frac{b^2-ac}{a^3x}-\frac{b\left(b^2-2ac\right)\log(x)}{a^4}+\frac{\int \frac{b^4-3ab^2c+a^2c^2+bc\left(b^2-2ac\right)x}{a+bx+cx^2}dx}{a^4}\\
&= -\frac{1}{3ax^3}+\frac{b}{2a^2x^2}-\frac{b^2-ac}{a^3x}-\frac{b\left(b^2-2ac\right)\log(x)}{a^4}+\frac{\left(b\left(b^2-2ac\right)\right)\int \frac{b+2cx}{a+bx+cx^2}dx}{2a^4}+\\
&= -\frac{1}{3ax^3}+\frac{b}{2a^2x^2}-\frac{b^2-ac}{a^3x}-\frac{b\left(b^2-2ac\right)\log(x)}{a^4}+\frac{b\left(b^2-2ac\right)\log\left(a+bx+cx^2\right.}{2a^4}\\
&= -\frac{1}{3ax^3}+\frac{b}{2a^2x^2}-\frac{b^2-ac}{a^3x}-\frac{\left(b^4-4ab^2c+2a^2c^2\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^4\sqrt{b^2-4ac}}-\frac{b\left(b^2-2a\right.}{a^4}\end{aligned}$$

Mathematica [A] time = 0.10, size = 131, normalized size = 0.96

$$\frac{-\frac{2a^3}{x^3}+\frac{6\left(2a^2c^2-4ab^2c+b^4\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+\frac{3a^2b}{x^2}-6\log(x)\left(b^3-2abc\right)+3\left(b^3-2abc\right)\log(a+x(b+cx))+\frac{6a\left(ac-b^2\right)}{x}}{6a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a*x^2 + b*x^3 + c*x^4)),x]
```

```
[Out] ((-2*a^3)/x^3 + (3*a^2*b)/x^2 + (6*a*(-b^2 + a*c))/x + (6*(b^4 - 4*a*b^2*c
+ 2*a^2*c^2)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] - 6
*(b^3 - 2*a*b*c)*Log[x] + 3*(b^3 - 2*a*b*c)*Log[a + x*(b + c*x)])/(6*a^4)
```

fricas [A] time = 0.87, size = 445, normalized size = 3.25

$$\left[\frac{3\left(b^4 - 4\,ab^2c + 2\,a^2c^2\right)\sqrt{b^2 - 4\,ac}\,x^3\log\left(\frac{2\,c^2x^2 + 2\,bcx + b^2 - 2\,ac - \sqrt{b^2 - 4\,ac}\,(2\,cx + b)}{cx^2 + bx + a}\right) - 2\,a^3b^2 + 8\,a^4c + 3\left(b^5 - 6\,ab^3c + 8\,a\right.}{6\left(a^4b^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^3+a*x^2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(b^4 - 4*a*b^2*c + 2*a^2*c^2)*sqrt(b^2 - 4*a*c)*x^3*log((2*c^2*x^2
+ 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a))
 - 2*a^3*b^2 + 8*a^4*c + 3*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*x^3*log(c*x^2 +
b*x + a) - 6*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*x^3*log(x) - 6*(a*b^4 - 5*a^2*
b^2*c + 4*a^3*c^2)*x^2 + 3*(a^2*b^3 - 4*a^3*b*c)*x)/((a^4*b^2 - 4*a^5*c)*x^
3), -1/6*(6*(b^4 - 4*a*b^2*c + 2*a^2*c^2)*sqrt(-b^2 + 4*a*c)*x^3*arctan(-sq
rt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*a^3*b^2 - 8*a^4*c - 3*(b^5
- 6*a*b^3*c + 8*a^2*b*c^2)*x^3*log(c*x^2 + b*x + a) + 6*(b^5 - 6*a*b^3*c +
8*a^2*b*c^2)*x^3*log(x) + 6*(a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*x^2 - 3*(a^2*
b^3 - 4*a^3*b*c)*x)/((a^4*b^2 - 4*a^5*c)*x^3)]
```

giac [A] time = 0.46, size = 136, normalized size = 0.99

$$\frac{\left(b^3 - 2\,abc\right)\log\left(cx^2 + bx + a\right)}{2\,a^4} - \frac{\left(b^3 - 2\,abc\right)\log\left(|x|\right)}{a^4} + \frac{\left(b^4 - 4\,ab^2c + 2\,a^2c^2\right)\arctan\left(\frac{2\,cx + b}{\sqrt{-b^2 + 4\,ac}}\right)}{\sqrt{-b^2 + 4\,ac}\,a^4} + \frac{3\,a^2bx - 2\,a^3 - \ldots}{6\,a\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^3+a*x^2),x, algorithm="giac")
```

```
[Out] 1/2*(b^3 - 2*a*b*c)*log(c*x^2 + b*x + a)/a^4 - (b^3 - 2*a*b*c)*log(abs(x))/
a^4 + (b^4 - 4*a*b^2*c + 2*a^2*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/
(sqrt(-b^2 + 4*a*c)*a^4) + 1/6*(3*a^2*b*x - 2*a^3 - 6*(a*b^2 - a^2*c)*x^2)/
(a^4*x^3)
```

maple [A] time = 0.01, size = 214, normalized size = 1.56

$$\frac{2c^2\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\;a^2} - \frac{4b^2c\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\;a^3} + \frac{b^4\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\;a^4} + \frac{2bc\ln(x)}{a^3} - \frac{bc\ln\left(c\,x^2 + bx + a\right)}{a^3} - \frac{b^3\ln(x)}{a^4} + \ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^4+b*x^3+a*x^2),x)
```

```
[Out] -1/3/a/x^3+1/a^2/x*c-1/a^3/x*b^2+2*b/a^3*ln(x)*c-b^3/a^4*ln(x)+1/2/a^2*b/x^
2-1/a^3*c*ln(c*x^2+b*x+a)*b+1/2/a^4*ln(c*x^2+b*x+a)*b^3+2/a^2/(4*a*c-b^2)^(
1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*c^2-4/a^3/(4*a*c-b^2)^(1/2)*arctan
((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^2*c+1/a^4/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b
)/(4*a*c-b^2)^(1/2))*b^4
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^3+a*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 2.60, size = 524, normalized size = 3.82

$$\ln\left(2\,a\,b^4\,\sqrt{b^2-4\,a\,c}-2\,b^6\,x-2\,a\,b^5+2\,b^5\,x\,\sqrt{b^2-4\,a\,c}+11\,a^2\,b^3\,c-13\,a^3\,b\,c^2+2\,a^3\,c^3\,x+a^3\,c^2\,\sqrt{b^2-4\,a\,c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a*x^2 + b*x^3 + c*x^4)),x)
```

```
[Out] log(2*a*b^4*(b^2 - 4*a*c)^(1/2) - 2*b^6*x - 2*a*b^5 + 2*b^5*x*(b^2 - 4*a*c)
^(1/2) + 11*a^2*b^3*c - 13*a^3*b*c^2 + 2*a^3*c^3*x + a^3*c^2*(b^2 - 4*a*c)^
(1/2) - 17*a^2*b^2*c^2*x + 12*a*b^4*c*x - 5*a^2*b^2*c*(b^2 - 4*a*c)^(1/2) -
 8*a*b^3*c*x*(b^2 - 4*a*c)^(1/2) + 7*a^2*b*c^2*x*(b^2 - 4*a*c)^(1/2))*(b^3/
(2*a^4) - (b^2*(b^2 - 4*a*c)^(1/2))/(2*a^4) - (b*c)/a^3 + (a^2*c^2*(b^2 - 4
*a*c)^(1/2))/(4*a^5*c - a^4*b^2)) + log(2*a*b^5 + 2*b^6*x + 2*a*b^4*(b^2 -
4*a*c)^(1/2) + 2*b^5*x*(b^2 - 4*a*c)^(1/2) - 11*a^2*b^3*c + 13*a^3*b*c^2 -
2*a^3*c^3*x + a^3*c^2*(b^2 - 4*a*c)^(1/2) + 17*a^2*b^2*c^2*x - 12*a*b^4*c*x
 - 5*a^2*b^2*c*(b^2 - 4*a*c)^(1/2) - 8*a*b^3*c*x*(b^2 - 4*a*c)^(1/2) + 7*a^
2*b*c^2*x*(b^2 - 4*a*c)^(1/2))*(b^3/(2*a^4) + (b^2*(b^2 - 4*a*c)^(1/2))/(2*
a^4) - (b*c)/a^3 - (a^2*c^2*(b^2 - 4*a*c)^(1/2))/(4*a^5*c - a^4*b^2)) + ((x
^2*(a*c - b^2))/a^3 - 1/(3*a) + (b*x)/(2*a^2))/x^3 + (b*log(x)*(2*a*c - b^2
))/a^4
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**4+b*x**3+a*x**2),x)
```

```
[Out] Timed out
```

3.19 $\int \frac{x^8}{(ax^2+bx^3+cx^4)^2}\,dx$

Optimal. Leaf size=150

$$-\frac{2(6a^2c^2-6ab^2c+b^4)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3(b^2-4ac)^{3/2}}+\frac{2x(b^2-3ac)}{c^2(b^2-4ac)}-\frac{bx^2}{c(b^2-4ac)}+\frac{x^3(2a+bx)}{(b^2-4ac)(a+bx+cx^2)}-\frac{b\log(a+bx}{c^3}$$

```
[Out] 2*(-3*a*c+b^2)*x/c^2/(-4*a*c+b^2)-b*x^2/c/(-4*a*c+b^2)+x^3*(b*x+2*a)/(-4*a*
c+b^2)/(c*x^2+b*x+a)-2*(6*a^2*c^2-6*a*b^2*c+b^4)*arctanh((2*c*x+b)/(-4*a*c+
b^2)^(1/2))/c^3/(-4*a*c+b^2)^(3/2)-b*ln(c*x^2+b*x+a)/c^3
```

Rubi [A] time = 0.16, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {1585, 738, 800, 634, 618, 206, 628}

$$-\frac{2(6a^2c^2-6ab^2c+b^4)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3(b^2-4ac)^{3/2}}+\frac{2x(b^2-3ac)}{c^2(b^2-4ac)}+\frac{x^3(2a+bx)}{(b^2-4ac)(a+bx+cx^2)}-\frac{bx^2}{c(b^2-4ac)}-\frac{b\log(a+bx}{c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] (2*(b^2 - 3*a*c)*x)/(c^2*(b^2 - 4*a*c)) - (b*x^2)/(c*(b^2 - 4*a*c)) + (x^3*
(2*a + b*x))/((b^2 - 4*a*c)*(a + b*x + c*x^2)) - (2*(b^4 - 6*a*b^2*c + 6*a^
2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^3*(b^2 - 4*a*c)^(3/2)) -
(b*Log[a + b*x + c*x^2])/c^3
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
```

```
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\int \frac{x^8}{\left(ax^2+bx^3+cx^4\right)^2}\,dx = \int \frac{x^4}{\left(a+bx+cx^2\right)^2}\,dx$$

$$= \frac{x^3(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} + \frac{\int \frac{x^2(6a+2bx)}{a+bx+cx^2}\,dx}{-b^2+4ac}$$

$$= \frac{x^3(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} + \frac{\int \left(-\frac{2\left(b^2-3ac\right)}{c^2}+\frac{2bx}{c}+\frac{2\left(a\left(b^2-3ac\right)+b\left(b^2-4ac\right)x\right)}{c^2\left(a+bx+cx^2\right)}\right)dx}{-b^2+4ac}$$

$$= \frac{2\left(b^2-3ac\right)x}{c^2\left(b^2-4ac\right)} - \frac{bx^2}{c\left(b^2-4ac\right)} + \frac{x^3(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} - \frac{2\int \frac{a\left(b^2-3ac\right)+b\left(b^2-4ac\right)x}{a+bx+cx^2}}{c^2\left(b^2-4ac\right)}$$

$$= \frac{2\left(b^2-3ac\right)x}{c^2\left(b^2-4ac\right)} - \frac{bx^2}{c\left(b^2-4ac\right)} + \frac{x^3(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} - \frac{b\int \frac{b+2cx}{a+bx+cx^2}\,dx}{c^3} + \frac{\left(b^4\right.}{}$$

$$= \frac{2\left(b^2-3ac\right)x}{c^2\left(b^2-4ac\right)} - \frac{bx^2}{c\left(b^2-4ac\right)} + \frac{x^3(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} - \frac{b\log\left(a+bx+cx^2\right)}{c^3} -$$

$$= \frac{2\left(b^2-3ac\right)x}{c^2\left(b^2-4ac\right)} - \frac{bx^2}{c\left(b^2-4ac\right)} + \frac{x^3(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} - \frac{2\left(b^4-6ab^2c+6a^2c^2\right.}{c^3\left(b^2-4\right.}$$

Mathematica [A] time = 0.20, size = 132, normalized size = 0.88

$$\frac{-\frac{2\left(6a^2c^2-6ab^2c+b^4\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{3/2}} + \frac{a^2c(3b-2cx)-ab^2(b-4cx)+b^4(-x)}{\left(b^2-4ac\right)(a+x(b+cx))} - b\log(a+x(b+cx)) + cx}{c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] (c*x + (-(b^4*x) - a*b^2*(b - 4*c*x) + a^2*c*(3*b - 2*c*x))/((b^2 - 4*a*c)*
(a + x*(b + c*x))) - (2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTan[(b + 2*c*x)/Sq
rt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) - b*Log[a + x*(b + c*x)])/c^3
```

fricas [B] time = 0.69, size = 837, normalized size = 5.58

$$\left[-\frac{ab^5 - 7\,a^2b^3c + 12\,a^3bc^2 - \left(b^4c^2 - 8\,ab^2c^3 + 16\,a^2c^4\right)x^3 - \left(b^5c - 8\,ab^3c^2 + 16\,a^2bc^3\right)x^2 + \left(ab^4 - 6\,a^2b^2c + 6\,a^3c^2 \cdots\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")
```

```
[Out] [-(a*b^5 - 7*a^2*b^3*c + 12*a^3*b*c^2 - (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4
)*x^3 - (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2 + (a*b^4 - 6*a^2*b^2*c + 6
*a^3*c^2 + (b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*x^2 + (b^5 - 6*a*b^3*c + 6*a^2
*b*c^2)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(
b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + (b^6 - 9*a*b^4*c + 26*a^2*b^
2*c^2 - 24*a^3*c^3)*x + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + (b^5*c - 8*a*
b^3*c^2 + 16*a^2*b*c^3)*x^2 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*x)*log(c*x
^2 + b*x + a))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5 + (b^4*c^4 - 8*a*b^2
*c^5 + 16*a^2*c^6)*x^2 + (b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x), -(a*b^5
 - 7*a^2*b^3*c + 12*a^3*b*c^2 - (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^3 -
(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2 + 2*(a*b^4 - 6*a^2*b^2*c + 6*a^3*c
^2 + (b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*x^2 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2
)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c
)) + (b^6 - 9*a*b^4*c + 26*a^2*b^2*c^2 - 24*a^3*c^3)*x + (a*b^5 - 8*a^2*b^3
*c + 16*a^3*b*c^2 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2 + (b^6 - 8*a*b
^4*c + 16*a^2*b^2*c^2)*x)*log(c*x^2 + b*x + a))/(a*b^4*c^3 - 8*a^2*b^2*c^4
+ 16*a^3*c^5 + (b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*x^2 + (b^5*c^3 - 8*a*b^
3*c^4 + 16*a^2*b*c^5)*x)]
```

giac [A] time = 0.57, size = 161, normalized size = 1.07

$$\frac{2\left(b^4 - 6\,ab^2c + 6\,a^2c^2\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^2c^3 - 4\,ac^4\right)\sqrt{-b^2+4\,ac}} + \frac{x}{c^2} - \frac{b\log\left(cx^2+bx+a\right)}{c^3} - \frac{\frac{\left(b^4-4\,ab^2c+2\,a^2c^2\right)x}{c} + \frac{ab^3-3\,a^2bc}{c}}{\left(cx^2+bx+a\right)\left(b^2-4\,ac\right)c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")
```

```
[Out] 2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^
2*c^3 - 4*a*c^4)*sqrt(-b^2 + 4*a*c)) + x/c^2 - b*log(c*x^2 + b*x + a)/c^3 -
 ((b^4 - 4*a*b^2*c + 2*a^2*c^2)*x/c + (a*b^3 - 3*a^2*b*c)/c)/((c*x^2 + b*x
+ a)*(b^2 - 4*a*c)*c^2)
```

maple [B] time = 0.01, size = 352, normalized size = 2.35

$$\frac{2a^2x}{\left(c\,x^2+bx+a\right)\left(4ac-b^2\right)c} - \frac{12a^2\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}c} - \frac{4a\,b^2x}{\left(c\,x^2+bx+a\right)\left(4ac-b^2\right)c^2} + \frac{12a\,b^2\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}c^2} + \frac{\cdots}{\left(c\,x^2 \cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(c*x^4+b*x^3+a*x^2)^2,x)
```

```
[Out] x/c^2+2/c/(c*x^2+b*x+a)/(4*a*c-b^2)*x*a^2-4/c^2/(c*x^2+b*x+a)/(4*a*c-b^2)*x
*a*b^2+1/c^3/(c*x^2+b*x+a)/(4*a*c-b^2)*x*b^4-3/c^2/(c*x^2+b*x+a)/(4*a*c-b^2
```

```
)*a^2*b+1/c^3/(c*x^2+b*x+a)/(4*a*c-b^2)*a*b^3-4/c^2/(4*a*c-b^2)*ln(c*x^2+b*
x+a)*a*b+1/c^3/(4*a*c-b^2)*ln(c*x^2+b*x+a)*b^3-12/c/(4*a*c-b^2)^(3/2)*arcta
n((2*c*x+b)/(4*a*c-b^2)^(1/2))*a^2+12/c^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b
)/(4*a*c-b^2)^(1/2))*a*b^2-2/c^3/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-
b^2)^(1/2))*b^4
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 2.46, size = 261, normalized size = 1.74

$$\frac{x}{c^2}+\frac{\frac{a\left(b^3-3\,a\,b\,c\right)}{c\left(4\,a\,c-b^2\right)}+\frac{x\left(2\,a^2\,c^2-4\,a\,b^2\,c+b^4\right)}{c\left(4\,a\,c-b^2\right)}}{c^3\,x^2+b\,c^2\,x+a\,c^2}+\frac{\ln\left(c\,x^2+b\,x+a\right)\left(-128\,a^3\,b\,c^3+96\,a^2\,b^3\,c^2-24\,a\,b^5\,c+2\,b^7\right)}{2\left(64\,a^3\,c^6-48\,a^2\,b^2\,c^5+12\,a\,b^4\,c^4-b^6\,c^3\right)}-\frac{2\,\mathrm{atan}\Big(\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(a*x^2 + b*x^3 + c*x^4)^2,x)
```

```
[Out] x/c^2 + ((a*(b^3 - 3*a*b*c))/(c*(4*a*c - b^2)) + (x*(b^4 + 2*a^2*c^2 - 4*a*
b^2*c))/(c*(4*a*c - b^2)))/(a*c^2 + c^3*x^2 + b*c^2*x) + (log(a + b*x + c*x
^2)*(2*b^7 - 128*a^3*b*c^3 + 96*a^2*b^3*c^2 - 24*a*b^5*c))/(2*(64*a^3*c^6 -
 b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) - (2*atan((2*c*x)/(4*a*c - b^2)^
(1/2) - (b^3*c^2 - 4*a*b*c^3)/(c^2*(4*a*c - b^2)^(3/2)))*(b^4 + 6*a^2*c^2 -
 6*a*b^2*c))/(c^3*(4*a*c - b^2)^(3/2))
```

sympy [B] time = 1.79, size = 842, normalized size = 5.61

$$\left(-\frac{b}{c^3}-\frac{\sqrt{-\left(4ac-b^2\right)^3}\left(6a^2c^2-6ab^2c+b^4\right)}{c^3\left(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6\right)}\right)\log\left(x+\frac{-10a^2bc-16a^2c^4\left(-\frac{b}{c^3}-\frac{\sqrt{-\left(4ac-b^2\right)^3}\left(6a^2c^2-6ab^2c+b^4\right)}{c^3\left(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6\right)}\right)+\cdots}{\cdots}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(c*x**4+b*x**3+a*x**2)**2,x)
```

```
[Out] (-b/c**3 - sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(c**3
*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))*log(x + (-10*a**
2*b*c - 16*a**2*c**4*(-b/c**3 - sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a
*b**2*c + b**4)/(c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**
6))) + 2*a*b**3 + 8*a*b**2*c**3*(-b/c**3 - sqrt(-(4*a*c - b**2)**3)*(6*a**2
*c**2 - 6*a*b**2*c + b**4)/(c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b
**4*c - b**6))) - b**4*c**2*(-b/c**3 - sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**
2 - 6*a*b**2*c + b**4)/(c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*
c - b**6))))/(12*a**2*c**2 - 12*a*b**2*c + 2*b**4)) + (-b/c**3 + sqrt(-(4*a
*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(c**3*(64*a**3*c**3 - 48*a
**2*b**2*c**2 + 12*a*b**4*c - b**6)))*log(x + (-10*a**2*b*c - 16*a**2*c**4*
```

```
(-b/c**3 + sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(c**3
*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) + 2*a*b**3 + 8*a
*b**2*c**3*(-b/c**3 + sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c +
b**4)/(c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) - b**
4*c**2*(-b/c**3 + sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4
)/(c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))))/(12*a**2
*c**2 - 12*a*b**2*c + 2*b**4)) + (-3*a**2*b*c + a*b**3 + x*(2*a**2*c**2 - 4
*a*b**2*c + b**4))/(4*a**2*c**4 - a*b**2*c**3 + x**2*(4*a*c**5 - b**2*c**4)
 + x*(4*a*b*c**4 - b**3*c**3)) + x/c**2
```

3.20 $\int \frac{x^7}{\left(ax^2+bx^3+cx^4\right)^2} \, dx$

Optimal. Leaf size=114

$$\frac{b\left(b^2-6ac\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\left(b^2-4ac\right)^{3/2}}+\frac{x^2(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{bx}{c\left(b^2-4ac\right)}+\frac{\log\left(a+bx+cx^2\right)}{2c^2}$$

```
[Out] -b*x/c/(-4*a*c+b^2)+x^2*(b*x+2*a)/(-4*a*c+b^2)/(c*x^2+b*x+a)+b*(-6*a*c+b^2)
*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c^2/(-4*a*c+b^2)^(3/2)+1/2*ln(c*x^2+
b*x+a)/c^2
```

Rubi [A] time = 0.10, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {1585, 738, 773, 634, 618, 206, 628}

$$\frac{b\left(b^2-6ac\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\left(b^2-4ac\right)^{3/2}}+\frac{x^2(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{bx}{c\left(b^2-4ac\right)}+\frac{\log\left(a+bx+cx^2\right)}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] -((b*x)/(c*(b^2 - 4*a*c))) + (x^2*(2*a + b*x))/((b^2 - 4*a*c)*(a + b*x + c*
x^2)) + (b*(b^2 - 6*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^2*(b^2
- 4*a*c)^(3/2)) + Log[a + b*x + c*x^2]/(2*c^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
```

```
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 773

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\int \frac{x^7}{\left(ax^2+bx^3+cx^4\right)^2}\,dx = \int \frac{x^3}{\left(a+bx+cx^2\right)^2}\,dx$$

$$= \frac{x^2(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} + \frac{\int \frac{x(4a+bx)}{a+bx+cx^2}\,dx}{-b^2+4ac}$$

$$= -\frac{bx}{c\left(b^2-4ac\right)} + \frac{x^2(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} - \frac{\int \frac{-ab+\left(-b^2+4ac\right)x}{a+bx+cx^2}\,dx}{c\left(b^2-4ac\right)}$$

$$= -\frac{bx}{c\left(b^2-4ac\right)} + \frac{x^2(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} + \frac{\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2c^2} - \frac{\left(b\left(b^2-6ac\right)\right)\int \frac{1}{a+bx+cx^2}}{2c^2\left(b^2-4ac\right)}$$

$$= -\frac{bx}{c\left(b^2-4ac\right)} + \frac{x^2(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} + \frac{\log\left(a+bx+cx^2\right)}{2c^2} + \frac{\left(b\left(b^2-6ac\right)\right)\text{Sub}}{c}$$

$$= -\frac{bx}{c\left(b^2-4ac\right)} + \frac{x^2(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} + \frac{b\left(b^2-6ac\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\left(b^2-4ac\right)^{3/2}} + \frac{\log(a}{}$$

Mathematica [A] time = 0.15, size = 109, normalized size = 0.96

$$\frac{\frac{2\left(-2a^2c+ab(b-3cx)+b^3x\right)}{\left(b^2-4ac\right)(a+x(b+cx))} + \frac{2b\left(b^2-6ac\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{3/2}} + \log(a+x(b+cx))}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] ((2*(-2*a^2*c + b^3*x + a*b*(b - 3*c*x)))/((b^2 - 4*a*c)*(a + x*(b + c*x)))
 + (2*b*(b^2 - 6*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c
)^(3/2) + Log[a + x*(b + c*x)])/(2*c^2)
```

fricas [B] time = 0.62, size = 635, normalized size = 5.57

$$\left[\frac{2\,ab^4 - 12\,a^2b^2c + 16\,a^3c^2 + \left(ab^3 - 6\,a^2bc + \left(b^3c - 6\,abc^2\right)x^2 + \left(b^4 - 6\,ab^2c\right)x\right)\sqrt{b^2-4\,ac}\,\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2}{cx^2}\right.}{2\left(ab^4c^2 - 8\,a^2b^2c^3 + 16\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + (a*b^3 - 6*a^2*b*c + (b^3*c - 6
*a*b*c^2)*x^2 + (b^4 - 6*a*b^2*c)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b
*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 2*
(b^5 - 7*a*b^3*c + 12*a^2*b*c^2)*x + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b
^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)*
log(c*x^2 + b*x + a))/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4 + (b^4*c^3 -
8*a*b^2*c^4 + 16*a^2*c^5)*x^2 + (b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x),
1/2*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + 2*(a*b^3 - 6*a^2*b*c + (b^3*c -
6*a*b*c^2)*x^2 + (b^4 - 6*a*b^2*c)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2
+ 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*(b^5 - 7*a*b^3*c + 12*a^2*b*c^2)*x
+ (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^
2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)*log(c*x^2 + b*x + a))/(a*b^4*c^2 -
8*a^2*b^2*c^3 + 16*a^3*c^4 + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^2 + (b^
5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x)]
```

giac [A] time = 0.60, size = 125, normalized size = 1.10

$$-\frac{\left(b^3 - 6\,abc\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^2c^2 - 4\,ac^3\right)\sqrt{-b^2+4\,ac}} + \frac{\log\left(cx^2+bx+a\right)}{2\,c^2} + \frac{ab^2 - 2\,a^2c + \left(b^3 - 3\,abc\right)x}{\left(cx^2+bx+a\right)\left(b^2-4\,ac\right)c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")
```

```
[Out] -(b^3 - 6*a*b*c)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2*c^2 - 4*a*c^3
)*sqrt(-b^2 + 4*a*c)) + 1/2*log(c*x^2 + b*x + a)/c^2 + (a*b^2 - 2*a^2*c + (
b^3 - 3*a*b*c)*x)/((c*x^2 + b*x + a)*(b^2 - 4*a*c)*c^2)
```

maple [A] time = 0.01, size = 209, normalized size = 1.83

$$-\frac{6ab\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}c} + \frac{b^3\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}c^2} + \frac{2a\ln\left(c\,x^2+bx+a\right)}{\left(4ac-b^2\right)c} - \frac{b^2\ln\left(c\,x^2+bx+a\right)}{2\left(4ac-b^2\right)c^2} + \frac{\frac{\left(3ac-b^2\right)bx}{\left(4ac-b^2\right)c^2} + \frac{\left(2ac-b^2\right.}{\left(4ac-b^2\right)}}{c\,x^2+bx+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(c*x^4+b*x^3+a*x^2)^2,x)
```

```
[Out] ((3*a*c-b^2)/(4*a*c-b^2)*b/c^2*x+(2*a*c-b^2)/(4*a*c-b^2)*a/c^2)/(c*x^2+b*x+
a)+2/c/(4*a*c-b^2)*ln(c*x^2+b*x+a)*a-1/2/c^2/(4*a*c-b^2)*ln(c*x^2+b*x+a)*b^
2-6/c/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b+1/c^2/(4*a*
c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^3
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 2.49, size = 279, normalized size = 2.45

$$\frac{\frac{a\left(2ac-b^2\right)}{c^2\left(4ac-b^2\right)}+\frac{bx\left(3ac-b^2\right)}{c^2\left(4ac-b^2\right)}}{cx^2+bx+a}-\frac{\ln\left(cx^2+bx+a\right)\left(-64a^3c^3+48a^2b^2c^2-12ab^4c+b^6\right)}{2\left(64a^3c^5-48a^2b^2c^4+12ab^4c^3-b^6c^2\right)}+\frac{b\,\mathrm{atan}\left(\frac{c^2\left(4ac-b^2\right)^{5/2}\left(\frac{2bx(6ac-}{c(4ac-b^2}\right.}{b^3-}\right.}{c^2\left(4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a*x^2 + b*x^3 + c*x^4)^2,x)
```

```
[Out] ((a*(2*a*c - b^2))/(c^2*(4*a*c - b^2)) + (b*x*(3*a*c - b^2))/(c^2*(4*a*c -
b^2)))/(a + b*x + c*x^2) - (log(a + b*x + c*x^2)*(b^6 - 64*a^3*c^3 + 48*a^2
*b^2*c^2 - 12*a*b^4*c))/(2*(64*a^3*c^5 - b^6*c^2 + 12*a*b^4*c^3 - 48*a^2*b^
2*c^4)) + (b*atan((c^2*(4*a*c - b^2)^(5/2)*((2*b*x*(6*a*c - b^2))/(c*(4*a*c
 - b^2)^3) + (b^2*(4*a*c^2 - b^2*c)*(6*a*c - b^2))/(c^3*(4*a*c - b^2)^4)))/
(b^3 - 6*a*b*c))*(6*a*c - b^2))/(c^2*(4*a*c - b^2)^(3/2))
```

sympy [B] time = 1.37, size = 729, normalized size = 6.39

$$\left(-\frac{b\sqrt{-\left(4ac-b^2\right)^3}\left(6ac-b^2\right)}{2c^2\left(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6\right)}+\frac{1}{2c^2}\right)\log\left(x+\frac{-16a^2c^3\left(-\frac{b\sqrt{-\left(4ac-b^2\right)^3}\left(6ac-b^2\right)}{2c^2\left(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6\right)}+\frac{1}{2c^2}\right)+8a^2c+}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(c*x**4+b*x**3+a*x**2)**2,x)
```

```
[Out] (-b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2
*b**2*c**2 + 12*a*b**4*c - b**6)) + 1/(2*c**2))*log(x + (-16*a**2*c**3*(-b*
sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**
2*c**2 + 12*a*b**4*c - b**6)) + 1/(2*c**2)) + 8*a**2*c + 8*a*b**2*c**2*(-b*
sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**
2*c**2 + 12*a*b**4*c - b**6)) + 1/(2*c**2)) - a*b**2 - b**4*c*(-b*sqrt(-(4*
a*c - b**2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 +
12*a*b**4*c - b**6)) + 1/(2*c**2)))/(6*a*b*c - b**3)) + (b*sqrt(-(4*a*c - b
**2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b*
*4*c - b**6)) + 1/(2*c**2))*log(x + (-16*a**2*c**3*(b*sqrt(-(4*a*c - b**2)*
*3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c
- b**6)) + 1/(2*c**2)) + 8*a**2*c + 8*a*b**2*c**2*(b*sqrt(-(4*a*c - b**2)**
3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c -
 b**6)) + 1/(2*c**2)) - a*b**2 - b**4*c*(b*sqrt(-(4*a*c - b**2)**3)*(6*a*c
- b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) +
1/(2*c**2)))/(6*a*b*c - b**3)) + (2*a**2*c - a*b**2 + x*(3*a*b*c - b**3))/(
4*a**2*c**3 - a*b**2*c**2 + x**2*(4*a*c**4 - b**2*c**3) + x*(4*a*b*c**3 - b
**3*c**2))
```

3.21 $\int \frac{x^6}{\left(ax^2+bx^3+cx^4\right)^2}\,dx$

Optimal. Leaf size=67

$$\frac{x(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{4a\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}$$

```
[Out] x*(b*x+2*a)/(-4*a*c+b^2)/(c*x^2+b*x+a)+4*a*arctanh((2*c*x+b)/(-4*a*c+b^2)^(
1/2))/(-4*a*c+b^2)^(3/2)
```

Rubi [A] time = 0.04, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1585, 722, 618, 206}

$$\frac{x(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{4a\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] (x*(2*a + b*x))/((b^2 - 4*a*c)*(a + b*x + c*x^2)) + (4*a*ArcTanh[(b + 2*c*x
)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 722

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*(2*p + 3)*(c*d^2 -
b*d*e + a*e^2))/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c
*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
0] && LtQ[p, -1]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\int \frac{x^6}{\left(ax^2+bx^3+cx^4\right)^2}\,dx=\int \frac{x^2}{\left(a+bx+cx^2\right)^2}\,dx$$
$$=\frac{x(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{(2a)\int \frac{1}{a+bx+cx^2}\,dx}{b^2-4ac}$$
$$=\frac{x(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{(4a)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx\right)}{b^2-4ac}$$
$$=\frac{x(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{4a\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}$$

Mathematica [A] time = 0.09, size = 81, normalized size = 1.21

$$\frac{a(b-2cx)+b^2x}{c\left(4ac-b^2\right)(a+x(b+cx))}+\frac{4a\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] (b^2*x + a*(b - 2*c*x))/(c*(-b^2 + 4*a*c)*(a + x*(b + c*x))) + (4*a*ArcTan[
(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2)
```

fricas [B] time = 0.68, size = 387, normalized size = 5.78

$$\left[-\frac{ab^3-4\,a^2bc+2\left(ac^2x^2+abcx+a^2c\right)\sqrt{b^2-4\,ac}\,\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}\,(2\,cx+b)}{cx^2+bx+a}\right)+\left(b^4-6\,ab^2c+8\,a^2c^2\right)}{ab^4c-8\,a^2b^2c^2+16\,a^3c^3+\left(b^4c^2-8\,ab^2c^3+16\,a^2c^4\right)x^2+\left(b^5c-8\,ab^3c^2+16\,a^2bc^3\right)x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")
```

```
[Out] [-(a*b^3 - 4*a^2*b*c + 2*(a*c^2*x^2 + a*b*c*x + a^2*c)*sqrt(b^2 - 4*a*c)*lo
g((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^
2 + b*x + a)) + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*x)/(a*b^4*c - 8*a^2*b^2*c^2 +
 16*a^3*c^3 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^2 + (b^5*c - 8*a*b^3*c
^2 + 16*a^2*b*c^3)*x), -(a*b^3 - 4*a^2*b*c - 4*(a*c^2*x^2 + a*b*c*x + a^2*c
)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c))
+ (b^4 - 6*a*b^2*c + 8*a^2*c^2)*x)/(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3 +
(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^2 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*
c^3)*x)]
```

giac [A] time = 0.49, size = 88, normalized size = 1.31

$$-\frac{4\,a\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^2-4\,ac\right)\sqrt{-b^2+4\,ac}}-\frac{b^2x-2\,acx+ab}{\left(b^2c-4\,ac^2\right)\left(cx^2+bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")
```

```
[Out] -4*a*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-b^2 + 4*a*
c)) - (b^2*x - 2*a*c*x + a*b)/((b^2*c - 4*a*c^2)*(c*x^2 + b*x + a))
```

maple [A] time = 0.01, size = 97, normalized size = 1.45

$$\frac{4a\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}}+\frac{\frac{ab}{\left(4ac-b^2\right)c}-\frac{\left(2ac-b^2\right)x}{\left(4ac-b^2\right)c}}{c\,x^2+bx+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(c*x^4+b*x^3+a*x^2)^2,x)
```

```
[Out] (-(2*a*c-b^2)/(4*a*c-b^2)/c*x+1/(4*a*c-b^2)*a*b/c)/(c*x^2+b*x+a)+4*a/(4*a*c
-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 2.13, size = 135, normalized size = 2.01

$$-\frac{\frac{x\left(2\,a\,c-b^2\right)}{c\left(4\,a\,c-b^2\right)}-\frac{a\,b}{c\left(4\,a\,c-b^2\right)}}{c\,x^2+b\,x+a}-\frac{4\,a\,\mathrm{atan}\left(\frac{\left(\frac{2\,a\left(b^3-4\,a\,b\,c\right)}{\left(4\,a\,c-b^2\right)^{5/2}}-\frac{4\,a\,c\,x}{\left(4\,a\,c-b^2\right)^{3/2}}\right)\left(4\,a\,c-b^2\right)}{2\,a}\right)}{\left(4\,a\,c-b^2\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a*x^2 + b*x^3 + c*x^4)^2,x)
```

```
[Out] - ((x*(2*a*c - b^2))/(c*(4*a*c - b^2)) - (a*b)/(c*(4*a*c - b^2)))/(a + b*x
+ c*x^2) - (4*a*atan((((2*a*(b^3 - 4*a*b*c))/(4*a*c - b^2)^(5/2) - (4*a*c*x
)/(4*a*c - b^2)^(3/2))*(4*a*c - b^2))/(2*a)))/(4*a*c - b^2)^(3/2)
```

sympy [B] time = 0.60, size = 280, normalized size = 4.18

$$-2a\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\log\left(x+\frac{-32a^3c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+16a^2b^2c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}-2ab^4\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+2ab}{4ac}\right)+2a\sqrt{-\frac{1}{(}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(c*x**4+b*x**3+a*x**2)**2,x)
```

```
[Out] -2*a*sqrt(-1/(4*a*c - b**2)**3)*log(x + (-32*a**3*c**2*sqrt(-1/(4*a*c - b**
2)**3) + 16*a**2*b**2*c*sqrt(-1/(4*a*c - b**2)**3) - 2*a*b**4*sqrt(-1/(4*a*
c - b**2)**3) + 2*a*b)/(4*a*c)) + 2*a*sqrt(-1/(4*a*c - b**2)**3)*log(x + (3
2*a**3*c**2*sqrt(-1/(4*a*c - b**2)**3) - 16*a**2*b**2*c*sqrt(-1/(4*a*c - b*
*2)**3) + 2*a*b**4*sqrt(-1/(4*a*c - b**2)**3) + 2*a*b)/(4*a*c)) + (a*b + x*
(-2*a*c + b**2))/(4*a**2*c**2 - a*b**2*c + x**2*(4*a*c**3 - b**2*c**2) + x*
(4*a*b*c**2 - b**3*c))
```

3.22 $\int \frac{x^5}{\left(ax^2+bx^3+cx^4\right)^2} dx$

Optimal. Leaf size=66

$$\frac{2a+bx}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{2b\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}$$

```
[Out] (b*x+2*a)/(-4*a*c+b^2)/(c*x^2+b*x+a)-2*b*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/
2))/(-4*a*c+b^2)^(3/2)
```

Rubi [A] time = 0.04, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1585, 638, 618, 206}

$$\frac{2a+bx}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{2b\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] (2*a + b*x)/((b^2 - 4*a*c)*(a + b*x + c*x^2)) - (2*b*ArcTanh[(b + 2*c*x)/Sq
rt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\int \frac{x^5}{\left(ax^2+bx^3+cx^4\right)^2}\,dx = \int \frac{x}{\left(a+bx+cx^2\right)^2}\,dx$$
$$= \frac{2a+bx}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} + \frac{b\int \frac{1}{a+bx+cx^2}\,dx}{b^2-4ac}$$
$$= \frac{2a+bx}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} - \frac{(2b)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx\right)}{b^2-4ac}$$
$$= \frac{2a+bx}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} - \frac{2b\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}$$

Mathematica [A] time = 0.07, size = 69, normalized size = 1.05

$$\frac{2a+bx}{\left(b^2-4ac\right)(a+x(b+cx))} - \frac{2b\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] (2*a + b*x)/((b^2 - 4*a*c)*(a + x*(b + c*x))) - (2*b*ArcTan[(b + 2*c*x)/Sqr
t[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2)
```

fricas [B] time = 0.59, size = 338, normalized size = 5.12

$$\left[\frac{2\,ab^2-8\,a^2c-\left(bcx^2+b^2x+ab\right)\sqrt{b^2-4\,ac}\,\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}\,(2\,cx+b)}{cx^2+bx+a}\right)+\left(b^3-4\,abc\right)x}{ab^4-8\,a^2b^2c+16\,a^3c^2+\left(b^4c-8\,ab^2c^2+16\,a^2c^3\right)x^2+\left(b^5-8\,ab^3c+16\,a^2bc^2\right)x}, \frac{2\,ab^2-8}{ab^4-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")
```

```
[Out] [(2*a*b^2 - 8*a^2*c - (b*c*x^2 + b^2*x + a*b)*sqrt(b^2 - 4*a*c)*log((2*c^2*
x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x +
 a)) + (b^3 - 4*a*b*c)*x)/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*
b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x), (2*a*b^2 -
 8*a^2*c - 2*(b*c*x^2 + b^2*x + a*b)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 +
 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + (b^3 - 4*a*b*c)*x)/(a*b^4 - 8*a^2*b^2*
c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c
+ 16*a^2*b*c^2)*x)]
```

giac [A] time = 0.38, size = 76, normalized size = 1.15

$$\frac{2\,b\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^2-4\,ac\right)\sqrt{-b^2+4\,ac}} + \frac{bx+2\,a}{\left(cx^2+bx+a\right)\left(b^2-4\,ac\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")
```

```
[Out] 2*b*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-b^2 + 4*a*c
)) + (b*x + 2*a)/((c*x^2 + b*x + a)*(b^2 - 4*a*c))
```

maple [A] time = 0.00, size = 70, normalized size = 1.06

$$-\frac{2b\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}}+\frac{-bx-2a}{\left(4ac-b^2\right)\left(c\,x^2+bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(c*x^4+b*x^3+a*x^2)^2,x)
```

```
[Out] (-b*x-2*a)/(4*a*c-b^2)/(c*x^2+b*x+a)-2*b/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)
/(4*a*c-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 2.18, size = 110, normalized size = 1.67

$$-\frac{\frac{2\,a}{4\,a\,c-b^2}+\frac{b\,x}{4\,a\,c-b^2}}{c\,x^2+b\,x+a}-\frac{2\,b\,\mathrm{atan}\left(\frac{\left(\frac{b^2}{{\left(4\,a\,c-b^2\right)}^{3/2}}+\frac{2\,b\,c\,x}{{\left(4\,a\,c-b^2\right)}^{3/2}}\right)\left(4\,a\,c-b^2\right)}{b}\right)}{{\left(4\,a\,c-b^2\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a*x^2 + b*x^3 + c*x^4)^2,x)
```

```
[Out] - ((2*a)/(4*a*c - b^2) + (b*x)/(4*a*c - b^2))/(a + b*x + c*x^2) - (2*b*atan
(((b^2/(4*a*c - b^2)^(3/2) + (2*b*c*x)/(4*a*c - b^2)^(3/2))*(4*a*c - b^2))/
b))/(4*a*c - b^2)^(3/2)
```

sympy [B] time = 0.56, size = 253, normalized size = 3.83

$$b\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\log\left(x+\frac{-16a^2bc^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+8ab^3c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}-b^5\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+b^2}{2bc}\right)-b\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\ \mathrm{l}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(c*x**4+b*x**3+a*x**2)**2,x)
```

```
[Out] b*sqrt(-1/(4*a*c - b**2)**3)*log(x + (-16*a**2*b*c**2*sqrt(-1/(4*a*c - b**2
)**3) + 8*a*b**3*c*sqrt(-1/(4*a*c - b**2)**3) - b**5*sqrt(-1/(4*a*c - b**2)
**3) + b**2)/(2*b*c)) - b*sqrt(-1/(4*a*c - b**2)**3)*log(x + (16*a**2*b*c**
2*sqrt(-1/(4*a*c - b**2)**3) - 8*a*b**3*c*sqrt(-1/(4*a*c - b**2)**3) + b**5
*sqrt(-1/(4*a*c - b**2)**3) + b**2)/(2*b*c)) + (-2*a - b*x)/(4*a**2*c - a*b
**2 + x**2*(4*a*c**2 - b**2*c) + x*(4*a*b*c - b**3))
```

3.23 $\int \frac{x^4}{(ax^2+bx^3+cx^4)^2}\,dx$

Optimal. Leaf size=66

$$\frac{4c\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{b+2cx}{(b^2-4ac)(a+bx+cx^2)}$$

```
[Out] (-2*c*x-b)/(-4*a*c+b^2)/(c*x^2+b*x+a)+4*c*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1
/2))/(-4*a*c+b^2)^(3/2)
```

Rubi [A] time = 0.03, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1585, 614, 618, 206}

$$\frac{4c\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{b+2cx}{(b^2-4ac)(a+bx+cx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] -((b + 2*c*x)/((b^2 - 4*a*c)*(a + b*x + c*x^2))) + (4*c*ArcTanh[(b + 2*c*x)
/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\int \frac{x^4}{\left(ax^2+bx^3+cx^4\right)^2}\,dx = \int \frac{1}{\left(a+bx+cx^2\right)^2}\,dx$$
$$= -\frac{b+2cx}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} - \frac{(2c)\int \frac{1}{a+bx+cx^2}\,dx}{b^2-4ac}$$
$$= -\frac{b+2cx}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} + \frac{(4c)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx\right)}{b^2-4ac}$$
$$= -\frac{b+2cx}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} + \frac{4c\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}$$

Mathematica [A] time = 0.07, size = 70, normalized size = 1.06

$$-\frac{\frac{4c\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}} + \frac{b+2cx}{a+x(b+cx)}}{b^2-4ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] -(((b + 2*c*x)/(a + x*(b + c*x)) + (4*c*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*
c]])/Sqrt[-b^2 + 4*a*c])/(b^2 - 4*a*c))
```

fricas [B] time = 0.59, size = 341, normalized size = 5.17

$$\left[-\frac{b^3-4\,abc+2\left(c^2x^2+bcx+ac\right)\sqrt{b^2-4\,ac}\,\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}\,(2\,cx+b)}{cx^2+bx+a}\right)+2\left(b^2c-4\,ac^2\right)x}{ab^4-8\,a^2b^2c+16\,a^3c^2+\left(b^4c-8\,ab^2c^2+16\,a^2c^3\right)x^2+\left(b^5-8\,ab^3c+16\,a^2bc^2\right)x}, -\frac{b^3-4\,ab}{ab^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")
```

```
[Out] [-(b^3 - 4*a*b*c + 2*(c^2*x^2 + b*c*x + a*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x
^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x +
a)) + 2*(b^2*c - 4*a*c^2)*x)/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8
*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x), -(b^3 -
 4*a*b*c - 4*(c^2*x^2 + b*c*x + a*c)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 +
 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*(b^2*c - 4*a*c^2)*x)/(a*b^4 - 8*a^2*
b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^
3*c + 16*a^2*b*c^2)*x)]
```

giac [A] time = 0.46, size = 76, normalized size = 1.15

$$-\frac{4\,c\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^2-4\,ac\right)\sqrt{-b^2+4\,ac}} - \frac{2\,cx+b}{\left(cx^2+bx+a\right)\left(b^2-4\,ac\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")
```

```
[Out] -4*c*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-b^2 + 4*a*
c)) - (2*c*x + b)/((c*x^2 + b*x + a)*(b^2 - 4*a*c))
```

maple [A] time = 0.00, size = 68, normalized size = 1.03

$$\frac{4c\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}}+\frac{2cx+b}{\left(4ac-b^2\right)\left(c\,x^2+bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^4+b*x^3+a*x^2)^2,x)
```

```
[Out] (2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)+4*c/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/
(4*a*c-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 0.08, size = 119, normalized size = 1.80

$$\frac{\frac{b}{4\,a\,c-b^2}+\frac{2\,c\,x}{4\,a\,c-b^2}}{c\,x^2+b\,x+a}-\frac{4\,c\,\mathrm{atan}\left(\frac{\left(\frac{2\,c\,\left(b^3-4\,a\,b\,c\right)}{\left(4\,a\,c-b^2\right)^{5/2}}-\frac{4\,c^2\,x}{\left(4\,a\,c-b^2\right)^{3/2}}\right)\left(4\,a\,c-b^2\right)}{2\,c}\right)}{\left(4\,a\,c-b^2\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a*x^2 + b*x^3 + c*x^4)^2,x)
```

```
[Out] (b/(4*a*c - b^2) + (2*c*x)/(4*a*c - b^2))/(a + b*x + c*x^2) - (4*c*atan((((
2*c*(b^3 - 4*a*b*c))/(4*a*c - b^2)^(5/2) - (4*c^2*x)/(4*a*c - b^2)^(3/2))*(
4*a*c - b^2))/(2*c)))/(4*a*c - b^2)^(3/2)
```

sympy [B] time = 0.59, size = 265, normalized size = 4.02

$$-2c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\log\left(x+\frac{-32a^2c^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+16ab^2c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}-2b^4c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+2bc}{4c^2}\right)+2c\sqrt{-\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**4+b*x**3+a*x**2)**2,x)
```

```
[Out] -2*c*sqrt(-1/(4*a*c - b**2)**3)*log(x + (-32*a**2*c**3*sqrt(-1/(4*a*c - b**
2)**3) + 16*a*b**2*c**2*sqrt(-1/(4*a*c - b**2)**3) - 2*b**4*c*sqrt(-1/(4*a*
c - b**2)**3) + 2*b*c)/(4*c**2)) + 2*c*sqrt(-1/(4*a*c - b**2)**3)*log(x + (
32*a**2*c**3*sqrt(-1/(4*a*c - b**2)**3) - 16*a*b**2*c**2*sqrt(-1/(4*a*c - b
**2)**3) + 2*b**4*c*sqrt(-1/(4*a*c - b**2)**3) + 2*b*c)/(4*c**2)) + (b + 2*
c*x)/(4*a**2*c - a*b**2 + x**2*(4*a*c**2 - b**2*c) + x*(4*a*b*c - b**3))
```

3.24 $\int \frac{x^3}{(ax^2+bx^3+cx^4)^2} dx$

Optimal. Leaf size=108

$$\frac{b(b^2-6ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2(b^2-4ac)^{3/2}}-\frac{\log(a+bx+cx^2)}{2a^2}+\frac{\log(x)}{a^2}+\frac{-2ac+b^2+bcx}{a(b^2-4ac)(a+bx+cx^2)}$$

```
[Out] (b*c*x-2*a*c+b^2)/a/(-4*a*c+b^2)/(c*x^2+b*x+a)+b*(-6*a*c+b^2)*arctanh((2*c*
x+b)/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^(3/2)+ln(x)/a^2-1/2*ln(c*x^2+b*x+
a)/a^2
```

Rubi [A] time = 0.15, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {1585, 740, 800, 634, 618, 206, 628}

$$\frac{b(b^2-6ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2(b^2-4ac)^{3/2}}-\frac{\log(a+bx+cx^2)}{2a^2}+\frac{\log(x)}{a^2}+\frac{-2ac+b^2+bcx}{a(b^2-4ac)(a+bx+cx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] (b^2 - 2*a*c + b*c*x)/(a*(b^2 - 4*a*c)*(a + b*x + c*x^2)) + (b*(b^2 - 6*a*c
)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^2*(b^2 - 4*a*c)^(3/2)) + Log[x
]/a^2 - Log[a + b*x + c*x^2]/(2*a^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
```

```
e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\int \frac{x^3}{\left(ax^2+bx^3+cx^4\right)^2}\,dx = \int \frac{1}{x\left(a+bx+cx^2\right)^2}\,dx$$

$$= \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)\left(a+bx+cx^2\right)} - \frac{\int \frac{-b^2+4ac-bcx}{x\left(a+bx+cx^2\right)}\,dx}{a\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)\left(a+bx+cx^2\right)} - \frac{\int \left(\frac{-b^2+4ac}{ax}+\frac{b\left(b^2-5ac\right)+c\left(b^2-4ac\right)x}{a\left(a+bx+cx^2\right)}\right)dx}{a\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)\left(a+bx+cx^2\right)} + \frac{\log(x)}{a^2} - \frac{\int \frac{b\left(b^2-5ac\right)+c\left(b^2-4ac\right)x}{a+bx+cx^2}\,dx}{a^2\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)\left(a+bx+cx^2\right)} + \frac{\log(x)}{a^2} - \frac{\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2a^2} - \frac{\left(b\left(b^2-6ac\right)\right)\int \frac{1}{a+bx+cx^2}\,d}{2a^2\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)\left(a+bx+cx^2\right)} + \frac{\log(x)}{a^2} - \frac{\log\left(a+bx+cx^2\right)}{2a^2} + \frac{\left(b\left(b^2-6ac\right)\right)\text{Subst}}{a^2(}$$

$$= \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)\left(a+bx+cx^2\right)} + \frac{b\left(b^2-6ac\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2\left(b^2-4ac\right)^{3/2}} + \frac{\log(x)}{a^2} - \frac{\log\left(a+b\right.}{2a}$$

Mathematica [A] time = 0.18, size = 107, normalized size = 0.99

$$\frac{\frac{2a\left(-2ac+b^2+bcx\right)}{\left(b^2-4ac\right)\left(a+x(b+cx)\right)} + \frac{2b\left(b^2-6ac\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{3/2}} - \log(a+x(b+cx)) + 2\log(x)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] ((2*a*(b^2 - 2*a*c + b*c*x))/((b^2 - 4*a*c)*(a + x*(b + c*x))) + (2*b*(b^2
- 6*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + 2*L
og[x] - Log[a + x*(b + c*x)])/(2*a^2)
```

fricas [B] time = 0.55, size = 781, normalized size = 7.23

$$\left[\frac{2\,ab^4 - 12\,a^2b^2c + 16\,a^3c^2 + \left(ab^3 - 6\,a^2bc + \left(b^3c - 6\,abc^2\right)x^2 + \left(b^4 - 6\,ab^2c\right)x\right)\sqrt{b^2-4\,ac}\,\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\cdots}{cx^2+bx\cdots}\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + (a*b^3 - 6*a^2*b*c + (b^3*c - 6
*a*b*c^2)*x^2 + (b^4 - 6*a*b^2*c)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b
*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 2*
(a*b^3*c - 4*a^2*b*c^2)*x - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*
a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)*log(c*x^2
 + b*x + a) + 2*(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 +
16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)*log(x))/(a^3*b^4 - 8*
a^4*b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*x^2 + (a^
2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x), 1/2*(2*a*b^4 - 12*a^2*b^2*c + 16*a^
3*c^2 + 2*(a*b^3 - 6*a^2*b*c + (b^3*c - 6*a*b*c^2)*x^2 + (b^4 - 6*a*b^2*c)*
x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c))
 + 2*(a*b^3*c - 4*a^2*b*c^2)*x - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c
 - 8*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)*log(
c*x^2 + b*x + a) + 2*(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c
^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)*log(x))/(a^3*b^4
 - 8*a^4*b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*x^2
+ (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x)]
```

giac [A] time = 0.51, size = 126, normalized size = 1.17

$$-\frac{\left(b^3 - 6\,abc\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(a^2b^2 - 4\,a^3c\right)\sqrt{-b^2+4\,ac}} - \frac{\log\left(cx^2+bx+a\right)}{2\,a^2} + \frac{\log\left(|x|\right)}{a^2} + \frac{abcx + ab^2 - 2\,a^2c}{\left(cx^2+bx+a\right)\left(b^2-4\,ac\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")
```

```
[Out] -(b^3 - 6*a*b*c)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((a^2*b^2 - 4*a^3*c
)*sqrt(-b^2 + 4*a*c)) - 1/2*log(c*x^2 + b*x + a)/a^2 + log(abs(x))/a^2 + (a
*b*c*x + a*b^2 - 2*a^2*c)/((c*x^2 + b*x + a)*(b^2 - 4*a*c)*a^2)
```

maple [B] time = 0.01, size = 237, normalized size = 2.19

$$-\frac{bcx}{\left(c\,x^2+bx+a\right)\left(4ac-b^2\right)a} - \frac{6bc\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}a} + \frac{b^3\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}a^2} - \frac{b^2}{\left(c\,x^2+bx+a\right)\left(4ac-b^2\right)a} - \frac{2c\ln\left(c\,x\cdots\right.}{\left(4ac\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^4+b*x^3+a*x^2)^2,x)
```

```
[Out] 1/a^2*ln(x)-1/a/(c*x^2+b*x+a)*b*c/(4*a*c-b^2)*x+2/(c*x^2+b*x+a)/(4*a*c-b^2)
*c-1/a/(c*x^2+b*x+a)/(4*a*c-b^2)*b^2-2/a/(4*a*c-b^2)*c*ln(c*x^2+b*x+a)+1/2/
a^2/(4*a*c-b^2)*ln(c*x^2+b*x+a)*b^2-6/a/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/
```

```
(4*a*c-b^2)^(1/2))*b*c+1/a^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)
^(1/2))*b^3
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 2.87, size = 620, normalized size = 5.74

$$\frac{\ln(x)}{a^2}+\frac{\frac{2\,a\,c-b^2}{a\,\left(4\,a\,c-b^2\right)}-\frac{b\,c\,x}{a\,\left(4\,a\,c-b^2\right)}}{c\,x^2+b\,x+a}+\frac{\ln\left(2\,a\,b^6+2\,b^7\,x-96\,a^4\,c^3+2\,a\,b^3\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-23\,a^2\,b^4\,c+2\,b^4\,x\,\sqrt{-\left(4\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*x^2 + b*x^3 + c*x^4)^2,x)
```

```
[Out] log(x)/a^2 + ((2*a*c - b^2)/(a*(4*a*c - b^2)) - (b*c*x)/(a*(4*a*c - b^2)))/
(a + b*x + c*x^2) + (log(2*a*b^6 + 2*b^7*x - 96*a^4*c^3 + 2*a*b^3*(-(4*a*c
- b^2)^3)^(1/2) - 23*a^2*b^4*c + 2*b^4*x*(-(4*a*c - b^2)^3)^(1/2) + 84*a^3*
b^2*c^2 + 94*a^2*b^3*c^2*x + 12*a^2*c^2*x*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b
^5*c*x - 9*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2) - 120*a^3*b*c^3*x - 12*a*b^2*c*
x*(-(4*a*c - b^2)^3)^(1/2))*(b^6 - 64*a^3*c^3 + b^3*(-(4*a*c - b^2)^3)^(1/2
) + 48*a^2*b^2*c^2 - 12*a*b^4*c - 6*a*b*c*(-(4*a*c - b^2)^3)^(1/2)))/(2*a^2
*(4*a*c - b^2)^3) + (log(96*a^4*c^3 - 2*b^7*x - 2*a*b^6 + 2*a*b^3*(-(4*a*c
- b^2)^3)^(1/2) + 23*a^2*b^4*c + 2*b^4*x*(-(4*a*c - b^2)^3)^(1/2) - 84*a^3*
b^2*c^2 - 94*a^2*b^3*c^2*x + 12*a^2*c^2*x*(-(4*a*c - b^2)^3)^(1/2) + 24*a*b
^5*c*x - 9*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2) + 120*a^3*b*c^3*x - 12*a*b^2*c*
x*(-(4*a*c - b^2)^3)^(1/2))*(b^6 - 64*a^3*c^3 - b^3*(-(4*a*c - b^2)^3)^(1/2
) + 48*a^2*b^2*c^2 - 12*a*b^4*c + 6*a*b*c*(-(4*a*c - b^2)^3)^(1/2)))/(2*a^2
*(4*a*c - b^2)^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**4+b*x**3+a*x**2)**2,x)
```

```
[Out] Timed out
```

3.25 $\int \frac{x^2}{\left(ax^2+bx^3+cx^4\right)^2} \, dx$

Optimal. Leaf size=148

$$\frac{b\log\left(a+bx+cx^2\right)}{a^3}-\frac{2b\log(x)}{a^3}-\frac{2\left(b^2-3ac\right)}{a^2x\left(b^2-4ac\right)}-\frac{2\left(6a^2c^2-6ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3\left(b^2-4ac\right)^{3/2}}+\frac{-2ac+b^2+bc}{ax\left(b^2-4ac\right)\left(a+bx\right.}$$

```
[Out] -2*(-3*a*c+b^2)/a^2/(-4*a*c+b^2)/x+(b*c*x-2*a*c+b^2)/a/(-4*a*c+b^2)/x/(c*x^
2+b*x+a)-2*(6*a^2*c^2-6*a*b^2*c+b^4)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/
a^3/(-4*a*c+b^2)^(3/2)-2*b*ln(x)/a^3+b*ln(c*x^2+b*x+a)/a^3
```

Rubi [A] time = 0.20, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {1585, 740, 800, 634, 618, 206, 628}

$$-\frac{2\left(6a^2c^2-6ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3\left(b^2-4ac\right)^{3/2}}-\frac{2\left(b^2-3ac\right)}{a^2x\left(b^2-4ac\right)}+\frac{b\log\left(a+bx+cx^2\right)}{a^3}-\frac{2b\log(x)}{a^3}+\frac{-2ac+b^2+b}{ax\left(b^2-4ac\right)\left(a+b\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] (-2*(b^2 - 3*a*c))/(a^2*(b^2 - 4*a*c)*x) + (b^2 - 2*a*c + b*c*x)/(a*(b^2 -
4*a*c)*x*(a + b*x + c*x^2)) - (2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTanh[(b +
 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^3*(b^2 - 4*a*c)^(3/2)) - (2*b*Log[x])/a^3 +
(b*Log[a + b*x + c*x^2])/a^3
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
```

```
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\int \frac{x^2}{\left(ax^2+bx^3+cx^4\right)^2}\,dx = \int \frac{1}{x^2\left(a+bx+cx^2\right)^2}\,dx$$

$$= \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x\left(a+bx+cx^2\right)} - \frac{\int \frac{-2\left(b^2-3ac\right)-2bcx}{x^2\left(a+bx+cx^2\right)}\,dx}{a\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x\left(a+bx+cx^2\right)} - \frac{\int \left(\frac{2\left(-b^2+3ac\right)}{ax^2} - \frac{2b\left(-b^2+4ac\right)}{a^2x} + \frac{2\left(-b^4+5ab^2c-3a^2c^2-bc\left(b^2-\cdots\right.\right.}{a^2\left(a+bx+cx^2\right)}\cdots\right.}{a\left(b^2-4ac\right)}$$

$$= -\frac{2\left(b^2-3ac\right)}{a^2\left(b^2-4ac\right)x} + \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x\left(a+bx+cx^2\right)} - \frac{2b\log(x)}{a^3} - \frac{2\int \frac{-b^4+5ab^2c-3a^2c^2-\cdots}{a+bx+c\cdots}}{a^3\left(b^2-4\cdots\right.}$$

$$= -\frac{2\left(b^2-3ac\right)}{a^2\left(b^2-4ac\right)x} + \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x\left(a+bx+cx^2\right)} - \frac{2b\log(x)}{a^3} + \frac{b\int \frac{b+2cx}{a+bx+cx^2}\,dx}{a^3} + \cdots$$

$$= -\frac{2\left(b^2-3ac\right)}{a^2\left(b^2-4ac\right)x} + \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x\left(a+bx+cx^2\right)} - \frac{2b\log(x)}{a^3} + \frac{b\log\left(a+bx+cx^{2}\cdots\right.}{a^3}$$

$$= -\frac{2\left(b^2-3ac\right)}{a^2\left(b^2-4ac\right)x} + \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x\left(a+bx+cx^2\right)} - \frac{2\left(b^4-6ab^2c+6a^2c^2\right)\tanh^{-1}\left(\cdots\right.}{a^3\left(b^2-4ac\right)^{3/2}}$$

Mathematica [A] time = 0.26, size = 131, normalized size = 0.89

$$-\frac{\frac{2\left(6a^2c^2-6ab^2c+b^4\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{3/2}} + \frac{a\left(-3abc-2ac^2x+b^3+b^2cx\right)}{\left(b^2-4ac\right)\left(a+x(b+cx)\right)} - b\log(a+x(b+cx)) + \frac{a}{x} + 2b\log(x)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] -((a/x + (a*(b^3 - 3*a*b*c + b^2*c*x - 2*a*c^2*x))/((b^2 - 4*a*c)*(a + x*(b
 + c*x))) + (2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 +
 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + 2*b*Log[x] - b*Log[a + x*(b + c*x)])/a^3)
```

fricas [B] time = 0.86, size = 975, normalized size = 6.59

$$\left[-\frac{a^2b^4 - 8\,a^3b^2c + 16\,a^4c^2 + 2\left(ab^4c - 7\,a^2b^2c^2 + 12\,a^3c^3\right)x^2 + \left(\left(b^4c - 6\,ab^2c^2 + 6\,a^2c^3\right)x^3 + \left(b^5 - 6\,ab^3c + 6\,a^2bc\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")
```

```
[Out] [-(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + 2*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3
*c^3)*x^2 + ((b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*x^3 + (b^5 - 6*a*b^3*c + 6*a
^2*b*c^2)*x^2 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*x)*sqrt(b^2 - 4*a*c)*log(
(2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2
+ b*x + a)) + (2*a*b^5 - 15*a^2*b^3*c + 28*a^3*b*c^2)*x - ((b^5*c - 8*a*b^3
*c^2 + 16*a^2*b*c^3)*x^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*x^2 + (a*b^5
- 8*a^2*b^3*c + 16*a^3*b*c^2)*x)*log(c*x^2 + b*x + a) + 2*((b^5*c - 8*a*b^3
*c^2 + 16*a^2*b*c^3)*x^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*x^2 + (a*b^5
- 8*a^2*b^3*c + 16*a^3*b*c^2)*x)*log(x))/((a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a
^5*c^3)*x^3 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^2 + (a^4*b^4 - 8*a^5
*b^2*c + 16*a^6*c^2)*x), -(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + 2*(a*b^4*c
- 7*a^2*b^2*c^2 + 12*a^3*c^3)*x^2 + 2*((b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*x^
3 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*x^2 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)
*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)
) + (2*a*b^5 - 15*a^2*b^3*c + 28*a^3*b*c^2)*x - ((b^5*c - 8*a*b^3*c^2 + 16*
a^2*b*c^3)*x^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*x^2 + (a*b^5 - 8*a^2*b^
3*c + 16*a^3*b*c^2)*x)*log(c*x^2 + b*x + a) + 2*((b^5*c - 8*a*b^3*c^2 + 16*
a^2*b*c^3)*x^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*x^2 + (a*b^5 - 8*a^2*b^
3*c + 16*a^3*b*c^2)*x)*log(x))/((a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*x^
3 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^2 + (a^4*b^4 - 8*a^5*b^2*c + 1
6*a^6*c^2)*x)]
```

giac [A] time = 0.45, size = 171, normalized size = 1.16

$$\frac{2\left(b^4 - 6\,ab^2c + 6\,a^2c^2\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(a^3b^2 - 4\,a^4c\right)\sqrt{-b^2 + 4\,ac}} - \frac{2\,b^2cx^2 - 6\,ac^2x^2 + 2\,b^3x - 7\,abcx + ab^2 - 4\,a^2c}{\left(a^2b^2 - 4\,a^3c\right)\left(cx^3 + bx^2 + ax\right)} + \frac{b\log\left(cx^2 + bx + a\right.}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")
```

```
[Out] 2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((a^
3*b^2 - 4*a^4*c)*sqrt(-b^2 + 4*a*c)) - (2*b^2*c*x^2 - 6*a*c^2*x^2 + 2*b^3*x
 - 7*a*b*c*x + a*b^2 - 4*a^2*c)/((a^2*b^2 - 4*a^3*c)*(c*x^3 + b*x^2 + a*x))
 + b*log(c*x^2 + b*x + a)/a^3 - 2*b*log(abs(x))/a^3
```

maple [B] time = 0.02, size = 328, normalized size = 2.22

$$-\frac{2c^2x}{\left(c\,x^2 + bx + a\right)\left(4ac - b^2\right)a} - \frac{12c^2\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac - b^2\right)^{\frac{3}{2}}a} + \frac{b^2cx}{\left(c\,x^2 + bx + a\right)\left(4ac - b^2\right)a^2} + \frac{12b^2c\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac - b^2\right)^{\frac{3}{2}}a^2} - \frac{2b^4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^4+b*x^3+a*x^2)^2,x)
```

```
[Out] -1/a^2/x-2/a^3*b*ln(x)-2/a/(c*x^2+b*x+a)*c^2/(4*a*c-b^2)*x+1/a^2/(c*x^2+b*x
+a)*c/(4*a*c-b^2)*x*b^2-3/a/(c*x^2+b*x+a)*b/(4*a*c-b^2)*c+1/a^2/(c*x^2+b*x+
a)*b^3/(4*a*c-b^2)+4/a^2/(4*a*c-b^2)*c*ln(c*x^2+b*x+a)*b-1/a^3/(4*a*c-b^2)*
ln(c*x^2+b*x+a)*b^3-12/a/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/
2))*c^2+12/a^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^2*c-
2/a^3/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^4
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [**B**] time = 2.83, size = 775, normalized size = 5.24

$$\ln\Bigg(2\,a\,b^7+2\,b^8\,x+2\,a\,b^4\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-23\,a^2\,b^5\,c-108\,a^4\,b\,c^3+24\,a^4\,c^4\,x+2\,b^5\,x\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}+87$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x^2 + b*x^3 + c*x^4)^2,x)
```

```
[Out] log(2*a*b^7 + 2*b^8*x + 2*a*b^4*(-(4*a*c - b^2)^3)^(1/2) - 23*a^2*b^5*c - 1
08*a^4*b*c^3 + 24*a^4*c^4*x + 2*b^5*x*(-(4*a*c - b^2)^3)^(1/2) + 87*a^3*b^3
*c^2 + 3*a^3*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^2*c*(-(4*a*c - b^2)^3)^
(1/2) + 97*a^2*b^4*c^2*x - 138*a^3*b^2*c^3*x - 24*a*b^6*c*x - 12*a*b^3*c*x*
(-(4*a*c - b^2)^3)^(1/2) + 15*a^2*b*c^2*x*(-(4*a*c - b^2)^3)^(1/2))*((b^4*(
-(4*a*c - b^2)^3)^(1/2) + 6*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*(-
(4*a*c - b^2)^3)^(1/2))/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c
^2) + b/a^3) - (1/a - (x*(2*b^3 - 7*a*b*c))/(a^2*(4*a*c - b^2)) + (2*c*x^2*
(3*a*c - b^2))/(a^2*(4*a*c - b^2)))/(a*x + b*x^2 + c*x^3) - log(2*a*b^4*(-(
4*a*c - b^2)^3)^(1/2) - 2*b^8*x - 2*a*b^7 + 23*a^2*b^5*c + 108*a^4*b*c^3 -
24*a^4*c^4*x + 2*b^5*x*(-(4*a*c - b^2)^3)^(1/2) - 87*a^3*b^3*c^2 + 3*a^3*c^
2*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^2*c*(-(4*a*c - b^2)^3)^(1/2) - 97*a^2*
b^4*c^2*x + 138*a^3*b^2*c^3*x + 24*a*b^6*c*x - 12*a*b^3*c*x*(-(4*a*c - b^2)
^3)^(1/2) + 15*a^2*b*c^2*x*(-(4*a*c - b^2)^3)^(1/2))*((b^4*(-(4*a*c - b^2)^
3)^(1/2) + 6*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*(-(4*a*c - b^2)^3
)^(1/2))/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) - b/a^3) -
(2*b*log(x))/a^3
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**4+b*x**3+a*x**2)**2,x)
```

```
[Out] Timed out
```

3.26 $\int \frac{x}{(ax^2+bx^3+cx^4)^2}\,dx$

Optimal. Leaf size=202

$$-\frac{(3b^2-2ac)\log(a+bx+cx^2)}{2a^4}+\frac{\log(x)(3b^2-2ac)}{a^4}+\frac{b(3b^2-11ac)}{a^3x(b^2-4ac)}-\frac{3b^2-8ac}{2a^2x^2(b^2-4ac)}+\frac{b(30a^2c^2-20ab^2c+3b}{a^4(b^2-4}$$

```
[Out] 1/2*(8*a*c-3*b^2)/a^2/(-4*a*c+b^2)/x^2+b*(-11*a*c+3*b^2)/a^3/(-4*a*c+b^2)/x
+(b*c*x-2*a*c+b^2)/a/(-4*a*c+b^2)/x^2/(c*x^2+b*x+a)+b*(30*a^2*c^2-20*a*b^2*
c+3*b^4)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/a^4/(-4*a*c+b^2)^(3/2)+(-2*a
*c+3*b^2)*ln(x)/a^4-1/2*(-2*a*c+3*b^2)*ln(c*x^2+b*x+a)/a^4
```

Rubi [A] time = 0.25, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1585, 740, 800, 634, 618, 206, 628}

$$\frac{b(30a^2c^2-20ab^2c+3b^4)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^4(b^2-4ac)^{3/2}}-\frac{3b^2-8ac}{2a^2x^2(b^2-4ac)}-\frac{(3b^2-2ac)\log(a+bx+cx^2)}{2a^4}+\frac{b(3b^2-11ac)}{a^3x(b^2-4ac)}+$$

Antiderivative was successfully verified.

```
[In] Int[x/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] -(3*b^2 - 8*a*c)/(2*a^2*(b^2 - 4*a*c)*x^2) + (b*(3*b^2 - 11*a*c))/(a^3*(b^2
 - 4*a*c)*x) + (b^2 - 2*a*c + b*c*x)/(a*(b^2 - 4*a*c)*x^2*(a + b*x + c*x^2)
) + (b*(3*b^4 - 20*a*b^2*c + 30*a^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a
*c]])/(a^4*(b^2 - 4*a*c)^(3/2)) + ((3*b^2 - 2*a*c)*Log[x])/a^4 - ((3*b^2 -
2*a*c)*Log[a + b*x + c*x^2])/(2*a^4)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
```

```
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\int \frac{x}{\left(ax^2+bx^3+cx^4\right)^2}\,dx = \int \frac{1}{x^3\left(a+bx+cx^2\right)^2}\,dx$$

$$= \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x^2\left(a+bx+cx^2\right)} - \frac{\int \frac{-3b^2+8ac-3bcx}{x^3\left(a+bx+cx^2\right)}\,dx}{a\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x^2\left(a+bx+cx^2\right)} - \frac{\int \left(\frac{-3b^2+8ac}{ax^3} + \frac{3b^3-11abc}{a^2x^2} + \frac{\left(b^2-4ac\right)\left(-3b^2+2ac\right)}{a^3x} + \frac{b\left(3b^4-\cdots\right.}{\cdots}\right.}{a\left(b^2-4ac\right)}$$

$$= -\frac{3b^2-8ac}{2a^2\left(b^2-4ac\right)x^2} + \frac{b\left(3b^2-11ac\right)}{a^3\left(b^2-4ac\right)x} + \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x^2\left(a+bx+cx^2\right)} + \frac{\left(3b^2-2a\cdots\right.}{a^{\cdots}}$$

$$= -\frac{3b^2-8ac}{2a^2\left(b^2-4ac\right)x^2} + \frac{b\left(3b^2-11ac\right)}{a^3\left(b^2-4ac\right)x} + \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x^2\left(a+bx+cx^2\right)} + \frac{\left(3b^2-2a\cdots\right.}{a^{\cdots}}$$

$$= -\frac{3b^2-8ac}{2a^2\left(b^2-4ac\right)x^2} + \frac{b\left(3b^2-11ac\right)}{a^3\left(b^2-4ac\right)x} + \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x^2\left(a+bx+cx^2\right)} + \frac{\left(3b^2-2a\cdots\right.}{a^{\cdots}}$$

$$= -\frac{3b^2-8ac}{2a^2\left(b^2-4ac\right)x^2} + \frac{b\left(3b^2-11ac\right)}{a^3\left(b^2-4ac\right)x} + \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x^2\left(a+bx+cx^2\right)} + \frac{b\left(3b^4-2\cdots\right.}{\cdots}$$

Mathematica [A] time = 0.34, size = 175, normalized size = 0.87

$$\frac{\frac{2b\left(30a^2c^2-20ab^2c+3b^4\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{3/2}} + \frac{2a\left(2a^2c^2-4ab^2c-3abc^2x+b^4+b^3cx\right)}{\left(b^2-4ac\right)\left(a+x(b+cx)\right)} - \frac{a^2}{x^2} + 2\log(x)\left(3b^2-2ac\right) + \left(2ac-3b^2\right)\log(a\cdots}{2a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] (-(a^2/x^2) + (4*a*b)/x + (2*a*(b^4 - 4*a*b^2*c + 2*a^2*c^2 + b^3*c*x - 3*a
*b*c^2*x))/((b^2 - 4*a*c)*(a + x*(b + c*x))) + (2*b*(3*b^4 - 20*a*b^2*c + 3
0*a^2*c^2)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + 2
*(3*b^2 - 2*a*c)*Log[x] + (-3*b^2 + 2*a*c)*Log[a + x*(b + c*x)])/(2*a^4)
```

fricas [B] time = 1.03, size = 1226, normalized size = 6.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 - 2*(3*a*b^5*c - 23*a^2*b^3*c^2 +
 44*a^3*b*c^3)*x^3 - (6*a*b^6 - 49*a^2*b^4*c + 108*a^3*b^2*c^2 - 32*a^4*c^3
)*x^2 + ((3*b^5*c - 20*a*b^3*c^2 + 30*a^2*b*c^3)*x^4 + (3*b^6 - 20*a*b^4*c
+ 30*a^2*b^2*c^2)*x^3 + (3*a*b^5 - 20*a^2*b^3*c + 30*a^3*b*c^2)*x^2)*sqrt(b
^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c
*x + b))/(c*x^2 + b*x + a)) - 3*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x +
((3*b^6*c - 26*a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*x^4 + (3*b^7 - 26*a
*b^5*c + 64*a^2*b^3*c^2 - 32*a^3*b*c^3)*x^3 + (3*a*b^6 - 26*a^2*b^4*c + 64*
a^3*b^2*c^2 - 32*a^4*c^3)*x^2)*log(c*x^2 + b*x + a) - 2*((3*b^6*c - 26*a*b^
4*c^2 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*x^4 + (3*b^7 - 26*a*b^5*c + 64*a^2*b^3
*c^2 - 32*a^3*b*c^3)*x^3 + (3*a*b^6 - 26*a^2*b^4*c + 64*a^3*b^2*c^2 - 32*a^
4*c^3)*x^2)*log(x))/((a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*x^4 + (a^4*b^
5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*x^3 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*
x^2), -1/2*(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 - 2*(3*a*b^5*c - 23*a^2*b^3*
c^2 + 44*a^3*b*c^3)*x^3 - (6*a*b^6 - 49*a^2*b^4*c + 108*a^3*b^2*c^2 - 32*a^
4*c^3)*x^2 - 2*((3*b^5*c - 20*a*b^3*c^2 + 30*a^2*b*c^3)*x^4 + (3*b^6 - 20*a
*b^4*c + 30*a^2*b^2*c^2)*x^3 + (3*a*b^5 - 20*a^2*b^3*c + 30*a^3*b*c^2)*x^2)
*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) -
 3*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x + ((3*b^6*c - 26*a*b^4*c^2 + 64
*a^2*b^2*c^3 - 32*a^3*c^4)*x^4 + (3*b^7 - 26*a*b^5*c + 64*a^2*b^3*c^2 - 32*
a^3*b*c^3)*x^3 + (3*a*b^6 - 26*a^2*b^4*c + 64*a^3*b^2*c^2 - 32*a^4*c^3)*x^2
)*log(c*x^2 + b*x + a) - 2*((3*b^6*c - 26*a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*a
^3*c^4)*x^4 + (3*b^7 - 26*a*b^5*c + 64*a^2*b^3*c^2 - 32*a^3*b*c^3)*x^3 + (3
*a*b^6 - 26*a^2*b^4*c + 64*a^3*b^2*c^2 - 32*a^4*c^3)*x^2)*log(x))/((a^4*b^4
*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*x^4 + (a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^
2)*x^3 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*x^2)]
```

giac [A] time = 0.44, size = 229, normalized size = 1.13

$$-\frac{\left(3\,b^{5}-20\,ab^{3}c+30\,a^{2}bc^{2}\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^{2}+4\,ac}}\right)}{\left(a^{4}b^{2}-4\,a^{5}c\right)\sqrt{-b^{2}+4\,ac}}-\frac{\left(3\,b^{2}-2\,ac\right)\log\left(cx^{2}+bx+a\right)}{2\,a^{4}}+\frac{\left(3\,b^{2}-2\,ac\right)\log\left(|x|\right)}{a^{4}}-\frac{a^{3}b^{2}-\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")
```

```
[Out] -(3*b^5 - 20*a*b^3*c + 30*a^2*b*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))
/((a^4*b^2 - 4*a^5*c)*sqrt(-b^2 + 4*a*c)) - 1/2*(3*b^2 - 2*a*c)*log(c*x^2 +
 b*x + a)/a^4 + (3*b^2 - 2*a*c)*log(abs(x))/a^4 - 1/2*(a^3*b^2 - 4*a^4*c -
2*(3*a*b^3*c - 11*a^2*b*c^2)*x^3 - (6*a*b^4 - 25*a^2*b^2*c + 8*a^3*c^2)*x^2
 - 3*(a^2*b^3 - 4*a^3*b*c)*x)/((c*x^2 + b*x + a)*(b^2 - 4*a*c)*a^4*x^2)
```

maple [B] time = 0.02, size = 418, normalized size = 2.07

$$\frac{3bc^{2}x}{\left(cx^{2}+bx+a\right)\left(4ac-b^{2}\right)a^{2}}+\frac{30bc^{2}\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^{2}}}\right)}{\left(4ac-b^{2}\right)^{\frac{3}{2}}a^{2}}-\frac{b^{3}cx}{\left(cx^{2}+bx+a\right)\left(4ac-b^{2}\right)a^{3}}-\frac{20b^{3}c\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^{2}}}\right)}{\left(4ac-b^{2}\right)^{\frac{3}{2}}a^{3}}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^4+b*x^3+a*x^2)^2,x)
```

```
[Out] -1/2/a^2/x^2-2/a^3*ln(x)*c+3/a^4*ln(x)*b^2+2/a^3*b/x+3/a^2/(c*x^2+b*x+a)*b*
c^2/(4*a*c-b^2)*x-1/a^3/(c*x^2+b*x+a)*b^3*c/(4*a*c-b^2)*x-2/a/(c*x^2+b*x+a)
/(4*a*c-b^2)*c^2+4/a^2/(c*x^2+b*x+a)/(4*a*c-b^2)*b^2*c-1/a^3/(c*x^2+b*x+a)/
(4*a*c-b^2)*b^4+4/a^2/(4*a*c-b^2)*c^2*ln(c*x^2+b*x+a)-7/a^3/(4*a*c-b^2)*c*l
n(c*x^2+b*x+a)*b^2+3/2/a^4/(4*a*c-b^2)*ln(c*x^2+b*x+a)*b^4+30/a^2/(4*a*c-b^
2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*c^2-20/a^3/(4*a*c-b^2)^(3/2)
*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^3*c+3/a^4/(4*a*c-b^2)^(3/2)*arctan((
2*c*x+b)/(4*a*c-b^2)^(1/2))*b^5
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 2.96, size = 914, normalized size = 4.52

$$\ln\left(6\,a\,b^8+6\,b^9\,x+192\,a^5\,c^4-6\,a\,b^5\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-73\,a^2\,b^6\,c-6\,b^6\,x\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}+307\,a^3\,b^4\,c^2-49$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x^2 + b*x^3 + c*x^4)^2,x)
```

```
[Out] (log(6*a*b^8 + 6*b^9*x + 192*a^5*c^4 - 6*a*b^5*(-(4*a*c - b^2)^3)^(1/2) - 7
3*a^2*b^6*c - 6*b^6*x*(-(4*a*c - b^2)^3)^(1/2) + 307*a^3*b^4*c^2 - 492*a^4*
b^2*c^3 + 31*a^2*b^3*c*(-(4*a*c - b^2)^3)^(1/2) - 27*a^3*b*c^2*(-(4*a*c - b
^2)^3)^(1/2) + 339*a^2*b^5*c^2*x - 602*a^3*b^3*c^3*x + 24*a^3*c^3*x*(-(4*a*
c - b^2)^3)^(1/2) - 76*a*b^7*c*x + 312*a^4*b*c^4*x + 40*a*b^4*c*x*(-(4*a*c
- b^2)^3)^(1/2) - 69*a^2*b^2*c^2*x*(-(4*a*c - b^2)^3)^(1/2))*(3*b^8 + 128*a
^4*c^4 - 3*b^5*(-(4*a*c - b^2)^3)^(1/2) + 168*a^2*b^4*c^2 - 288*a^3*b^2*c^3
 - 38*a*b^6*c - 30*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) + 20*a*b^3*c*(-(4*a*c
 - b^2)^3)^(1/2)))/(2*a^4*(4*a*c - b^2)^3) - (log(x)*(2*a*c - 3*b^2))/a^4 -
 (1/(2*a) - (3*b*x)/(2*a^2) + (x^2*(6*b^4 + 8*a^2*c^2 - 25*a*b^2*c))/(2*a^3
*(4*a*c - b^2)) - (b*c*x^3*(11*a*c - 3*b^2))/(a^3*(4*a*c - b^2)))/(a*x^2 +
b*x^3 + c*x^4) + (log(6*a*b^8 + 6*b^9*x + 192*a^5*c^4 + 6*a*b^5*(-(4*a*c -
b^2)^3)^(1/2) - 73*a^2*b^6*c + 6*b^6*x*(-(4*a*c - b^2)^3)^(1/2) + 307*a^3*b
^4*c^2 - 492*a^4*b^2*c^3 - 31*a^2*b^3*c*(-(4*a*c - b^2)^3)^(1/2) + 27*a^3*b
*c^2*(-(4*a*c - b^2)^3)^(1/2) + 339*a^2*b^5*c^2*x - 602*a^3*b^3*c^3*x - 24*
a^3*c^3*x*(-(4*a*c - b^2)^3)^(1/2) - 76*a*b^7*c*x + 312*a^4*b*c^4*x - 40*a*
b^4*c*x*(-(4*a*c - b^2)^3)^(1/2) + 69*a^2*b^2*c^2*x*(-(4*a*c - b^2)^3)^(1/2
))*(3*b^8 + 128*a^4*c^4 + 3*b^5*(-(4*a*c - b^2)^3)^(1/2) + 168*a^2*b^4*c^2
- 288*a^3*b^2*c^3 - 38*a*b^6*c + 30*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) - 20
*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2)))/(2*a^4*(4*a*c - b^2)^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**4+b*x**3+a*x**2)**2,x)

[Out] Timed out
```

3.27 $\int \frac{1}{\left(ax^2+bx^3+cx^4\right)^2}\,dx$

Optimal. Leaf size=252

$$\frac{b\left(2b^2-3ac\right)\log\left(a+bx+cx^2\right)}{a^5}-\frac{2b\log(x)\left(2b^2-3ac\right)}{a^5}+\frac{b\left(2b^2-7ac\right)}{a^3x^2\left(b^2-4ac\right)}-\frac{2\left(2b^2-5ac\right)}{3a^2x^3\left(b^2-4ac\right)}-\frac{2\left(5a^2c^2-9ab^2c\right.}{a^4x\left(b^2-4a\right.}$$

```
[Out] -2/3*(-5*a*c+2*b^2)/a^2/(-4*a*c+b^2)/x^3+b*(-7*a*c+2*b^2)/a^3/(-4*a*c+b^2)/
x^2-2*(5*a^2*c^2-9*a*b^2*c+2*b^4)/a^4/(-4*a*c+b^2)/x+(b*c*x-2*a*c+b^2)/a/(-
4*a*c+b^2)/x^3/(c*x^2+b*x+a)-2*(-10*a^3*c^3+30*a^2*b^2*c^2-15*a*b^4*c+2*b^6
)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/a^5/(-4*a*c+b^2)^(3/2)-2*b*(-3*a*c+
2*b^2)*ln(x)/a^5+b*(-3*a*c+2*b^2)*ln(c*x^2+b*x+a)/a^5
```

Rubi [A] time = 0.32, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {1594, 740, 800, 634, 618, 206, 628}

$$-\frac{2\left(5a^2c^2-9ab^2c+2b^4\right)}{a^4x\left(b^2-4ac\right)}-\frac{2\left(30a^2b^2c^2-10a^3c^3-15ab^4c+2b^6\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^5\left(b^2-4ac\right)^{3/2}}+\frac{b\left(2b^2-7ac\right)}{a^3x^2\left(b^2-4ac\right)}-\frac{2\left(2b^2\right.}{3a^2x^3\left(b\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3 + c*x^4)^(-2),x]
```

```
[Out] (-2*(2*b^2 - 5*a*c))/(3*a^2*(b^2 - 4*a*c)*x^3) + (b*(2*b^2 - 7*a*c))/(a^3*(
b^2 - 4*a*c)*x^2) - (2*(2*b^4 - 9*a*b^2*c + 5*a^2*c^2))/(a^4*(b^2 - 4*a*c)*
x) + (b^2 - 2*a*c + b*c*x)/(a*(b^2 - 4*a*c)*x^3*(a + b*x + c*x^2)) - (2*(2*
b^6 - 15*a*b^4*c + 30*a^2*b^2*c^2 - 10*a^3*c^3)*ArcTanh[(b + 2*c*x)/Sqrt[b^
2 - 4*a*c]])/(a^5*(b^2 - 4*a*c)^(3/2)) - (2*b*(2*b^2 - 3*a*c)*Log[x])/a^5 +
 (b*(2*b^2 - 3*a*c)*Log[a + b*x + c*x^2])/a^5
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rubi steps

$$\int \frac{1}{\left(ax^2+bx^3+cx^4\right)^2}\,dx = \int \frac{1}{x^4\left(a+bx+cx^2\right)^2}\,dx$$

$$= \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x^3\left(a+bx+cx^2\right)} - \frac{\int \frac{-2\left(2b^2-5ac\right)-4bcx}{x^4\left(a+bx+cx^2\right)}\,dx}{a\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x^3\left(a+bx+cx^2\right)} - \frac{\int\left(\frac{2\left(-2b^2+5ac\right)}{ax^4} - \frac{2\left(-2b^3+7abc\right)}{a^2x^3} - \frac{2\left(2b^4-9ab^2c+5a^2c^2\right)}{a^3x^2} + \frac{2b(b}{}\right.}{}$$

$$= -\frac{2\left(2b^2-5ac\right)}{3a^2\left(b^2-4ac\right)x^3} + \frac{b\left(2b^2-7ac\right)}{a^3\left(b^2-4ac\right)x^2} - \frac{2\left(2b^4-9ab^2c+5a^2c^2\right)}{a^4\left(b^2-4ac\right)x} + \frac{b^2-2ac+}{a\left(b^2-4ac\right)x^3(a}$$

$$= -\frac{2\left(2b^2-5ac\right)}{3a^2\left(b^2-4ac\right)x^3} + \frac{b\left(2b^2-7ac\right)}{a^3\left(b^2-4ac\right)x^2} - \frac{2\left(2b^4-9ab^2c+5a^2c^2\right)}{a^4\left(b^2-4ac\right)x} + \frac{b^2-2ac+}{a\left(b^2-4ac\right)x^3(a}$$

$$= -\frac{2\left(2b^2-5ac\right)}{3a^2\left(b^2-4ac\right)x^3} + \frac{b\left(2b^2-7ac\right)}{a^3\left(b^2-4ac\right)x^2} - \frac{2\left(2b^4-9ab^2c+5a^2c^2\right)}{a^4\left(b^2-4ac\right)x} + \frac{b^2-2ac+}{a\left(b^2-4ac\right)x^3(a}$$

$$= -\frac{2\left(2b^2-5ac\right)}{3a^2\left(b^2-4ac\right)x^3} + \frac{b\left(2b^2-7ac\right)}{a^3\left(b^2-4ac\right)x^2} - \frac{2\left(2b^4-9ab^2c+5a^2c^2\right)}{a^4\left(b^2-4ac\right)x} + \frac{b^2-2ac+}{a\left(b^2-4ac\right)x^3(a}$$

Mathematica [A] time = 0.32, size = 218, normalized size = 0.87

$$\frac{-\frac{a^3}{x^3} - \frac{3a\left(5a^2bc^2+2a^2c^3x-5ab^3c-4ab^2c^2x+b^5+b^4cx\right)}{\left(b^2-4ac\right)(a+x(b+cx))} + \frac{3a^2b}{x^2} - \frac{6\left(-10a^3c^3+30a^2b^2c^2-15ab^4c+2b^6\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{3/2}} + 6\log(x)\left(3abc-2b^3\right.}{3a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3 + c*x^4)^(-2),x]
```

```
[Out] (-(a^3/x^3) + (3*a^2*b)/x^2 + (3*a*(-3*b^2 + 2*a*c))/x - (3*a*(b^5 - 5*a*b^
3*c + 5*a^2*b*c^2 + b^4*c*x - 4*a*b^2*c^2*x + 2*a^2*c^3*x))/((b^2 - 4*a*c)*
(a + x*(b + c*x))) - (6*(2*b^6 - 15*a*b^4*c + 30*a^2*b^2*c^2 - 10*a^3*c^3)*
ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + 6*(-2*b^3 +
3*a*b*c)*Log[x] + 3*(2*b^3 - 3*a*b*c)*Log[a + x*(b + c*x)])/(3*a^5)
```

fricas [B] time = 1.32, size = 1407, normalized size = 5.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/3*(a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 6*(2*a*b^6*c - 17*a^2*b^4*c^2 +
 41*a^3*b^2*c^3 - 20*a^4*c^4)*x^4 + 3*(4*a*b^7 - 36*a^2*b^5*c + 97*a^3*b^3*
c^2 - 68*a^4*b*c^3)*x^3 + (6*a^2*b^6 - 53*a^3*b^4*c + 136*a^4*b^2*c^2 - 80*
a^5*c^3)*x^2 - 3*((2*b^6*c - 15*a*b^4*c^2 + 30*a^2*b^2*c^3 - 10*a^3*c^4)*x^
5 + (2*b^7 - 15*a*b^5*c + 30*a^2*b^3*c^2 - 10*a^3*b*c^3)*x^4 + (2*a*b^6 - 1
5*a^2*b^4*c + 30*a^3*b^2*c^2 - 10*a^4*c^3)*x^3)*sqrt(b^2 - 4*a*c)*log((2*c^
2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x
 + a)) - 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x - 3*((2*b^7*c - 19*a*b^
5*c^2 + 56*a^2*b^3*c^3 - 48*a^3*b*c^4)*x^5 + (2*b^8 - 19*a*b^6*c + 56*a^2*b
^4*c^2 - 48*a^3*b^2*c^3)*x^4 + (2*a*b^7 - 19*a^2*b^5*c + 56*a^3*b^3*c^2 - 4
8*a^4*b*c^3)*x^3)*log(c*x^2 + b*x + a) + 6*((2*b^7*c - 19*a*b^5*c^2 + 56*a^
2*b^3*c^3 - 48*a^3*b*c^4)*x^5 + (2*b^8 - 19*a*b^6*c + 56*a^2*b^4*c^2 - 48*a
^3*b^2*c^3)*x^4 + (2*a*b^7 - 19*a^2*b^5*c + 56*a^3*b^3*c^2 - 48*a^4*b*c^3)*
x^3)*log(x))/((a^5*b^4*c - 8*a^6*b^2*c^2 + 16*a^7*c^3)*x^5 + (a^5*b^5 - 8*a
^6*b^3*c + 16*a^7*b*c^2)*x^4 + (a^6*b^4 - 8*a^7*b^2*c + 16*a^8*c^2)*x^3), -
1/3*(a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 6*(2*a*b^6*c - 17*a^2*b^4*c^2 + 4
1*a^3*b^2*c^3 - 20*a^4*c^4)*x^4 + 3*(4*a*b^7 - 36*a^2*b^5*c + 97*a^3*b^3*c^
2 - 68*a^4*b*c^3)*x^3 + (6*a^2*b^6 - 53*a^3*b^4*c + 136*a^4*b^2*c^2 - 80*a^
5*c^3)*x^2 + 6*((2*b^6*c - 15*a*b^4*c^2 + 30*a^2*b^2*c^3 - 10*a^3*c^4)*x^5
+ (2*b^7 - 15*a*b^5*c + 30*a^2*b^3*c^2 - 10*a^3*b*c^3)*x^4 + (2*a*b^6 - 15*
a^2*b^4*c + 30*a^3*b^2*c^2 - 10*a^4*c^3)*x^3)*sqrt(-b^2 + 4*a*c)*arctan(-sq
rt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - 2*(a^3*b^5 - 8*a^4*b^3*c + 16
*a^5*b*c^2)*x - 3*((2*b^7*c - 19*a*b^5*c^2 + 56*a^2*b^3*c^3 - 48*a^3*b*c^4)
*x^5 + (2*b^8 - 19*a*b^6*c + 56*a^2*b^4*c^2 - 48*a^3*b^2*c^3)*x^4 + (2*a*b^
7 - 19*a^2*b^5*c + 56*a^3*b^3*c^2 - 48*a^4*b*c^3)*x^3)*log(c*x^2 + b*x + a)
 + 6*((2*b^7*c - 19*a*b^5*c^2 + 56*a^2*b^3*c^3 - 48*a^3*b*c^4)*x^5 + (2*b^8
 - 19*a*b^6*c + 56*a^2*b^4*c^2 - 48*a^3*b^2*c^3)*x^4 + (2*a*b^7 - 19*a^2*b^
5*c + 56*a^3*b^3*c^2 - 48*a^4*b*c^3)*x^3)*log(x))/((a^5*b^4*c - 8*a^6*b^2*c
^2 + 16*a^7*c^3)*x^5 + (a^5*b^5 - 8*a^6*b^3*c + 16*a^7*b*c^2)*x^4 + (a^6*b^
4 - 8*a^7*b^2*c + 16*a^8*c^2)*x^3)]
```

giac [A] time = 0.55, size = 282, normalized size = 1.12

$$\frac{2\left(2\,b^{6}-15\,ab^{4}c+30\,a^{2}b^{2}c^{2}-10\,a^{3}c^{3}\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^{2}+4\,ac}}\right)}{\left(a^{5}b^{2}-4\,a^{6}c\right)\sqrt{-b^{2}+4\,ac}}+\frac{\left(2\,b^{3}-3\,abc\right)\log\left(cx^{2}+bx+a\right)}{a^{5}}-\frac{2\left(2\,b^{3}-3\,abc\right)}{a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")
```

```
[Out] 2*(2*b^6 - 15*a*b^4*c + 30*a^2*b^2*c^2 - 10*a^3*c^3)*arctan((2*c*x + b)/sqr
t(-b^2 + 4*a*c))/((a^5*b^2 - 4*a^6*c)*sqrt(-b^2 + 4*a*c)) + (2*b^3 - 3*a*b*
c)*log(c*x^2 + b*x + a)/a^5 - 2*(2*b^3 - 3*a*b*c)*log(abs(x))/a^5 - 1/3*(a^
```

```
4*b^2 - 4*a^5*c + 6*(2*a*b^4*c - 9*a^2*b^2*c^2 + 5*a^3*c^3)*x^4 + 3*(4*a*b^
5 - 20*a^2*b^3*c + 17*a^3*b*c^2)*x^3 + (6*a^2*b^4 - 29*a^3*b^2*c + 20*a^4*c
^2)*x^2 - 2*(a^3*b^3 - 4*a^4*b*c)*x)/((c*x^2 + b*x + a)*(b^2 - 4*a*c)*a^5*x
^3)
```

maple [B] time = 0.02, size = 515, normalized size = 2.04

$$\frac{2c^3x}{\left(cx^2+bx+a\right)\left(4ac-b^2\right)a^2}+\frac{20c^3\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}a^2}-\frac{4b^2c^2x}{\left(cx^2+bx+a\right)\left(4ac-b^2\right)a^3}-\frac{60b^2c^2\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}a^3}+\frac{}{(c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+b*x^3+a*x^2)^2,x)
```

```
[Out] -1/3/a^2/x^3+2/a^3/x*c-3/a^4/x*b^2+1/a^3*b/x^2+6*b/a^4*ln(x)*c-4*b^3/a^5*ln
(x)+2/a^2/(c*x^2+b*x+a)*c^3/(4*a*c-b^2)*x-4/a^3/(c*x^2+b*x+a)*c^2/(4*a*c-b^
2)*x*b^2+1/a^4/(c*x^2+b*x+a)*c/(4*a*c-b^2)*x*b^4+5/a^2/(c*x^2+b*x+a)*b/(4*a
*c-b^2)*c^2-5/a^3/(c*x^2+b*x+a)*b^3/(4*a*c-b^2)*c+1/a^4/(c*x^2+b*x+a)*b^5/(
4*a*c-b^2)-12/a^3/(4*a*c-b^2)*c^2*ln(c*x^2+b*x+a)*b+11/a^4/(4*a*c-b^2)*c*ln
(c*x^2+b*x+a)*b^3-2/a^5/(4*a*c-b^2)*ln(c*x^2+b*x+a)*b^5+20/a^2/(4*a*c-b^2)^
(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*c^3-60/a^3/(4*a*c-b^2)^(3/2)*arct
an((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^2*c^2+30/a^4/(4*a*c-b^2)^(3/2)*arctan((2*
c*x+b)/(4*a*c-b^2)^(1/2))*b^4*c-4/a^5/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4
*a*c-b^2)^(1/2))*b^6
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 3.06, size = 1120, normalized size = 4.44

$$\frac{\frac{x^2\left(5ac-6b^2\right)}{3a^3}-\frac{1}{3a}+\frac{2bx}{3a^2}+\frac{x^3\left(17a^2bc^2-20ab^3c+4b^5\right)}{a^4\left(4ac-b^2\right)}+\frac{2cx^4\left(5a^2c^2-9ab^2c+2b^4\right)}{a^4\left(4ac-b^2\right)}}{cx^5+bx^4+ax^3}+\frac{\ln\left(4ab^9+4b^{10}x-4ab^6\sqrt{-\left(4ac-\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^2 + b*x^3 + c*x^4)^2,x)
```

```
[Out] ((x^2*(5*a*c - 6*b^2))/(3*a^3) - 1/(3*a) + (2*b*x)/(3*a^2) + (x^3*(4*b^5 +
17*a^2*b*c^2 - 20*a*b^3*c))/(a^4*(4*a*c - b^2)) + (2*c*x^4*(2*b^4 + 5*a^2*c
^2 - 9*a*b^2*c))/(a^4*(4*a*c - b^2)))/(a*x^3 + b*x^4 + c*x^5) + (log(4*a*b^
9 + 4*b^10*x - 4*a*b^6*(-(4*a*c - b^2)^3)^(1/2) - 52*a^2*b^7*c + 308*a^5*b*
c^4 - 40*a^5*c^5*x - 4*b^7*x*(-(4*a*c - b^2)^3)^(1/2) + 243*a^3*b^5*c^2 - 4
73*a^4*b^3*c^3 + 5*a^4*c^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^4*c*(-(4*a*c
 - b^2)^3)^(1/2) + 266*a^2*b^6*c^2*x - 563*a^3*b^4*c^3*x + 438*a^4*b^2*c^4*
x - 54*a*b^8*c*x - 33*a^3*b^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 30*a*b^5*c*x*(
-(4*a*c - b^2)^3)^(1/2) + 41*a^3*b*c^3*x*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*
b^3*c^2*x*(-(4*a*c - b^2)^3)^(1/2))*(a^2*(132*b^5*c^2 - 30*b^2*c^2*(-(4*a*c
 - b^2)^3)^(1/2)) - a^3*(272*b^3*c^3 - 10*c^3*(-(4*a*c - b^2)^3)^(1/2)) + 2
*b^9 - 2*b^6*(-(4*a*c - b^2)^3)^(1/2) - a*(27*b^7*c - 15*b^4*c*(-(4*a*c - b
```

```
^2)^3)^(1/2)) + 192*a^4*b*c^4))/(a^5*b^6 - 64*a^8*c^3 - 12*a^6*b^4*c + 48*a
^7*b^2*c^2) + (log(4*a*b^9 + 4*b^10*x + 4*a*b^6*(-(4*a*c - b^2)^3)^(1/2) -
52*a^2*b^7*c + 308*a^5*b*c^4 - 40*a^5*c^5*x + 4*b^7*x*(-(4*a*c - b^2)^3)^(1
/2) + 243*a^3*b^5*c^2 - 473*a^4*b^3*c^3 - 5*a^4*c^3*(-(4*a*c - b^2)^3)^(1/2
) - 24*a^2*b^4*c*(-(4*a*c - b^2)^3)^(1/2) + 266*a^2*b^6*c^2*x - 563*a^3*b^4
*c^3*x + 438*a^4*b^2*c^4*x - 54*a*b^8*c*x + 33*a^3*b^2*c^2*(-(4*a*c - b^2)^
3)^(1/2) - 30*a*b^5*c*x*(-(4*a*c - b^2)^3)^(1/2) - 41*a^3*b*c^3*x*(-(4*a*c
- b^2)^3)^(1/2) + 66*a^2*b^3*c^2*x*(-(4*a*c - b^2)^3)^(1/2))*(a^2*(132*b^5*
c^2 + 30*b^2*c^2*(-(4*a*c - b^2)^3)^(1/2)) - a^3*(272*b^3*c^3 + 10*c^3*(-(4
*a*c - b^2)^3)^(1/2)) + 2*b^9 + 2*b^6*(-(4*a*c - b^2)^3)^(1/2) - a*(27*b^7*
c + 15*b^4*c*(-(4*a*c - b^2)^3)^(1/2)) + 192*a^4*b*c^4))/(a^5*b^6 - 64*a^8*
c^3 - 12*a^6*b^4*c + 48*a^7*b^2*c^2) + (2*b*log(x)*(3*a*c - 2*b^2))/a^5
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+b*x**3+a*x**2)**2,x)
[Out] Timed out
```

3.28 $\int \frac{1}{x(ax^2+bx^3+cx^4)^2}\,dx$

Optimal. Leaf size=318

$$\frac{b(5b^2-17ac)}{3a^3x^3(b^2-4ac)}-\frac{5b^2-12ac}{4a^2x^4(b^2-4ac)}-\frac{(3a^2c^2-12ab^2c+5b^4)\log(a+bx+cx^2)}{2a^6}+\frac{\log(x)(3a^2c^2-12ab^2c+5b^4)}{a^6}+$$

```
[Out] 1/4*(12*a*c-5*b^2)/a^2/(-4*a*c+b^2)/x^4+1/3*b*(-17*a*c+5*b^2)/a^3/(-4*a*c+b
^2)/x^3+1/2*(-12*a^2*c^2+22*a*b^2*c-5*b^4)/a^4/(-4*a*c+b^2)/x^2+b*(29*a^2*c
^2-27*a*b^2*c+5*b^4)/a^5/(-4*a*c+b^2)/x+(b*c*x-2*a*c+b^2)/a/(-4*a*c+b^2)/x^
4/(c*x^2+b*x+a)+b*(-70*a^3*c^3+105*a^2*b^2*c^2-42*a*b^4*c+5*b^6)*arctanh((2
*c*x+b)/(-4*a*c+b^2)^(1/2))/a^6/(-4*a*c+b^2)^(3/2)+(3*a^2*c^2-12*a*b^2*c+5*
b^4)*ln(x)/a^6-1/2*(3*a^2*c^2-12*a*b^2*c+5*b^4)*ln(c*x^2+b*x+a)/a^6
```

Rubi [A] time = 0.39, antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {1585, 740, 800, 634, 618, 206, 628}

$$-\frac{12a^2c^2-22ab^2c+5b^4}{2a^4x^2(b^2-4ac)}-\frac{(3a^2c^2-12ab^2c+5b^4)\log(a+bx+cx^2)}{2a^6}+\frac{b(29a^2c^2-27ab^2c+5b^4)}{a^5x(b^2-4ac)}+\frac{\log(x)(3a^2c^2-}{a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a*x^2 + b*x^3 + c*x^4)^2),x]
```

```
[Out] -(5*b^2 - 12*a*c)/(4*a^2*(b^2 - 4*a*c)*x^4) + (b*(5*b^2 - 17*a*c))/(3*a^3*(
b^2 - 4*a*c)*x^3) - (5*b^4 - 22*a*b^2*c + 12*a^2*c^2)/(2*a^4*(b^2 - 4*a*c)*
x^2) + (b*(5*b^4 - 27*a*b^2*c + 29*a^2*c^2))/(a^5*(b^2 - 4*a*c)*x) + (b^2 -
 2*a*c + b*c*x)/(a*(b^2 - 4*a*c)*x^4*(a + b*x + c*x^2)) + (b*(5*b^6 - 42*a*
b^4*c + 105*a^2*b^2*c^2 - 70*a^3*c^3)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]
])/(a^6*(b^2 - 4*a*c)^(3/2)) + ((5*b^4 - 12*a*b^2*c + 3*a^2*c^2)*Log[x])/a^
6 - ((5*b^4 - 12*a*b^2*c + 3*a^2*c^2)*Log[a + b*x + c*x^2])/(2*a^6)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\int \frac{1}{x\left(ax^2+bx^3+cx^4\right)^2}\,dx = \int \frac{1}{x^5\left(a+bx+cx^2\right)^2}\,dx$$

$$= \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x^4\left(a+bx+cx^2\right)} - \frac{\int \frac{-5b^2+12ac-5bcx}{x^5\left(a+bx+cx^2\right)}\,dx}{a\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x^4\left(a+bx+cx^2\right)} - \frac{\int \left(\frac{-5b^2+12ac}{ax^5} + \frac{5b^3-17abc}{a^2x^4} + \frac{-5b^4+22ab^2c-12a^2c^2}{a^3x^3} + 5b$$

$$= -\frac{5b^2-12ac}{4a^2\left(b^2-4ac\right)x^4} + \frac{b\left(5b^2-17ac\right)}{3a^3\left(b^2-4ac\right)x^3} - \frac{5b^4-22ab^2c+12a^2c^2}{2a^4\left(b^2-4ac\right)x^2} + \frac{b\left(5b^4-27ab}{a^5\left(b^2-$$

$$= -\frac{5b^2-12ac}{4a^2\left(b^2-4ac\right)x^4} + \frac{b\left(5b^2-17ac\right)}{3a^3\left(b^2-4ac\right)x^3} - \frac{5b^4-22ab^2c+12a^2c^2}{2a^4\left(b^2-4ac\right)x^2} + \frac{b\left(5b^4-27ab}{a^5\left(b^2-$$

$$= -\frac{5b^2-12ac}{4a^2\left(b^2-4ac\right)x^4} + \frac{b\left(5b^2-17ac\right)}{3a^3\left(b^2-4ac\right)x^3} - \frac{5b^4-22ab^2c+12a^2c^2}{2a^4\left(b^2-4ac\right)x^2} + \frac{b\left(5b^4-27ab}{a^5\left(b^2-$$

$$= -\frac{5b^2-12ac}{4a^2\left(b^2-4ac\right)x^4} + \frac{b\left(5b^2-17ac\right)}{3a^3\left(b^2-4ac\right)x^3} - \frac{5b^4-22ab^2c+12a^2c^2}{2a^4\left(b^2-4ac\right)x^2} + \frac{b\left(5b^4-27ab}{a^5\left(b^2-$$

Mathematica [A] time = 0.38, size = 272, normalized size = 0.86

$$\frac{-\frac{3a^4}{x^4}+\frac{8a^3b}{x^3}+\frac{6a^2(2ac-3b^2)}{x^2}+12\log(x)\left(3a^2c^2-12ab^2c+5b^4\right)-6\left(3a^2c^2-12ab^2c+5b^4\right)\log(a+x(b+cx))+\frac{12b(}{}}{12a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a*x^2 + b*x^3 + c*x^4)^2),x]
```

```
[Out] ((-3*a^4)/x^4 + (8*a^3*b)/x^3 + (6*a^2*(-3*b^2 + 2*a*c))/x^2 - (24*a*b*(-2*
b^2 + 3*a*c))/x - (12*a*(-b^6 + 6*a*b^4*c - 9*a^2*b^2*c^2 + 2*a^3*c^3 - b^5
*c*x + 5*a*b^3*c^2*x - 5*a^2*b*c^3*x))/((b^2 - 4*a*c)*(a + x*(b + c*x))) +
(12*b*(5*b^6 - 42*a*b^4*c + 105*a^2*b^2*c^2 - 70*a^3*c^3)*ArcTan[(b + 2*c*x
)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + 12*(5*b^4 - 12*a*b^2*c + 3*a^
2*c^2)*Log[x] - 6*(5*b^4 - 12*a*b^2*c + 3*a^2*c^2)*Log[a + x*(b + c*x)])/(1
2*a^6)
```

fricas [B] time = 2.07, size = 1640, normalized size = 5.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/12*(3*a^5*b^4 - 24*a^6*b^2*c + 48*a^7*c^2 - 12*(5*a*b^7*c - 47*a^2*b^5*
c^2 + 137*a^3*b^3*c^3 - 116*a^4*b*c^4)*x^5 - 6*(10*a*b^8 - 99*a^2*b^6*c + 3
16*a^3*b^4*c^2 - 332*a^4*b^2*c^3 + 48*a^5*c^4)*x^4 - 2*(15*a^2*b^7 - 146*a^
3*b^5*c + 448*a^4*b^3*c^2 - 416*a^5*b*c^3)*x^3 + (10*a^3*b^6 - 89*a^4*b^4*c
 + 232*a^5*b^2*c^2 - 144*a^6*c^3)*x^2 - 6*((5*b^7*c - 42*a*b^5*c^2 + 105*a^
2*b^3*c^3 - 70*a^3*b*c^4)*x^6 + (5*b^8 - 42*a*b^6*c + 105*a^2*b^4*c^2 - 70*
a^3*b^2*c^3)*x^5 + (5*a*b^7 - 42*a^2*b^5*c + 105*a^3*b^3*c^2 - 70*a^4*b*c^3
)*x^4)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2
- 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - 5*(a^4*b^5 - 8*a^5*b^3*c + 16*a^
6*b*c^2)*x + 6*((5*b^8*c - 52*a*b^6*c^2 + 179*a^2*b^4*c^3 - 216*a^3*b^2*c^4
 + 48*a^4*c^5)*x^6 + (5*b^9 - 52*a*b^7*c + 179*a^2*b^5*c^2 - 216*a^3*b^3*c^
3 + 48*a^4*b*c^4)*x^5 + (5*a*b^8 - 52*a^2*b^6*c + 179*a^3*b^4*c^2 - 216*a^4
*b^2*c^3 + 48*a^5*c^4)*x^4)*log(c*x^2 + b*x + a) - 12*((5*b^8*c - 52*a*b^6*
c^2 + 179*a^2*b^4*c^3 - 216*a^3*b^2*c^4 + 48*a^4*c^5)*x^6 + (5*b^9 - 52*a*b
^7*c + 179*a^2*b^5*c^2 - 216*a^3*b^3*c^3 + 48*a^4*b*c^4)*x^5 + (5*a*b^8 - 5
2*a^2*b^6*c + 179*a^3*b^4*c^2 - 216*a^4*b^2*c^3 + 48*a^5*c^4)*x^4)*log(x))/
((a^6*b^4*c - 8*a^7*b^2*c^2 + 16*a^8*c^3)*x^6 + (a^6*b^5 - 8*a^7*b^3*c + 16
*a^8*b*c^2)*x^5 + (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*x^4), -1/12*(3*a^5*b
^4 - 24*a^6*b^2*c + 48*a^7*c^2 - 12*(5*a*b^7*c - 47*a^2*b^5*c^2 + 137*a^3*b
^3*c^3 - 116*a^4*b*c^4)*x^5 - 6*(10*a*b^8 - 99*a^2*b^6*c + 316*a^3*b^4*c^2
- 332*a^4*b^2*c^3 + 48*a^5*c^4)*x^4 - 2*(15*a^2*b^7 - 146*a^3*b^5*c + 448*a
^4*b^3*c^2 - 416*a^5*b*c^3)*x^3 + (10*a^3*b^6 - 89*a^4*b^4*c + 232*a^5*b^2*
c^2 - 144*a^6*c^3)*x^2 - 12*((5*b^7*c - 42*a*b^5*c^2 + 105*a^2*b^3*c^3 - 70
*a^3*b*c^4)*x^6 + (5*b^8 - 42*a*b^6*c + 105*a^2*b^4*c^2 - 70*a^3*b^2*c^3)*x
^5 + (5*a*b^7 - 42*a^2*b^5*c + 105*a^3*b^3*c^2 - 70*a^4*b*c^3)*x^4)*sqrt(-b
^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - 5*(a^4*
b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*x + 6*((5*b^8*c - 52*a*b^6*c^2 + 179*a^2*
b^4*c^3 - 216*a^3*b^2*c^4 + 48*a^4*c^5)*x^6 + (5*b^9 - 52*a*b^7*c + 179*a^2
*b^5*c^2 - 216*a^3*b^3*c^3 + 48*a^4*b*c^4)*x^5 + (5*a*b^8 - 52*a^2*b^6*c +
179*a^3*b^4*c^2 - 216*a^4*b^2*c^3 + 48*a^5*c^4)*x^4)*log(c*x^2 + b*x + a) -
 12*((5*b^8*c - 52*a*b^6*c^2 + 179*a^2*b^4*c^3 - 216*a^3*b^2*c^4 + 48*a^4*c
^5)*x^6 + (5*b^9 - 52*a*b^7*c + 179*a^2*b^5*c^2 - 216*a^3*b^3*c^3 + 48*a^4*
b*c^4)*x^5 + (5*a*b^8 - 52*a^2*b^6*c + 179*a^3*b^4*c^2 - 216*a^4*b^2*c^3 +
48*a^5*c^4)*x^4)*log(x))/((a^6*b^4*c - 8*a^7*b^2*c^2 + 16*a^8*c^3)*x^6 + (a
```

```
^6*b^5 - 8*a^7*b^3*c + 16*a^8*b*c^2)*x^5 + (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*
c^2)*x^4)]
```

giac [A] time = 0.44, size = 347, normalized size = 1.09

$$-\frac{\left(5 b^7-42 a b^5 c+105 a^2 b^3 c^2-70 a^3 b c^3\right) \arctan\left(\frac{2 c x+b}{\sqrt{-b^2+4 a c}}\right)}{\left(a^6 b^2-4 a^7 c\right) \sqrt{-b^2+4 a c}}-\frac{\left(5 b^4-12 a b^2 c+3 a^2 c^2\right) \log\left(c x^2+b x+a\right)}{2 a^6}+\frac{(5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")
```

```
[Out] -(5*b^7 - 42*a*b^5*c + 105*a^2*b^3*c^2 - 70*a^3*b*c^3)*arctan((2*c*x + b)/s
qrt(-b^2 + 4*a*c))/((a^6*b^2 - 4*a^7*c)*sqrt(-b^2 + 4*a*c)) - 1/2*(5*b^4 -
12*a*b^2*c + 3*a^2*c^2)*log(c*x^2 + b*x + a)/a^6 + (5*b^4 - 12*a*b^2*c + 3*
a^2*c^2)*log(abs(x))/a^6 - 1/12*(3*a^5*b^2 - 12*a^6*c - 12*(5*a*b^5*c - 27*
a^2*b^3*c^2 + 29*a^3*b*c^3)*x^5 - 6*(10*a*b^6 - 59*a^2*b^4*c + 80*a^3*b^2*c
^2 - 12*a^4*c^3)*x^4 - 2*(15*a^2*b^5 - 86*a^3*b^3*c + 104*a^4*b*c^2)*x^3 +
(10*a^3*b^4 - 49*a^4*b^2*c + 36*a^5*c^2)*x^2 - 5*(a^4*b^3 - 4*a^5*b*c)*x)/(
(c*x^2 + b*x + a)*(b^2 - 4*a*c)*a^6*x^4)
```

maple [B] time = 0.02, size = 619, normalized size = 1.95

$$-\frac{5 b c^3 x}{\left(c x^2+b x+a\right) \left(4 a c-b^2\right) a^3}-\frac{70 b c^3 \arctan\left(\frac{2 c x+b}{\sqrt{4 a c-b^2}}\right)}{\left(4 a c-b^2\right)^{\frac{3}{2}} a^3}+\frac{5 b^3 c^2 x}{\left(c x^2+b x+a\right) \left(4 a c-b^2\right) a^4}+\frac{105 b^3 c^2 \arctan\left(\frac{2 c x+}{\sqrt{4 a c-}}\right.}{\left(4 a c-b^2\right)^{\frac{3}{2}} a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^4+b*x^3+a*x^2)^2,x)
```

```
[Out] -1/4/a^2/x^4+1/a^3/x^2*c-3/2/a^4/x^2*b^2+3/a^4*ln(x)*c^2-12/a^5*ln(x)*b^2*c
+5/a^6*ln(x)*b^4+2/3/a^3*b/x^3-6*b/a^4/x*c+4*b^3/a^5/x-5/a^3/(c*x^2+b*x+a)*
b*c^3/(4*a*c-b^2)*x+5/a^4/(c*x^2+b*x+a)*b^3*c^2/(4*a*c-b^2)*x-1/a^5/(c*x^2+
b*x+a)*b^5*c/(4*a*c-b^2)*x+2/a^2/(c*x^2+b*x+a)/(4*a*c-b^2)*c^3-9/a^3/(c*x^2
+b*x+a)/(4*a*c-b^2)*b^2*c^2+6/a^4/(c*x^2+b*x+a)/(4*a*c-b^2)*b^4*c-1/a^5/(c*
x^2+b*x+a)/(4*a*c-b^2)*b^6-6/a^3/(4*a*c-b^2)*c^3*ln(c*x^2+b*x+a)+51/2/a^4/(
4*a*c-b^2)*c^2*ln(c*x^2+b*x+a)*b^2-16/a^5/(4*a*c-b^2)*c*ln(c*x^2+b*x+a)*b^4
+5/2/a^6/(4*a*c-b^2)*ln(c*x^2+b*x+a)*b^6-70/a^3/(4*a*c-b^2)^(3/2)*arctan((2
*c*x+b)/(4*a*c-b^2)^(1/2))*b*c^3+105/a^4/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)
/(4*a*c-b^2)^(1/2))*b^3*c^2-42/a^5/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*
c-b^2)^(1/2))*b^5*c+5/a^6/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1
/2))*b^7
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 3.14, size = 1260, normalized size = 3.96

$$\frac{\ln(x) \left(3 a^2 c^2-12 a b^2 c+5 b^4\right)}{a^6}-\frac{\frac{1}{4 a}-\frac{x^2 \left(9 a c-10 b^2\right)}{12 a^3}-\frac{5 b x}{12 a^2}+\frac{x^4 \left(-12 a^3 c^3+80 a^2 b^2 c^2-59 a b^4 c+10 b^6\right)}{2 a^5 \left(4 a c-b^2\right)}+\frac{b x^3 \left(26 a c-15 b^2\right)}{6 a^4}}{c x^6+b x^5+a x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a*x^2 + b*x^3 + c*x^4)^2),x)
```

```
[Out] (log(x)*(5*b^4 + 3*a^2*c^2 - 12*a*b^2*c))/a^6 - (1/(4*a) - (x^2*(9*a*c - 10
*b^2))/(12*a^3) - (5*b*x)/(12*a^2) + (x^4*(10*b^6 - 12*a^3*c^3 + 80*a^2*b^2
*c^2 - 59*a*b^4*c))/(2*a^5*(4*a*c - b^2)) + (b*x^3*(26*a*c - 15*b^2))/(6*a^
4) + (b*c*x^5*(5*b^4 + 29*a^2*c^2 - 27*a*b^2*c))/(a^5*(4*a*c - b^2)))/(a*x^
4 + b*x^5 + c*x^6) + (log(288*a^6*c^5 - 10*b^11*x - 10*a*b^10 + 10*a*b^7*(-
(4*a*c - b^2)^3)^(1/2) + 139*a^2*b^8*c + 10*b^8*x*(-(4*a*c - b^2)^3)^(1/2)
- 717*a^3*b^6*c^2 + 1643*a^4*b^4*c^3 - 1508*a^5*b^2*c^4 - 69*a^2*b^5*c*(-(4
*a*c - b^2)^3)^(1/2) - 53*a^4*b*c^3*(-(4*a*c - b^2)^3)^(1/2) - 779*a^2*b^7*
c^2*x + 1916*a^3*b^5*c^3*x - 1998*a^4*b^3*c^4*x + 36*a^4*c^4*x*(-(4*a*c - b
^2)^3)^(1/2) + 144*a*b^9*c*x + 129*a^3*b^3*c^2*(-(4*a*c - b^2)^3)^(1/2) + 5
68*a^5*b*c^5*x - 84*a*b^6*c*x*(-(4*a*c - b^2)^3)^(1/2) + 225*a^2*b^4*c^2*x*
(-(4*a*c - b^2)^3)^(1/2) - 206*a^3*b^2*c^3*x*(-(4*a*c - b^2)^3)^(1/2))*(a^3
*(466*b^4*c^3 - 35*b*c^3*(-(4*a*c - b^2)^3)^(1/2)) - a^2*((387*b^6*c^2)/2 -
 (105*b^3*c^2*(-(4*a*c - b^2)^3)^(1/2))/2) - (5*b^10)/2 + 96*a^5*c^5 + (5*b
^7*(-(4*a*c - b^2)^3)^(1/2))/2 + a*(36*b^8*c - 21*b^5*c*(-(4*a*c - b^2)^3)^
(1/2)) - 456*a^4*b^2*c^4))/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^
2*c^2) - (log(10*a*b^10 + 10*b^11*x - 288*a^6*c^5 + 10*a*b^7*(-(4*a*c - b^2
)^3)^(1/2) - 139*a^2*b^8*c + 10*b^8*x*(-(4*a*c - b^2)^3)^(1/2) + 717*a^3*b^
6*c^2 - 1643*a^4*b^4*c^3 + 1508*a^5*b^2*c^4 - 69*a^2*b^5*c*(-(4*a*c - b^2)^
3)^(1/2) - 53*a^4*b*c^3*(-(4*a*c - b^2)^3)^(1/2) + 779*a^2*b^7*c^2*x - 1916
*a^3*b^5*c^3*x + 1998*a^4*b^3*c^4*x + 36*a^4*c^4*x*(-(4*a*c - b^2)^3)^(1/2)
 - 144*a*b^9*c*x + 129*a^3*b^3*c^2*(-(4*a*c - b^2)^3)^(1/2) - 568*a^5*b*c^5
*x - 84*a*b^6*c*x*(-(4*a*c - b^2)^3)^(1/2) + 225*a^2*b^4*c^2*x*(-(4*a*c - b
^2)^3)^(1/2) - 206*a^3*b^2*c^3*x*(-(4*a*c - b^2)^3)^(1/2))*(a^2*((387*b^6*c
^2)/2 + (105*b^3*c^2*(-(4*a*c - b^2)^3)^(1/2))/2) - a^3*(466*b^4*c^3 + 35*b
*c^3*(-(4*a*c - b^2)^3)^(1/2)) + (5*b^10)/2 - 96*a^5*c^5 + (5*b^7*(-(4*a*c
- b^2)^3)^(1/2))/2 - a*(36*b^8*c + 21*b^5*c*(-(4*a*c - b^2)^3)^(1/2)) + 456
*a^4*b^2*c^4))/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**4+b*x**3+a*x**2)**2,x)
```

```
[Out] Timed out
```

3.29 $\int x^2\sqrt{ax^2+bx^3+cx^4}\,dx$

Optimal. Leaf size=257

$$-\frac{\left(256a^2c^2-460ab^2c+105b^4\right)\sqrt{ax^2+bx^3+cx^4}}{1920c^4x}+\frac{bx\left(7b^2-12ac\right)\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\,\tanh^{-1}\left(\frac{b+2c}{2\sqrt{c}\sqrt{a+b}}\right.}{256c^{9/2}\sqrt{ax^2+bx^3+cx^4}}$$

```
[Out] 1/256*b*(-12*a*c+7*b^2)*(-4*a*c+b^2)*x*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2
+b*x+a)^(1/2))*(c*x^2+b*x+a)^(1/2)/c^(9/2)/(c*x^4+b*x^3+a*x^2)^(1/2)+1/960*
b*(-116*a*c+35*b^2)*(c*x^4+b*x^3+a*x^2)^(1/2)/c^3-1/1920*(256*a^2*c^2-460*a
*b^2*c+105*b^4)*(c*x^4+b*x^3+a*x^2)^(1/2)/c^4/x-1/240*(-16*a*c+7*b^2)*x*(c*
x^4+b*x^3+a*x^2)^(1/2)/c^2+1/40*x^2*(8*c*x+b)*(c*x^4+b*x^3+a*x^2)^(1/2)/c
```

Rubi [A] time = 0.59, antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1919, 1949, 12, 1914, 621, 206}

$$-\frac{\left(256a^2c^2-460ab^2c+105b^4\right)\sqrt{ax^2+bx^3+cx^4}}{1920c^4x}-\frac{x\left(7b^2-16ac\right)\sqrt{ax^2+bx^3+cx^4}}{240c^2}+\frac{b\left(35b^2-116ac\right)\sqrt{ax^2}}{960c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a*x^2 + b*x^3 + c*x^4],x]
```

```
[Out] (b*(35*b^2 - 116*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(960*c^3) - ((105*b^4 -
460*a*b^2*c + 256*a^2*c^2)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(1920*c^4*x) - ((7*
b^2 - 16*a*c)*x*Sqrt[a*x^2 + b*x^3 + c*x^4])/(240*c^2) + (x^2*(b + 8*c*x)*S
qrt[a*x^2 + b*x^3 + c*x^4])/(40*c) + (b*(7*b^2 - 12*a*c)*(b^2 - 4*a*c)*x*Sq
rt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])
/(256*c^(9/2)*Sqrt[a*x^2 + b*x^3 + c*x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1914

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))])/Sqrt[
a*x^q + b*x^n + c*x^(2*n - q)], Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

Rule 1919

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(x^(m - n + q + 1)*(b*(n - q)*p + c*(m + p*q + (n - q)
*(2*p - 1) + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(c*(m + p*(2*
n - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1)), x] + Dist[((n - q)*p)/(c*(m
 + p*(2*n - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1)), Int[x^(m - (n - 2*q
))*Simp[-(a*b*(m + p*q - n + q + 1)) + (2*a*c*(m + p*q + (n - q)*(2*p - 1)
+ 1) - b^2*(m + p*q + (n - q)*(p - 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n +
c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] &&
 PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p
, 0] && RationalQ[m, q] && GtQ[m + p*q + 1, n - q] && NeQ[m + p*(2*n - q) +
 1, 0] && NeQ[m + p*q + (n - q)*(2*p - 1) + 1, 0]
```

Rule 1949

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[(B*x^(m - n + 1)*(a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1))/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), x] -
Dist[1/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(m - n + q)*Simp[a*B*(m
 + p*q - n + q + 1) + (b*B*(m + p*q + (n - q)*p + 1) - A*c*(m + p*q + (n -
q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x]
/; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Integ
erQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[p, -1] && LtQ[p, 0] && R
ationalQ[m, q] && GeQ[m + p*q, n - q - 1] && NeQ[m + p*q + (n - q)*(2*p + 1
) + 1, 0]
```

Rubi steps

$$\int x^2\sqrt{ax^2+bx^3+cx^4}\,dx = \frac{x^2(b+8cx)\sqrt{ax^2+bx^3+cx^4}}{40c} + \frac{\int \frac{x^3\left(-3ab-\frac{1}{2}\left(7b^2-16ac\right)x\right)}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{40c}$$

$$= -\frac{\left(7b^2-16ac\right)x\sqrt{ax^2+bx^3+cx^4}}{240c^2} + \frac{x^2(b+8cx)\sqrt{ax^2+bx^3+cx^4}}{40c} - \frac{\int \frac{x^2\left(-a\left(7b^2-16a\right.\right.}{\sqrt{ax}}$$

$$= \frac{b\left(35b^2-116ac\right)\sqrt{ax^2+bx^3+cx^4}}{960c^3} - \frac{\left(7b^2-16ac\right)x\sqrt{ax^2+bx^3+cx^4}}{240c^2} + \frac{x^2(b+8c}{}$$

$$= \frac{b\left(35b^2-116ac\right)\sqrt{ax^2+bx^3+cx^4}}{960c^3} - \frac{\left(105b^4-460ab^2c+256a^2c^2\right)\sqrt{ax^2+bx^3+c}}{1920c^4x}$$

$$= \frac{b\left(35b^2-116ac\right)\sqrt{ax^2+bx^3+cx^4}}{960c^3} - \frac{\left(105b^4-460ab^2c+256a^2c^2\right)\sqrt{ax^2+bx^3+c}}{1920c^4x}$$

$$= \frac{b\left(35b^2-116ac\right)\sqrt{ax^2+bx^3+cx^4}}{960c^3} - \frac{\left(105b^4-460ab^2c+256a^2c^2\right)\sqrt{ax^2+bx^3+c}}{1920c^4x}$$

$$= \frac{b\left(35b^2-116ac\right)\sqrt{ax^2+bx^3+cx^4}}{960c^3} - \frac{\left(105b^4-460ab^2c+256a^2c^2\right)\sqrt{ax^2+bx^3+c}}{1920c^4x}$$

$$= \frac{b\left(35b^2-116ac\right)\sqrt{ax^2+bx^3+cx^4}}{960c^3} - \frac{\left(105b^4-460ab^2c+256a^2c^2\right)\sqrt{ax^2+bx^3+c}}{1920c^4x}$$

Mathematica [A] time = 0.24, size = 180, normalized size = 0.70

$$\frac{15x\left(48a^2bc^2-40ab^3c+7b^5\right)\sqrt{a+x(b+cx)}\,\log\left(2\sqrt{c}\sqrt{a+x(b+cx)}+b+2cx\right)+2\sqrt{c}\,x(a+x(b+cx))\left(128c^2\right.}{3840c^{9/2}\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a*x^2 + b*x^3 + c*x^4],x]
```

```
[Out] (2*Sqrt[c]*x*(a + x*(b + c*x))*(-105*b^4 + 70*b^3*c*x + 4*b^2*c*(115*a - 14
*c*x^2) + 8*b*c^2*x*(-29*a + 6*c*x^2) + 128*c^2*(-2*a^2 + a*c*x^2 + 3*c^2*x
^4)) + 15*(7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)*x*Sqrt[a + x*(b + c*x)]*Log[b
 + 2*c*x + 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/(3840*c^(9/2)*Sqrt[x^2*(a + x*
(b + c*x))])
```

fricas [A] time = 0.85, size = 390, normalized size = 1.52

$$\left[\frac{15\left(7\,b^5-40\,ab^3c+48\,a^2bc^2\right)\sqrt{c}\,x\log\left(-\frac{8\,c^2x^3+8\,bcx^2+4\,\sqrt{cx^4+bx^3+ax^2}\,(2\,cx+b)\sqrt{c}+\left(b^2+4\,ac\right)x}{x}\right)+4\left(384\,c^5x^4+48\,bc^4x\right.}{7680\,c^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/7680*(15*(7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)*sqrt(c)*x*log(-(8*c^2*x^3 +
 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a
*c)*x)/x) + 4*(384*c^5*x^4 + 48*b*c^4*x^3 - 105*b^4*c + 460*a*b^2*c^2 - 256
*a^2*c^3 - 8*(7*b^2*c^3 - 16*a*c^4)*x^2 + 2*(35*b^3*c^2 - 116*a*b*c^3)*x)*s
qrt(c*x^4 + b*x^3 + a*x^2))/(c^5*x), -1/3840*(15*(7*b^5 - 40*a*b^3*c + 48*a
^2*b*c^2)*sqrt(-c)*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqr
t(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) - 2*(384*c^5*x^4 + 48*b*c^4*x^3 - 105*b^
4*c + 460*a*b^2*c^2 - 256*a^2*c^3 - 8*(7*b^2*c^3 - 16*a*c^4)*x^2 + 2*(35*b^
3*c^2 - 116*a*b*c^3)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^5*x)]
```

giac [A] time = 0.87, size = 283, normalized size = 1.10

$$\frac{1}{1920}\sqrt{cx^2+bx+a}\left(2\left(4\left(6\left(8\,x\mathrm{sgn}(x)+\frac{b\mathrm{sgn}(x)}{c}\right)x-\frac{7\,b^2c^2\mathrm{sgn}(x)-16\,ac^3\mathrm{sgn}(x)}{c^4}\right)x+\frac{35\,b^3c\mathrm{sgn}(x)-116\,a}{c^4}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/1920*sqrt(c*x^2 + b*x + a)*(2*(4*(6*(8*x*sgn(x) + b*sgn(x)/c)*x - (7*b^2*
c^2*sgn(x) - 16*a*c^3*sgn(x))/c^4)*x + (35*b^3*c*sgn(x) - 116*a*b*c^2*sgn(x
))/c^4)*x - (105*b^4*sgn(x) - 460*a*b^2*c*sgn(x) + 256*a^2*c^2*sgn(x))/c^4)
 - 1/256*(7*b^5*sgn(x) - 40*a*b^3*c*sgn(x) + 48*a^2*b*c^2*sgn(x))*log(abs(-
2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(9/2) + 1/3840*(105*b
^5*log(abs(-b + 2*sqrt(a)*sqrt(c))) - 600*a*b^3*c*log(abs(-b + 2*sqrt(a)*sq
rt(c))) + 720*a^2*b*c^2*log(abs(-b + 2*sqrt(a)*sqrt(c))) + 210*sqrt(a)*b^4*
sqrt(c) - 920*a^(3/2)*b^2*c^(3/2) + 512*a^(5/2)*c^(5/2))*sgn(x)/c^(9/2)
```

maple [A] time = 0.01, size = 310, normalized size = 1.21

$$\frac{\sqrt{c\,x^4+b\,x^3+a\,x^2}\,\left(720a^2b\,c^3\ln\left(\frac{2cx+b+2\sqrt{c\,x^2+bx+a}\,\sqrt{c}}{2\sqrt{c}}\right)-600a\,b^3c^2\ln\left(\frac{2cx+b+2\sqrt{c\,x^2+bx+a}\,\sqrt{c}}{2\sqrt{c}}\right)+105b^5c\ln\left(\frac{2cx+\cdots}{}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^4+b*x^3+a*x^2)^(1/2),x)
```

```
[Out] 1/3840*(c*x^4+b*x^3+a*x^2)^(1/2)*(768*x^2*(c*x^2+b*x+a)^(3/2)*c^(9/2)-672*c
^(7/2)*(c*x^2+b*x+a)^(3/2)*x*b-512*c^(7/2)*(c*x^2+b*x+a)^(3/2)*a+560*c^(5/2
```

```
)*(c*x^2+b*x+a)^(3/2)*b^2+720*c^(7/2)*(c*x^2+b*x+a)^(1/2)*x*a*b-420*c^(5/2)
*(c*x^2+b*x+a)^(1/2)*x*b^3+360*c^(5/2)*(c*x^2+b*x+a)^(1/2)*a*b^2-210*c^(3/2
)*(c*x^2+b*x+a)^(1/2)*b^4+720*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b
)/c^(1/2))*a^2*b*c^3-600*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(
1/2))*a*b^3*c^2+105*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))
*b^5*c)/x/(c*x^2+b*x+a)^(1/2)/c^(11/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx^4 + bx^3 + ax^2}\, x^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^3 + a*x^2)*x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\, \sqrt{c\,x^4 + b\,x^3 + a\,x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a*x^2 + b*x^3 + c*x^4)^(1/2),x)
```

```
[Out] int(x^2*(a*x^2 + b*x^3 + c*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \sqrt{x^2 \left(a + bx + cx^2\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**4+b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(x**2*(a + b*x + c*x**2)), x)
```

3.30 $\int x\sqrt{ax^2+bx^3+cx^4}\,dx$

Optimal. Leaf size=205

$$-\frac{x\left(b^2-4ac\right)\left(5b^2-4ac\right)\sqrt{a+bx+cx^2}\,\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{128c^{7/2}\sqrt{ax^2+bx^3+cx^4}}+\frac{b\left(15b^2-52ac\right)\sqrt{ax^2+bx^3+cx^4}}{192c^3x}-\frac{\left(5b^2-1\right.}{}$$

```
[Out] -1/128*(-4*a*c+b^2)*(-4*a*c+5*b^2)*x*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b
*x+a)^(1/2))*(c*x^2+b*x+a)^(1/2)/c^(7/2)/(c*x^4+b*x^3+a*x^2)^(1/2)-1/96*(-1
2*a*c+5*b^2)*(c*x^4+b*x^3+a*x^2)^(1/2)/c^2+1/192*b*(-52*a*c+15*b^2)*(c*x^4+
b*x^3+a*x^2)^(1/2)/c^3/x+1/24*x*(6*c*x+b)*(c*x^4+b*x^3+a*x^2)^(1/2)/c
```

Rubi [A] time = 0.37, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1919, 1949, 12, 1914, 621, 206}

$$\frac{b\left(15b^2-52ac\right)\sqrt{ax^2+bx^3+cx^4}}{192c^3x}-\frac{\left(5b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{96c^2}-\frac{x\left(b^2-4ac\right)\left(5b^2-4ac\right)\sqrt{a+bx+cx^2}\,\tanh}{128c^{7/2}\sqrt{ax^2+bx^3+c}}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a*x^2 + b*x^3 + c*x^4],x]
```

```
[Out] -((5*b^2 - 12*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(96*c^2) + (b*(15*b^2 - 52*
a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(192*c^3*x) + (x*(b + 6*c*x)*Sqrt[a*x^2 +
 b*x^3 + c*x^4])/(24*c) - ((b^2 - 4*a*c)*(5*b^2 - 4*a*c)*x*Sqrt[a + b*x + c
*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(128*c^(7/2)*
Sqrt[a*x^2 + b*x^3 + c*x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1914

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))])/Sqrt[
a*x^q + b*x^n + c*x^(2*n - q)], Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

Rule 1919

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(x^(m - n + q + 1)*(b*(n - q)*p + c*(m + p*q + (n - q)
*(2*p - 1) + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(c*(m + p*(2*
n - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1)), x] + Dist[((n - q)*p)/(c*(m
 + p*(2*n - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1)), Int[x^(m - (n - 2*q
))*Simp[-(a*b*(m + p*q - n + q + 1)) + (2*a*c*(m + p*q + (n - q)*(2*p - 1)
+ 1) - b^2*(m + p*q + (n - q)*(p - 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n +
c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] &&
 PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p
, 0] && RationalQ[m, q] && GtQ[m + p*q + 1, n - q] && NeQ[m + p*(2*n - q) +
 1, 0] && NeQ[m + p*q + (n - q)*(2*p - 1) + 1, 0]
```

Rule 1949

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[(B*x^(m - n + 1)*(a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1))/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), x] -
Dist[1/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(m - n + q)*Simp[a*B*(m
 + p*q - n + q + 1) + (b*B*(m + p*q + (n - q)*p + 1) - A*c*(m + p*q + (n -
q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x]
/; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Integ
erQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[p, -1] && LtQ[p, 0] && R
ationalQ[m, q] && GeQ[m + p*q, n - q - 1] && NeQ[m + p*q + (n - q)*(2*p + 1
) + 1, 0]
```

Rubi steps

$$\int x\sqrt{ax^2+bx^3+cx^4}\,dx = \frac{x(b+6cx)\sqrt{ax^2+bx^3+cx^4}}{24c} + \frac{\int \frac{x^2\left(-2ab-\frac{1}{2}\left(5b^2-12ac\right)x\right)}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{24c}$$

$$= -\frac{\left(5b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{96c^2} + \frac{x(b+6cx)\sqrt{ax^2+bx^3+cx^4}}{24c} - \frac{\int \frac{x\left(-\frac{1}{2}a\left(5b^2-12ac\right)-\right.}{\sqrt{ax^2+bx}}}{48}$$

$$= -\frac{\left(5b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{96c^2} + \frac{b\left(15b^2-52ac\right)\sqrt{ax^2+bx^3+cx^4}}{192c^3x} + \frac{x(b+6cx)\sqrt{}}{}$$

$$= -\frac{\left(5b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{96c^2} + \frac{b\left(15b^2-52ac\right)\sqrt{ax^2+bx^3+cx^4}}{192c^3x} + \frac{x(b+6cx)\sqrt{}}{}$$

$$= -\frac{\left(5b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{96c^2} + \frac{b\left(15b^2-52ac\right)\sqrt{ax^2+bx^3+cx^4}}{192c^3x} + \frac{x(b+6cx)\sqrt{}}{}$$

$$= -\frac{\left(5b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{96c^2} + \frac{b\left(15b^2-52ac\right)\sqrt{ax^2+bx^3+cx^4}}{192c^3x} + \frac{x(b+6cx)\sqrt{}}{}$$

$$= -\frac{\left(5b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{96c^2} + \frac{b\left(15b^2-52ac\right)\sqrt{ax^2+bx^3+cx^4}}{192c^3x} + \frac{x(b+6cx)\sqrt{}}{}$$

Mathematica [A] time = 0.16, size = 150, normalized size = 0.73

$$\frac{2\sqrt{c}\,x(a+x(b+cx))\left(b\left(8c^2x^2-52ac\right)+24c^2x\left(a+2cx^2\right)+15b^3-10b^2cx\right)-3x\left(16a^2c^2-24ab^2c+5b^4\right)\sqrt{a+x}}{384c^{7/2}\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a*x^2 + b*x^3 + c*x^4],x]
```

```
[Out] (2*Sqrt[c]*x*(a + x*(b + c*x))*(15*b^3 - 10*b^2*c*x + 24*c^2*x*(a + 2*c*x^2
) + b*(-52*a*c + 8*c^2*x^2)) - 3*(5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*x*Sqrt[a
 + x*(b + c*x)]*Log[b + 2*c*x + 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/(384*c^(7
/2)*Sqrt[x^2*(a + x*(b + c*x))])
```

fricas [A] time = 0.83, size = 326, normalized size = 1.59

$$\left[\frac{3\left(5\,b^4-24\,ab^2c+16\,a^2c^2\right)\sqrt{c}\,x\log\left(-\frac{8\,c^2x^3+8\,bcx^2-4\,\sqrt{cx^4+bx^3+ax^2}\,(2\,cx+b)\sqrt{c}+\left(b^2+4\,ac\right)x}{x}\right)+4\left(48\,c^4x^3+8\,bc^3x^2+1\right.}{768\,c^4x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/768*(3*(5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*sqrt(c)*x*log(-(8*c^2*x^3 + 8*b
*c*x^2 - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*
x)/x) + 4*(48*c^4*x^3 + 8*b*c^3*x^2 + 15*b^3*c - 52*a*b*c^2 - 2*(5*b^2*c^2
- 12*a*c^3)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^4*x), 1/384*(3*(5*b^4 - 24*a
*b^2*c + 16*a^2*c^2)*sqrt(-c)*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c
*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) + 2*(48*c^4*x^3 + 8*b*c^3*x^2
 + 15*b^3*c - 52*a*b*c^2 - 2*(5*b^2*c^2 - 12*a*c^3)*x)*sqrt(c*x^4 + b*x^3 +
 a*x^2))/(c^4*x)]
```

giac [A] time = 0.75, size = 230, normalized size = 1.12

$$\frac{1}{192}\sqrt{cx^2+bx+a}\left(2\left(4\left(6\,x\mathrm{sgn}(x)+\frac{b\mathrm{sgn}(x)}{c}\right)x-\frac{5\,b^2c\mathrm{sgn}(x)-12\,ac^2\mathrm{sgn}(x)}{c^3}\right)x+\frac{15\,b^3\mathrm{sgn}(x)-52\,abc\mathrm{sgn}(}{c^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/192*sqrt(c*x^2 + b*x + a)*(2*(4*(6*x*sgn(x) + b*sgn(x)/c)*x - (5*b^2*c*sg
n(x) - 12*a*c^2*sgn(x))/c^3)*x + (15*b^3*sgn(x) - 52*a*b*c*sgn(x))/c^3) + 1
/128*(5*b^4*sgn(x) - 24*a*b^2*c*sgn(x) + 16*a^2*c^2*sgn(x))*log(abs(-2*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(7/2) - 1/384*(15*b^4*log(a
bs(-b + 2*sqrt(a)*sqrt(c))) - 72*a*b^2*c*log(abs(-b + 2*sqrt(a)*sqrt(c))) +
 48*a^2*c^2*log(abs(-b + 2*sqrt(a)*sqrt(c))) + 30*sqrt(a)*b^3*sqrt(c) - 104
*a^(3/2)*b*c^(3/2))*sgn(x)/c^(7/2)
```

maple [A] time = 0.01, size = 265, normalized size = 1.29

$$\frac{\sqrt{c\,x^4+b\,x^3+a\,x^2}\,\left(-48a^2c^3\ln\left(\frac{2cx+b+2\sqrt{c\,x^2+bx+a}\,\sqrt{c}}{2\sqrt{c}}\right)+72a\,b^2c^2\ln\left(\frac{2cx+b+2\sqrt{c\,x^2+bx+a}\,\sqrt{c}}{2\sqrt{c}}\right)-15b^4c\ln\left(\frac{2cx+b+2}{}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^4+b*x^3+a*x^2)^(1/2),x)
```

```
[Out] 1/384*(c*x^4+b*x^3+a*x^2)^(1/2)*(96*x*(c*x^2+b*x+a)^(3/2)*c^(7/2)-80*c^(5/2
)*(c*x^2+b*x+a)^(3/2)*b-48*c^(7/2)*(c*x^2+b*x+a)^(1/2)*x*a+60*c^(5/2)*(c*x^
2+b*x+a)^(1/2)*x*b^2-24*c^(5/2)*(c*x^2+b*x+a)^(1/2)*a*b+30*c^(3/2)*(c*x^2+b
*x+a)^(1/2)*b^3-48*ln(1/2*(2*c*x+b+2*(c*x^2+b*x+a)^(1/2)*c^(1/2))/c^(1/2))*
a^2*c^3+72*ln(1/2*(2*c*x+b+2*(c*x^2+b*x+a)^(1/2)*c^(1/2))/c^(1/2))*a*b^2*c^
2-15*ln(1/2*(2*c*x+b+2*(c*x^2+b*x+a)^(1/2)*c^(1/2))/c^(1/2))*b^4*c)/x/(c*x^
2+b*x+a)^(1/2)/c^(9/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx^4 + bx^3 + ax^2}\, x\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^3 + a*x^2)*x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\sqrt{c\,x^4 + b\,x^3 + a\,x^2}\; dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a*x^2 + b*x^3 + c*x^4)^(1/2),x)
```

```
[Out] int(x*(a*x^2 + b*x^3 + c*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{x^2\left(a + bx + cx^2\right)}\; dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**4+b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(x*sqrt(x**2*(a + b*x + c*x**2)), x)
```

3.31 $\int \sqrt{ax^2 + bx^3 + cx^4}\, dx$

Optimal. Leaf size=163

$$\frac{b\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{5/2}x\sqrt{a+bx+cx^2}}-\frac{b(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{8c^2x}+\frac{\left(a+bx+cx^2\right)\sqrt{ax^2+b}}{3cx}$$

```
[Out] -1/8*b*(2*c*x+b)*(c*x^4+b*x^3+a*x^2)^(1/2)/c^2/x+1/3*(c*x^2+b*x+a)*(c*x^4+b
*x^3+a*x^2)^(1/2)/c/x+1/16*b*(-4*a*c+b^2)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*
x^2+b*x+a)^(1/2))*(c*x^4+b*x^3+a*x^2)^(1/2)/c^(5/2)/x/(c*x^2+b*x+a)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1903, 640, 612, 621, 206}

$$\frac{b\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{5/2}x\sqrt{a+bx+cx^2}}-\frac{b(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{8c^2x}+\frac{\left(a+bx+cx^2\right)\sqrt{ax^2+b}}{3cx}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^2 + b*x^3 + c*x^4],x]
```

```
[Out] -(b*(b + 2*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(8*c^2*x) + ((a + b*x + c*x^2)
*Sqrt[a*x^2 + b*x^3 + c*x^4])/(3*c*x) + (b*(b^2 - 4*a*c)*Sqrt[a*x^2 + b*x^3
 + c*x^4]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(5/
2)*x*Sqrt[a + b*x + c*x^2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1903

```
Int[Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol]
 :> Dist[Sqrt[a*x^q + b*x^n + c*x^(2*n - q)]/(x^(q/2)*Sqrt[a + b*x^(n - q)
+ c*x^(2*(n - q))]), Int[x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))], x
```

```
], x] /; FreeQ[{a, b, c, n, q}, x] && EqQ[r, 2*n - q] && PosQ[n - q]
```

Rubi steps

$$\int \sqrt{ax^2+bx^3+cx^4}\,dx = \frac{\sqrt{ax^2+bx^3+cx^4}\int x\sqrt{a+bx+cx^2}\,dx}{x\sqrt{a+bx+cx^2}}$$
$$= \frac{\left(a+bx+cx^2\right)\sqrt{ax^2+bx^3+cx^4}}{3cx} - \frac{\left(b\sqrt{ax^2+bx^3+cx^4}\right)\int \sqrt{a+bx+cx^2}\,dx}{2cx\sqrt{a+bx+cx^2}}$$
$$= -\frac{b(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{8c^2x} + \frac{\left(a+bx+cx^2\right)\sqrt{ax^2+bx^3+cx^4}}{3cx} + \frac{\left(b\left(b^2-4ac\right)\sqrt{a\ldots}\right.}{16c\ldots}$$
$$= -\frac{b(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{8c^2x} + \frac{\left(a+bx+cx^2\right)\sqrt{ax^2+bx^3+cx^4}}{3cx} + \frac{\left(b\left(b^2-4ac\right)\sqrt{a\ldots}\right.}{\ldots}$$
$$= -\frac{b(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{8c^2x} + \frac{\left(a+bx+cx^2\right)\sqrt{ax^2+bx^3+cx^4}}{3cx} + \frac{b\left(b^2-4ac\right)\sqrt{ax\ldots}}{1\ldots}$$

Mathematica [A] time = 0.21, size = 119, normalized size = 0.73

$$\frac{2\sqrt{c}\,x(a+x(b+cx))\left(8c\left(a+cx^2\right)-3b^2+2bcx\right)+3bx\left(b^2-4ac\right)\sqrt{a+x(b+cx)}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{48c^{5/2}\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^2 + b*x^3 + c*x^4],x]
```

```
[Out] (2*Sqrt[c]*x*(a + x*(b + c*x))*(-3*b^2 + 2*b*c*x + 8*c*(a + c*x^2)) + 3*b*(
b^2 - 4*a*c)*x*Sqrt[a + x*(b + c*x)]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a
+ x*(b + c*x)])])/(48*c^(5/2)*Sqrt[x^2*(a + x*(b + c*x))])
```

fricas [A] time = 0.71, size = 260, normalized size = 1.60

$$\left[-\frac{3\left(b^3-4\,abc\right)\sqrt{c}\,x\log\left(-\frac{8\,c^2x^3+8\,bcx^2-4\,\sqrt{cx^4+bx^3+ax^2}\,(2\,cx+b)\sqrt{c}+\left(b^2+4\,ac\right)x}{x}\right)-4\left(8\,c^3x^2+2\,bc^2x-3\,b^2c+8\,ac^2\right)\sqrt{cx^4\ldots}}{96\,c^3x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(b^3 - 4*a*b*c)*sqrt(c)*x*log(-(8*c^2*x^3 + 8*b*c*x^2 - 4*sqrt(c*
x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) - 4*(8*c^3*x
^2 + 2*b*c^2*x - 3*b^2*c + 8*a*c^2)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^3*x), -
1/48*(3*(b^3 - 4*a*b*c)*sqrt(-c)*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(
2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) - 2*(8*c^3*x^2 + 2*b*c^2*x
 - 3*b^2*c + 8*a*c^2)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^3*x)]
```

giac [A] time = 0.88, size = 166, normalized size = 1.02

$$\frac{1}{24}\sqrt{cx^2+bx+a}\left(2\left(4\,x\mathrm{sgn}(x)+\frac{b\mathrm{sgn}(x)}{c}\right)x-\frac{3\,b^2\mathrm{sgn}(x)-8\,ac\mathrm{sgn}(x)}{c^2}\right)-\frac{\left(b^3\mathrm{sgn}(x)-4\,abc\mathrm{sgn}(x)\right)\log\left(\left|-2\left(\sqrt{\ldots}\right.\right.\right.}{16\,c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*sqrt(c*x^2 + b*x + a)*(2*(4*x*sgn(x) + b*sgn(x)/c)*x - (3*b^2*sgn(x) -
 8*a*c*sgn(x))/c^2) - 1/16*(b^3*sgn(x) - 4*a*b*c*sgn(x))*log(abs(-2*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(5/2) + 1/48*(3*b^3*log(abs(-b
 + 2*sqrt(a)*sqrt(c))) - 12*a*b*c*log(abs(-b + 2*sqrt(a)*sqrt(c))) + 6*sqrt
(a)*b^2*sqrt(c) - 16*a^(3/2)*c^(3/2))*sgn(x)/c^(5/2)
```

maple [A] time = 0.01, size = 167, normalized size = 1.02

$$\frac{\sqrt{c\,x^4+b\,x^3+a\,x^2}\,\left(-12ab\,c^2\ln\left(\frac{2cx+b+2\sqrt{c\,x^2+bx+a}\,\sqrt{c}}{2\sqrt{c}}\right)+3b^3c\ln\left(\frac{2cx+b+2\sqrt{c\,x^2+bx+a}\,\sqrt{c}}{2\sqrt{c}}\right)-12\sqrt{c\,x^2+bx+a}\;b\,c \cdots\right.}{48\sqrt{c\,x^2+bx+a}\;c^{\frac{7}{2}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^3+a*x^2)^(1/2),x)
```

```
[Out] 1/48*(c*x^4+b*x^3+a*x^2)^(1/2)*(16*(c*x^2+b*x+a)^(3/2)*c^(5/2)-12*c^(5/2)*(
c*x^2+b*x+a)^(1/2)*x*b-6*c^(3/2)*(c*x^2+b*x+a)^(1/2)*b^2-12*ln(1/2*(2*c*x+b
+2*(c*x^2+b*x+a)^(1/2)*c^(1/2))/c^(1/2))*a*b*c^2+3*ln(1/2*(2*c*x+b+2*(c*x^2
+b*x+a)^(1/2)*c^(1/2))/c^(1/2))*b^3*c)/x/(c*x^2+b*x+a)^(1/2)/c^(7/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx^4 + bx^3 + ax^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^3 + a*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{c\,x^4 + b\,x^3 + a\,x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3 + c*x^4)^(1/2),x)
```

```
[Out] int((a*x^2 + b*x^3 + c*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ax^2 + bx^3 + cx^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a*x**2 + b*x**3 + c*x**4), x)
```

3.32 $\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x}\,dx$

Optimal. Leaf size=119

$$\frac{(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{4cx}-\frac{x\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\,\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\,\sqrt{a+bx+cx^2}}\right)}{8c^{3/2}\sqrt{ax^2+bx^3+cx^4}}$$

```
[Out] -1/8*(-4*a*c+b^2)*x*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))*(c*x
^2+b*x+a)^(1/2)/c^(3/2)/(c*x^4+b*x^3+a*x^2)^(1/2)+1/4*(2*c*x+b)*(c*x^4+b*x^
3+a*x^2)^(1/2)/c/x
```

Rubi [A] time = 0.08, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1918, 1914, 621, 206}

$$\frac{(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{4cx}-\frac{x\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\,\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\,\sqrt{a+bx+cx^2}}\right)}{8c^{3/2}\sqrt{ax^2+bx^3+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^2 + b*x^3 + c*x^4]/x,x]
```

```
[Out] ((b + 2*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(4*c*x) - ((b^2 - 4*a*c)*x*Sqrt[a
 + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*
c^(3/2)*Sqrt[a*x^2 + b*x^3 + c*x^4])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1914

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))])/Sqrt[
a*x^q + b*x^n + c*x^(2*n - q)], Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

Rule 1918

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(x^(m - n + q + 1)*(b + 2*c*x^(n - q))*(a*x^q + b*x^n
+ c*x^(2*n - q))^p)/(2*c*(n - q)*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2
*c*(2*p + 1)), Int[x^(m + q)*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x
] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p]
 && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && Eq
Q[m + p*q + 1, n - q]
```

Rubi steps

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x}\,dx = \frac{(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{4cx} - \frac{\left(b^2-4ac\right)\int \frac{x}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{8c}$$
$$= \frac{(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{4cx} - \frac{\left(\left(b^2-4ac\right)x\sqrt{a+bx+cx^2}\right)\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{8c\sqrt{ax^2+bx^3+cx^4}}$$
$$= \frac{(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{4cx} - \frac{\left(\left(b^2-4ac\right)x\sqrt{a+bx+cx^2}\right)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, -\right.}{4c\sqrt{ax^2+bx^3+cx^4}}$$
$$= \frac{(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{4cx} - \frac{\left(b^2-4ac\right)x\sqrt{a+bx+cx^2}\,\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{3/2}\sqrt{ax^2+bx^3+cx^4}}$$

Mathematica [A] time = 0.13, size = 100, normalized size = 0.84

$$\frac{x\left(2\sqrt{c}(b+2cx)(a+x(b+cx))-\left(b^2-4ac\right)\sqrt{a+x(b+cx)}\log\left(2\sqrt{c}\sqrt{a+x(b+cx)}+b+2cx\right)\right)}{8c^{3/2}\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^2 + b*x^3 + c*x^4]/x,x]
```

```
[Out] (x*(2*Sqrt[c]*(b + 2*c*x)*(a + x*(b + c*x)) - (b^2 - 4*a*c)*Sqrt[a + x*(b +
 c*x)]*Log[b + 2*c*x + 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]]))/(8*c^(3/2)*Sqrt[x
^2*(a + x*(b + c*x))])
```

fricas [A] time = 0.61, size = 220, normalized size = 1.85

$$\left[-\frac{\left(b^2-4\,ac\right)\sqrt{c}\,x\log\left(-\frac{8\,c^2x^3+8\,bcx^2+4\,\sqrt{cx^4+bx^3+ax^2}\,(2\,cx+b)\sqrt{c}+\left(b^2+4\,ac\right)x}{x}\right)-4\,\sqrt{cx^4+bx^3+ax^2}\left(2\,c^2x+bc\right)}{16\,c^2x}, \frac{\left(b^2-\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [-1/16*((b^2 - 4*a*c)*sqrt(c)*x*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4
+ b*x^3 + a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) - 4*sqrt(c*x^4 +
 b*x^3 + a*x^2)*(2*c^2*x + b*c))/(c^2*x), 1/8*((b^2 - 4*a*c)*sqrt(-c)*x*arc
tan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2
 + a*c*x)) + 2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c^2*x + b*c))/(c^2*x)]
```

giac [A] time = 0.95, size = 125, normalized size = 1.05

$$\frac{1}{8}\left(2\sqrt{cx^2+bx+a}\left(2\,x+\frac{b}{c}\right)+\frac{\left(b^2-4\,ac\right)\log\left(\left|-2\left(\sqrt{c}\,x-\sqrt{cx^2+bx+a}\right)\sqrt{c}-b\right|\right)}{c^{\frac{3}{2}}}\right)\text{sgn}(x)-\frac{\left(b^2\log\left(\left|-b+\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 1/8*(2*sqrt(c*x^2 + b*x + a)*(2*x + b/c) + (b^2 - 4*a*c)*log(abs(-2*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(3/2))*sgn(x) - 1/8*(b^2*log(a
```

```
bs(-b + 2*sqrt(a)*sqrt(c))) - 4*a*c*log(abs(-b + 2*sqrt(a)*sqrt(c))) + 2*sq
rt(a)*b*sqrt(c))*sgn(x)/c^(3/2)
```

maple [A] time = 0.00, size = 146, normalized size = 1.23

$$\frac{\sqrt{c\,x^4+b\,x^3+a\,x^2}\,\left(4a\,c^2\ln\left(\frac{2cx+b+2\sqrt{c\,x^2+bx+a}\,\sqrt{c}}{2\sqrt{c}}\right)-b^2c\ln\left(\frac{2cx+b+2\sqrt{c\,x^2+bx+a}\,\sqrt{c}}{2\sqrt{c}}\right)+4\sqrt{c\,x^2+bx+a}\,c^{\frac{5}{2}}x+2\sqrt{c}\right.}{8\sqrt{c\,x^2+bx+a}\,c^{\frac{5}{2}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^3+a*x^2)^(1/2)/x,x)
```

```
[Out] 1/8*(c*x^4+b*x^3+a*x^2)^(1/2)*(4*(c*x^2+b*x+a)^(1/2)*c^(5/2)*x+2*(c*x^2+b*x
+a)^(1/2)*c^(3/2)*b+4*ln(1/2*(2*c*x+b+2*(c*x^2+b*x+a)^(1/2)*c^(1/2))/c^(1/2
))*a*c^2-ln(1/2*(2*c*x+b+2*(c*x^2+b*x+a)^(1/2)*c^(1/2))/c^(1/2))*b^2*c)/(c*
x^2+b*x+a)^(1/2)/c^(5/2)/x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^4+bx^3+ax^2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^3 + a*x^2)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x^4+b\,x^3+a\,x^2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3 + c*x^4)^(1/2)/x,x)
```

```
[Out] int((a*x^2 + b*x^3 + c*x^4)^(1/2)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2\left(a+bx+cx^2\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**3+a*x**2)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(x**2*(a + b*x + c*x**2))/x, x)
```

3.33 $\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^2} dx$

Optimal. Leaf size=173

$$\frac{\sqrt{ax^2+bx^3+cx^4}}{x}-\frac{\sqrt{a}\,x\sqrt{a+bx+cx^2}\,\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\,\sqrt{a+bx+cx^2}}\right)}{\sqrt{ax^2+bx^3+cx^4}}+\frac{bx\sqrt{a+bx+cx^2}\,\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\,\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}\,\sqrt{ax^2+bx^3+cx^4}}$$

```
[Out] -x*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))*a^(1/2)*(c*x^2+b*x+a)
^(1/2)/(c*x^4+b*x^3+a*x^2)^(1/2)+1/2*b*x*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x
^2+b*x+a)^(1/2))*(c*x^2+b*x+a)^(1/2)/c^(1/2)/(c*x^4+b*x^3+a*x^2)^(1/2)+(c*x
^4+b*x^3+a*x^2)^(1/2)/x
```

Rubi [A] time = 0.13, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1921, 1933, 843, 621, 206, 724}

$$\frac{\sqrt{ax^2+bx^3+cx^4}}{x}-\frac{\sqrt{a}\,x\sqrt{a+bx+cx^2}\,\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\,\sqrt{a+bx+cx^2}}\right)}{\sqrt{ax^2+bx^3+cx^4}}+\frac{bx\sqrt{a+bx+cx^2}\,\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\,\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}\,\sqrt{ax^2+bx^3+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^2 + b*x^3 + c*x^4]/x^2,x]
```

```
[Out] Sqrt[a*x^2 + b*x^3 + c*x^4]/x - (Sqrt[a]*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(2
*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/Sqrt[a*x^2 + b*x^3 + c*x^4] +
 (b*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c
*x^2])])/(2*Sqrt[c]*Sqrt[a*x^2 + b*x^3 + c*x^4])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1921

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(x^(m + 1)*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(m + p*(
2*n - q) + 1), x] + Dist[((n - q)*p)/(m + p*(2*n - q) + 1), Int[x^(m + q)*(
2*a + b*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ
[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^
2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && GtQ[m + p*q
+ 1, -(n - q)] && NeQ[m + p*(2*n - q) + 1, 0]
```

Rule 1933

```
Int[((A_) + (B_.)*(x_)^(j_.))/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c
_.)*(x_)^(r_.)], x_Symbol] :> Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(
n - q))])/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)], Int[(A + B*x^(n - q))/(x^(q/
2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]), x], x] /; FreeQ[{a, b, c, A, B
, n, q}, x] && EqQ[j, n - q] && EqQ[r, 2*n - q] && PosQ[n - q] && EqQ[n, 3]
 && EqQ[q, 2]
```

Rubi steps

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^2}\,dx = \frac{\sqrt{ax^2+bx^3+cx^4}}{x} + \frac{1}{2}\int \frac{2a+bx}{\sqrt{ax^2+bx^3+cx^4}}\,dx$$

$$= \frac{\sqrt{ax^2+bx^3+cx^4}}{x} + \frac{\left(x\sqrt{a+bx+cx^2}\right)\int \frac{2a+bx}{x\sqrt{a+bx+cx^2}}\,dx}{2\sqrt{ax^2+bx^3+cx^4}}$$

$$= \frac{\sqrt{ax^2+bx^3+cx^4}}{x} + \frac{\left(ax\sqrt{a+bx+cx^2}\right)\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx}{\sqrt{ax^2+bx^3+cx^4}} + \frac{\left(bx\sqrt{a+bx+cx^2}\right)\int \frac{}{\sqrt{a+}}}{2\sqrt{ax^2+bx^3+cx}}$$

$$= \frac{\sqrt{ax^2+bx^3+cx^4}}{x} - \frac{\left(2ax\sqrt{a+bx+cx^2}\right)\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx}{\sqrt{a+bx+cx^2}}\right)}{\sqrt{ax^2+bx^3+cx^4}} + \frac{\left(bx\sqrt{a}\right.}{}$$

$$= \frac{\sqrt{ax^2+bx^3+cx^4}}{x} - \frac{\sqrt{a}\,x\sqrt{a+bx+cx^2}\,\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{ax^2+bx^3+cx^4}} + \frac{bx\sqrt{a+bx+cx^2}}{2\sqrt{c}\sqrt{ax}}$$

Mathematica [A] time = 0.10, size = 134, normalized size = 0.77

$$\frac{x\sqrt{a+x(b+cx)}\left(2\sqrt{c}\sqrt{a+x(b+cx)}-2\sqrt{a}\sqrt{c}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right)+b\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\right)}{2\sqrt{c}\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^2 + b*x^3 + c*x^4]/x^2,x]
```

```
[Out] (x*Sqrt[a + x*(b + c*x)]*(2*Sqrt[c]*Sqrt[a + x*(b + c*x)] - 2*Sqrt[a]*Sqrt[
c]*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])] + b*ArcTanh[(b +
2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))/(2*Sqrt[c]*Sqrt[x^2*(a + x*(b +
 c*x))])
```

fricas [A] time = 0.67, size = 638, normalized size = 3.69

$$\left[\frac{b\sqrt{c}\,x\log\left(-\frac{8c^2x^3+8bcx^2+4\sqrt{cx^4+bx^3+ax^2}\,(2cx+b)\sqrt{c}+\left(b^2+4ac\right)x}{x}\right)+2\sqrt{a}\,cx\log\left(-\frac{8abx^2+\left(b^2+4ac\right)x^3+8a^2x-4\sqrt{cx^4+bx^3+ax^2}\,(bx+}{x^3}\right.}{4cx}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/4*(b*sqrt(c)*x*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^
2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) + 2*sqrt(a)*c*x*log(-(8*a*b*x^
2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)
*sqrt(a))/x^3) + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*c)/(c*x), -1/2*(b*sqrt(-c)*x
*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c
*x^2 + a*c*x)) - sqrt(a)*c*x*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x
- 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) - 2*sqrt(c*x^4 +
b*x^3 + a*x^2)*c)/(c*x), 1/4*(4*sqrt(-a)*c*x*arctan(1/2*sqrt(c*x^4 + b*x^3
+ a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) + b*sqrt(c)*x*lo
g(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(
c) + (b^2 + 4*a*c)*x)/x) + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*c)/(c*x), 1/2*(2*s
qrt(-a)*c*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*
c*x^3 + a*b*x^2 + a^2*x)) - b*sqrt(-c)*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*
x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) + 2*sqrt(c*x^4 + b*x
^3 + a*x^2)*c)/(c*x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
index.cc index_m operator + Error: Bad Argument Value
```

maple [A] time = 0.01, size = 126, normalized size = 0.73

$$-\frac{\sqrt{c\,x^{4}+b\,x^{3}+a\,x^{2}}\,\left(2\sqrt{a}\,\sqrt{c}\,\ln\left(\frac{bx+2a+2\sqrt{c\,x^{2}+bx+a}\,\sqrt{a}}{x}\right)-b\ln\left(\frac{2cx+b+2\sqrt{c\,x^{2}+bx+a}\,\sqrt{c}}{2\sqrt{c}}\right)-2\sqrt{c\,x^{2}+bx+a}\,\sqrt{c}\right)}{2\sqrt{c\,x^{2}+bx+a}\,\sqrt{c}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^3+a*x^2)^(1/2)/x^2,x)
```

```
[Out] -1/2*(c*x^4+b*x^3+a*x^2)^(1/2)*(2*a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+
a)^(1/2))/x)*c^(1/2)-2*(c*x^2+b*x+a)^(1/2)*c^(1/2)-b*ln(1/2*(2*c*x+b+2*(c*x
^2+b*x+a)^(1/2)*c^(1/2))/c^(1/2)))/x/(c*x^2+b*x+a)^(1/2)/c^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^{4}+bx^{3}+ax^{2}}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^3 + a*x^2)/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x^{4}+b\,x^{3}+a\,x^{2}}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3 + c*x^4)^(1/2)/x^2,x)
[Out] int((a*x^2 + b*x^3 + c*x^4)^(1/2)/x^2, x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2 \left(a + bx + cx^2\right)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**3+a*x**2)**(1/2)/x**2,x)
[Out] Integral(sqrt(x**2*(a + b*x + c*x**2))/x**2, x)
```

3.34 $\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^3} \, dx$

Optimal. Leaf size=173

$$-\frac{\sqrt{ax^2+bx^3+cx^4}}{x^2}-\frac{bx\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}+\frac{\sqrt{c}\,x\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{ax^2+bx^3+cx^4}}$$

```
[Out] -1/2*b*x*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))*(c*x^2+b*x+a)^(
1/2)/a^(1/2)/(c*x^4+b*x^3+a*x^2)^(1/2)+x*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x
^2+b*x+a)^(1/2))*c^(1/2)*(c*x^2+b*x+a)^(1/2)/(c*x^4+b*x^3+a*x^2)^(1/2)-(c*x
^4+b*x^3+a*x^2)^(1/2)/x^2
```

Rubi [A] time = 0.12, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = $\{1920, 1933, 843, 621, 206, 724\}$

$$-\frac{\sqrt{ax^2+bx^3+cx^4}}{x^2}-\frac{bx\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}+\frac{\sqrt{c}\,x\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{ax^2+bx^3+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^2 + b*x^3 + c*x^4]/x^3,x]
```

```
[Out] -(Sqrt[a*x^2 + b*x^3 + c*x^4]/x^2) - (b*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(2*
a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3
+ c*x^4]) + (Sqrt[c]*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]
*Sqrt[a + b*x + c*x^2])])/Sqrt[a*x^2 + b*x^3 + c*x^4]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1920

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(x^(m + 1)*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(m + p*q
 + 1), x] - Dist[((n - q)*p)/(m + p*q + 1), Int[x^(m + n)*(b + 2*c*x^(n - q
))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &
& EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] &&
 IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q + 1, -(n - q) +
1] && NeQ[m + p*q + 1, 0]
```

Rule 1933

```
Int[((A_) + (B_.)*(x_)^(j_.))/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c
_.)*(x_)^(r_.)], x_Symbol] :> Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(
n - q))])/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)], Int[(A + B*x^(n - q))/(x^(q/
2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]), x], x] /; FreeQ[{a, b, c, A, B
, n, q}, x] && EqQ[j, n - q] && EqQ[r, 2*n - q] && PosQ[n - q] && EqQ[n, 3]
 && EqQ[q, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^3}\,dx &= -\frac{\sqrt{ax^2+bx^3+cx^4}}{x^2}+\frac{1}{2}\int \frac{b+2cx}{\sqrt{ax^2+bx^3+cx^4}}\,dx\\ &= -\frac{\sqrt{ax^2+bx^3+cx^4}}{x^2}+\frac{\left(x\sqrt{a+bx+cx^2}\right)\int \frac{b+2cx}{x\sqrt{a+bx+cx^2}}\,dx}{2\sqrt{ax^2+bx^3+cx^4}}\\ &= -\frac{\sqrt{ax^2+bx^3+cx^4}}{x^2}+\frac{\left(bx\sqrt{a+bx+cx^2}\right)\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx}{2\sqrt{ax^2+bx^3+cx^4}}+\frac{\left(cx\sqrt{a+bx+cx^2}\right)\int \frac{}{\sqrt{}}}{\sqrt{ax^2+bx^3+c}}\\ &= -\frac{\sqrt{ax^2+bx^3+cx^4}}{x^2}-\frac{\left(bx\sqrt{a+bx+cx^2}\right)\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx,x,\frac{2a+bx}{\sqrt{a+bx+cx^2}}\right)}{\sqrt{ax^2+bx^3+cx^4}}+\frac{\left(2cx\sqrt{a}\right.}{}\\ &= -\frac{\sqrt{ax^2+bx^3+cx^4}}{x^2}-\frac{bx\sqrt{a+bx+cx^2}\,\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}+\frac{\sqrt{c}\,x\sqrt{a+bx+cx^2}}{\sqrt{ax^2}}\end{aligned}$$

Mathematica [A] time = 0.12, size = 131, normalized size = 0.76

$$-\frac{\sqrt{a+x(b+cx)}\left(bx\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right)+2\sqrt{a}\left(\sqrt{a+x(b+cx)}-\sqrt{c}\,x\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\right)\right)}{2\sqrt{a}\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^2 + b*x^3 + c*x^4]/x^3,x]
```

```
[Out] -1/2*(Sqrt[a + x*(b + c*x)]*(b*x*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*
(b + c*x)])] + 2*Sqrt[a]*(Sqrt[a + x*(b + c*x)] - Sqrt[c]*x*ArcTanh[(b + 2*
c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])))/(Sqrt[a]*Sqrt[x^2*(a + x*(b + c*
x))])
```

fricas [A] time = 0.79, size = 653, normalized size = 3.77

$$\left[\frac{2\,a\sqrt{c}\,x^2\log\left(-\frac{8\,c^2x^3+8\,bcx^2+4\,\sqrt{cx^4+bx^3+ax^2}\,(2\,cx+b)\sqrt{c}+\left(b^2+4\,ac\right)x}{x}\right)+\sqrt{a}\,bx^2\log\left(-\frac{8\,abx^2+\left(b^2+4\,ac\right)x^3+8\,a^2x-4\,\sqrt{cx^4+bx^3+ax^2}\,(b}{x^3}\right.}{4\,ax^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/4*(2*a*sqrt(c)*x^2*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 +
a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) + sqrt(a)*b*x^2*log(-(8*a*
b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x +
2*a)*sqrt(a))/x^3) - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*a)/(a*x^2), -1/4*(4*a*sq
rt(-c)*x^2*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2
*x^3 + b*c*x^2 + a*c*x)) - sqrt(a)*b*x^2*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^
3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) + 4*s
qrt(c*x^4 + b*x^3 + a*x^2)*a)/(a*x^2), 1/2*(sqrt(-a)*b*x^2*arctan(1/2*sqrt(
c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) +
a*sqrt(c)*x^2*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(
2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) - 2*sqrt(c*x^4 + b*x^3 + a*x^2)*a)
/(a*x^2), 1/2*(sqrt(-a)*b*x^2*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x +
 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) - 2*a*sqrt(-c)*x^2*arctan(1/2*s
qrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)
) - 2*sqrt(c*x^4 + b*x^3 + a*x^2)*a)/(a*x^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Warning, choosing root of [1,0,%%%{-2,[1,0,0,2]%%%}+%%%{-2,[0,1,0,1]%%%}+%%
%{-4,[0,0,1,0]%%%},0,%%%{1,[2,0,0,4]%%%}+%%%{2,[1,1,0,3]%%%}+%%%{1,[0,2,0,2
]%%%}] at parameters values [-97,-82,63.4443001123,-27]Warning, choosing ro
ot of [1,0,%%%{-2,[1,0,0,2]%%%}+%%%{-2,[0,1,0,1]%%%}+%%%{-4,[0,0,1,0]%%%},0
,%%%{1,[2,0,0,4]%%%}+%%%{2,[1,1,0,3]%%%}+%%%{1,[0,2,0,2]%%%}] at parameters
 values [63,-49,35.2935628123,-64]Warning, choosing root of [1,0,%%%{-2,[2,
1,0,0]%%%}+%%%{-2,[1,0,1,0]%%%}+%%%{-4,[0,0,0,1]%%%},0,%%%{1,[4,2,0,0]%%%}+
%%%{2,[3,1,1,0]%%%}+%%%{1,[2,0,2,0]%%%}] at parameters values [22,42,56,43.
9628838282]Sign error (%%%{b-2*sqrt(a)*sqrt(c),0%%%}+%%%{-(-2*a*c+b*sqrt(a)
*sqrt(c))/a,1%%%}+%%%{-(4*a*c*sqrt(a)*sqrt(c)-b^2*sqrt(a)*sqrt(c))/(4*a^2),
2%%%}+%%%{undef,3%%%})Limit: Max order reached or unable to make series exp
ansion Error: Bad Argument Value
```

maple [A] time = 0.01, size = 173, normalized size = 1.00

$$-\frac{\sqrt{c x^4+b x^3+a x^2}\,\left(-2 a c^2 x \ln\left(\frac{2cx+b+2\sqrt{c x^2+bx+a}\,\sqrt{c}}{2\sqrt{c}}\right)+\sqrt{a}\, b c^{\frac{3}{2}} x \ln\left(\frac{bx+2a+2\sqrt{c x^2+bx+a}\,\sqrt{a}}{x}\right)-2\sqrt{c x^2+bx+}\right.}{2\sqrt{c x^2+bx+a}\, a c^{\frac{3}{2}} x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^3+a*x^2)^(1/2)/x^3,x)
```

```
[Out] -1/2*(c*x^4+b*x^3+a*x^2)^(1/2)*(-2*(c*x^2+b*x+a)^(1/2)*c^(5/2)*x^2+c^(3/2)*
a^(1/2)*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)*x*b+2*(c*x^2+b*x+a)^(
3/2)*c^(3/2)-2*(c*x^2+b*x+a)^(1/2)*c^(3/2)*x*b-2*ln(1/2*(2*c*x+b+2*(c*x^2+b
*x+a)^(1/2)*c^(1/2))/c^(1/2))*x*a*c^2)/x^2/(c*x^2+b*x+a)^(1/2)/a/c^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^4+bx^3+ax^2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^3 + a*x^2)/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x^4+b\,x^3+a\,x^2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3 + c*x^4)^(1/2)/x^3,x)
```

```
[Out] int((a*x^2 + b*x^3 + c*x^4)^(1/2)/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2\left(a+bx+cx^2\right)}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**3+a*x**2)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(x**2*(a + b*x + c*x**2))/x**3, x)
```

3.35 $\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^4} \, dx$

Optimal. Leaf size=114

$$\frac{(b^2-4ac)\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{8a^{3/2}}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{4ax^2}-\frac{\sqrt{ax^2+bx^3+cx^4}}{2x^3}$$

```
[Out] 1/8*(-4*a*c+b^2)*arctanh(1/2*x*(b*x+2*a)/a^(1/2)/(c*x^4+b*x^3+a*x^2)^(1/2))
/a^(3/2)-1/2*(c*x^4+b*x^3+a*x^2)^(1/2)/x^3-1/4*b*(c*x^4+b*x^3+a*x^2)^(1/2)/
a/x^2
```

Rubi [A] time = 0.15, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1920, 1951, 12, 1904, 206}

$$\frac{(b^2-4ac)\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{8a^{3/2}}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{4ax^2}-\frac{\sqrt{ax^2+bx^3+cx^4}}{2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^2 + b*x^3 + c*x^4]/x^4,x]
```

```
[Out] -Sqrt[a*x^2 + b*x^3 + c*x^4]/(2*x^3) - (b*Sqrt[a*x^2 + b*x^3 + c*x^4])/(4*a
*x^2) + ((b^2 - 4*a*c)*ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^
3 + c*x^4])])/(8*a^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1904

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, (x*(2*a + b*x^(n - 2)))/
Sqrt[a*x^2 + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n
 - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1920

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(x^(m + 1)*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(m + p*q
 + 1), x] - Dist[((n - q)*p)/(m + p*q + 1), Int[x^(m + n)*(b + 2*c*x^(n - q
))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &
& EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] &&
 IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q + 1, -(n - q) +
1] && NeQ[m + p*q + 1, 0]
```

Rule 1951

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[(A*x^(m - q + 1)*(a*x^q + b
```

```
*x^n + c*x^(2*n - q))^(p + 1))/(a*(m + p*q + 1)), x] + Dist[1/(a*(m + p*q +
 1)), Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p
+ 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b*
x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q
] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
&& RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n - q
)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^4}\,dx &= -\frac{\sqrt{ax^2+bx^3+cx^4}}{2x^3}+\frac{1}{4}\int \frac{b+2cx}{x\sqrt{ax^2+bx^3+cx^4}}\,dx\\
&= -\frac{\sqrt{ax^2+bx^3+cx^4}}{2x^3}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{4ax^2}-\frac{\int \frac{b^2-4ac}{2\sqrt{ax^2+bx^3+cx^4}}\,dx}{4a}\\
&= -\frac{\sqrt{ax^2+bx^3+cx^4}}{2x^3}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{4ax^2}-\frac{\left(b^2-4ac\right)\int \frac{1}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{8a}\\
&= -\frac{\sqrt{ax^2+bx^3+cx^4}}{2x^3}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{4ax^2}+\frac{\left(b^2-4ac\right)\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx,x,\frac{x(2a+b}{\sqrt{ax^2+bx^3}}\right.}{4a}\\
&= -\frac{\sqrt{ax^2+bx^3+cx^4}}{2x^3}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{4ax^2}+\frac{\left(b^2-4ac\right)\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{8a^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.10, size = 112, normalized size = 0.98

$$\frac{\sqrt{x^2(a+x(b+cx))}\left(x^2\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right)-2\sqrt{a}(2a+bx)\sqrt{a+x(b+cx)}\right)}{8a^{3/2}x^3\sqrt{a+x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^2 + b*x^3 + c*x^4]/x^4,x]
```

```
[Out] (Sqrt[x^2*(a + x*(b + c*x))]*(-2*Sqrt[a]*(2*a + b*x)*Sqrt[a + x*(b + c*x)]
+ (b^2 - 4*a*c)*x^2*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])])
)/(8*a^(3/2)*x^3*Sqrt[a + x*(b + c*x)])
```

fricas [A] time = 0.77, size = 226, normalized size = 1.98

$$\left[-\frac{\left(b^2-4\,ac\right)\sqrt{a}\,x^3\log\left(-\frac{8\,abx^2+\left(b^2+4\,ac\right)x^3+8\,a^2x-4\,\sqrt{cx^4+bx^3+ax^2}\,(bx+2\,a)\sqrt{a}}{x^3}\right)+4\,\sqrt{cx^4+bx^3+ax^2}\left(abx+2\,a^2\right)}{16\,a^2x^3},-\frac{\left(b^2-\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] [-1/16*((b^2 - 4*a*c)*sqrt(a)*x^3*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a
^2*x - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) + 4*sqrt(c*x
^4 + b*x^3 + a*x^2)*(a*b*x + 2*a^2))/(a^2*x^3), -1/8*((b^2 - 4*a*c)*sqrt(-a
)*x^3*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3
+ a*b*x^2 + a^2*x)) + 2*sqrt(c*x^4 + b*x^3 + a*x^2)*(a*b*x + 2*a^2))/(a^2*x
^3)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.01, size = 207, normalized size = 1.82

$$-\frac{\sqrt{c\,x^{4}+b\,x^{3}+a\,x^{2}}\,\left(4a^{\frac{3}{2}}c\,x^{2}\ln\left(\frac{bx+2a+2\sqrt{c\,x^{2}+bx+a}\,\sqrt{a}}{x}\right)-\sqrt{a}\,b^{2}x^{2}\ln\left(\frac{bx+2a+2\sqrt{c\,x^{2}+bx+a}\,\sqrt{a}}{x}\right)+2\sqrt{c\,x^{2}+bx+}\right.}{8\sqrt{c\,x^{2}+bx+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^3+a*x^2)^(1/2)/x^4,x)
```

```
[Out] -1/8*(c*x^4+b*x^3+a*x^2)^(1/2)*(4*c*a^(3/2)*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/
2)*a^(1/2))/x)*x^2+2*c*(c*x^2+b*x+a)^(1/2)*x^3*b-4*c*(c*x^2+b*x+a)^(1/2)*x^
2*a-a^(1/2)*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)*x^2*b^2-2*(c*x^2+
b*x+a)^(3/2)*x*b+2*(c*x^2+b*x+a)^(1/2)*x^2*b^2+4*(c*x^2+b*x+a)^(3/2)*a)/x^3
/(c*x^2+b*x+a)^(1/2)/a^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^{4}+bx^{3}+ax^{2}}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^3 + a*x^2)/x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x^{4}+b\,x^{3}+a\,x^{2}}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3 + c*x^4)^(1/2)/x^4,x)
```

```
[Out] int((a*x^2 + b*x^3 + c*x^4)^(1/2)/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^{2}\left(a+bx+cx^{2}\right)}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**3+a*x**2)**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(x**2*(a + b*x + c*x**2))/x**4, x)
```

3.36 $\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^5}\,dx$

Optimal. Leaf size=155

$$-\frac{b\left(b^2-4ac\right)\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{16a^{5/2}}+\frac{\left(3b^2-8ac\right)\sqrt{ax^2+bx^3+cx^4}}{24a^2x^2}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{12ax^3}-\frac{\sqrt{ax^2+bx^3+cx^4}}{3x^4}$$

```
[Out] -1/16*b*(-4*a*c+b^2)*arctanh(1/2*x*(b*x+2*a)/a^(1/2)/(c*x^4+b*x^3+a*x^2)^(1
/2))/a^(5/2)-1/3*(c*x^4+b*x^3+a*x^2)^(1/2)/x^4-1/12*b*(c*x^4+b*x^3+a*x^2)^(
1/2)/a/x^3+1/24*(-8*a*c+3*b^2)*(c*x^4+b*x^3+a*x^2)^(1/2)/a^2/x^2
```

Rubi [A] time = 0.26, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1920, 1951, 12, 1904, 206}

$$\frac{\left(3b^2-8ac\right)\sqrt{ax^2+bx^3+cx^4}}{24a^2x^2}-\frac{b\left(b^2-4ac\right)\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{16a^{5/2}}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{12ax^3}-\frac{\sqrt{ax^2+bx^3+cx^4}}{3x^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^2 + b*x^3 + c*x^4]/x^5,x]
```

```
[Out] -Sqrt[a*x^2 + b*x^3 + c*x^4]/(3*x^4) - (b*Sqrt[a*x^2 + b*x^3 + c*x^4])/(12*
a*x^3) + ((3*b^2 - 8*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(24*a^2*x^2) - (b*(b
^2 - 4*a*c)*ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])
])/(16*a^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1904

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, (x*(2*a + b*x^(n - 2)))/
Sqrt[a*x^2 + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n
 - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1920

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(x^(m + 1)*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(m + p*q
 + 1), x] - Dist[((n - q)*p)/(m + p*q + 1), Int[x^(m + n)*(b + 2*c*x^(n - q
))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &
& EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] &&
 IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q + 1, -(n - q) +
1] && NeQ[m + p*q + 1, 0]
```

Rule 1951

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[(A*x^(m - q + 1)*(a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1))/(a*(m + p*q + 1)), x] + Dist[1/(a*(m + p*q +
 1)), Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p
+ 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b*
x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q
] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
&& RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n - q
)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]
```

Rubi steps

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^5}\,dx = -\frac{\sqrt{ax^2+bx^3+cx^4}}{3x^4}+\frac{1}{6}\int \frac{b+2cx}{x^2\sqrt{ax^2+bx^3+cx^4}}\,dx$$

$$= -\frac{\sqrt{ax^2+bx^3+cx^4}}{3x^4}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{12ax^3}-\frac{\int \frac{\frac{1}{2}\left(3b^2-8ac\right)+bcx}{x\sqrt{ax^2+bx^3+cx^4}}\,dx}{12a}$$

$$= -\frac{\sqrt{ax^2+bx^3+cx^4}}{3x^4}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{12ax^3}+\frac{\left(3b^2-8ac\right)\sqrt{ax^2+bx^3+cx^4}}{24a^2x^2}+\frac{\int \frac{}{4\cdots}}{}$$

$$= -\frac{\sqrt{ax^2+bx^3+cx^4}}{3x^4}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{12ax^3}+\frac{\left(3b^2-8ac\right)\sqrt{ax^2+bx^3+cx^4}}{24a^2x^2}+\frac{\left(b\left(\cdots\right.\right.}{}$$

$$= -\frac{\sqrt{ax^2+bx^3+cx^4}}{3x^4}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{12ax^3}+\frac{\left(3b^2-8ac\right)\sqrt{ax^2+bx^3+cx^4}}{24a^2x^2}-\frac{\left(b\left(\cdots\right.\right.}{}$$

$$= -\frac{\sqrt{ax^2+bx^3+cx^4}}{3x^4}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{12ax^3}+\frac{\left(3b^2-8ac\right)\sqrt{ax^2+bx^3+cx^4}}{24a^2x^2}-\frac{b\left(b^{\cdots}\right.}{}$$

Mathematica [A] time = 0.12, size = 131, normalized size = 0.85

$$\frac{\sqrt{x^2(a+x(b+cx))}\left(-2\sqrt{a}\sqrt{a+x(b+cx)}\left(8a^2+2ax(b+4cx)-3b^2x^2\right)-3bx^3\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+\cdots}}\right.\right.}{48a^{5/2}x^4\sqrt{a+x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^2 + b*x^3 + c*x^4]/x^5,x]
```

```
[Out] (Sqrt[x^2*(a + x*(b + c*x))]*(-2*Sqrt[a]*Sqrt[a + x*(b + c*x)]*(8*a^2 - 3*b
^2*x^2 + 2*a*x*(b + 4*c*x)) - 3*b*(b^2 - 4*a*c)*x^3*ArcTanh[(2*a + b*x)/(2*
Sqrt[a]*Sqrt[a + x*(b + c*x)])]))/(48*a^(5/2)*x^4*Sqrt[a + x*(b + c*x)])
```

fricas [A] time = 0.74, size = 272, normalized size = 1.75

$$\left[-\frac{3\left(b^3-4\,abc\right)\sqrt{a}\,x^4\log\left(-\frac{8\,abx^2+\left(b^2+4\,ac\right)x^3+8\,a^2x+4\,\sqrt{cx^4+bx^3+ax^2}\,(bx+2\,a)\sqrt{a}}{x^3}\right)+4\,\sqrt{cx^4+bx^3+ax^2}\left(2\,a^2bx+8\,a^3\cdots\right.}{96\,a^3x^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(b^3 - 4*a*b*c)*sqrt(a)*x^4*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 +
 8*a^2*x + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) + 4*sqrt
```

```
(c*x^4 + b*x^3 + a*x^2)*(2*a^2*b*x + 8*a^3 - (3*a*b^2 - 8*a^2*c)*x^2))/(a^3
*x^4), 1/48*(3*(b^3 - 4*a*b*c)*sqrt(-a)*x^4*arctan(1/2*sqrt(c*x^4 + b*x^3 +
 a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) - 2*sqrt(c*x^4 +
b*x^3 + a*x^2)*(2*a^2*b*x + 8*a^3 - (3*a*b^2 - 8*a^2*c)*x^2))/(a^3*x^4)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.01, size = 234, normalized size = 1.51

$$\frac{\sqrt{c\,x^4+b\,x^3+a\,x^2}\,\left(12 a^{\frac{3}{2}} b c\,x^3 \ln\left(\frac{bx+2a+2\sqrt{c\,x^2+bx+a}\,\sqrt{a}}{x}\right)-3\sqrt{a}\,b^3 x^3 \ln\left(\frac{bx+2a+2\sqrt{c\,x^2+bx+a}\,\sqrt{a}}{x}\right)+6\sqrt{c\,x^2+bx+a}\cdots\right)}{4\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^3+a*x^2)^(1/2)/x^5,x)
```

```
[Out] 1/48*(c*x^4+b*x^3+a*x^2)^(1/2)*(12*c*a^(3/2)*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1
/2)*a^(1/2))/x)*x^3*b+6*c*(c*x^2+b*x+a)^(1/2)*x^4*b^2-12*c*(c*x^2+b*x+a)^(1
/2)*x^3*a*b-3*a^(1/2)*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)*x^3*b^3
-6*(c*x^2+b*x+a)^(3/2)*x^2*b^2+6*(c*x^2+b*x+a)^(1/2)*x^3*b^3+12*(c*x^2+b*x+
a)^(3/2)*x*a*b-16*(c*x^2+b*x+a)^(3/2)*a^2)/x^4/(c*x^2+b*x+a)^(1/2)/a^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^4+bx^3+ax^2}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^3 + a*x^2)/x^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x^4+b\,x^3+a\,x^2}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3 + c*x^4)^(1/2)/x^5,x)
```

```
[Out] int((a*x^2 + b*x^3 + c*x^4)^(1/2)/x^5, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2\left(a+bx+cx^2\right)}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**3+a*x**2)**(1/2)/x**5,x)
```

```
[Out] Integral(sqrt(x**2*(a + b*x + c*x**2))/x**5, x)
```

3.37 $\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^6}\,dx$

Optimal. Leaf size=205

$$\frac{\left(b^2-4ac\right)\left(5b^2-4ac\right)\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{128a^{7/2}}-\frac{b\left(15b^2-52ac\right)\sqrt{ax^2+bx^3+cx^4}}{192a^3x^2}+\frac{\left(5b^2-12ac\right)\sqrt{ax^2+bx}}{96a^2x^3}$$

```
[Out] 1/128*(-4*a*c+b^2)*(-4*a*c+5*b^2)*arctanh(1/2*x*(b*x+2*a)/a^(1/2)/(c*x^4+b*
x^3+a*x^2)^(1/2))/a^(7/2)-1/4*(c*x^4+b*x^3+a*x^2)^(1/2)/x^5-1/24*b*(c*x^4+b
*x^3+a*x^2)^(1/2)/a/x^4+1/96*(-12*a*c+5*b^2)*(c*x^4+b*x^3+a*x^2)^(1/2)/a^2/
x^3-1/192*b*(-52*a*c+15*b^2)*(c*x^4+b*x^3+a*x^2)^(1/2)/a^3/x^2
```

Rubi [A] time = 0.39, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1920, 1951, 12, 1904, 206}

$$-\frac{b\left(15b^2-52ac\right)\sqrt{ax^2+bx^3+cx^4}}{192a^3x^2}+\frac{\left(5b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{96a^2x^3}+\frac{\left(b^2-4ac\right)\left(5b^2-4ac\right)\tanh^{-1}\left(\frac{x(2a}{2\sqrt{a}\sqrt{ax^2}}\right.}{128a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^2 + b*x^3 + c*x^4]/x^6,x]
```

```
[Out] -Sqrt[a*x^2 + b*x^3 + c*x^4]/(4*x^5) - (b*Sqrt[a*x^2 + b*x^3 + c*x^4])/(24*
a*x^4) + ((5*b^2 - 12*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(96*a^2*x^3) - (b*(
15*b^2 - 52*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(192*a^3*x^2) + ((b^2 - 4*a*c
)*(5*b^2 - 4*a*c)*ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c
*x^4])])/(128*a^(7/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1904

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, (x*(2*a + b*x^(n - 2)))/
Sqrt[a*x^2 + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n
 - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1920

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(x^(m + 1)*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(m + p*q
 + 1), x] - Dist[((n - q)*p)/(m + p*q + 1), Int[x^(m + n)*(b + 2*c*x^(n - q
))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &
& EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] &&
 IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q + 1, -(n - q) +
1] && NeQ[m + p*q + 1, 0]
```

Rule 1951

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[(A*x^(m - q + 1)*(a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1))/(a*(m + p*q + 1)), x] + Dist[1/(a*(m + p*q +
 1)), Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p
+ 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b*
x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q
] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
&& RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n - q
)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]
```

Rubi steps

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^6}\,dx = -\frac{\sqrt{ax^2+bx^3+cx^4}}{4x^5} + \frac{1}{8}\int \frac{b+2cx}{x^3\sqrt{ax^2+bx^3+cx^4}}\,dx$$

$$= -\frac{\sqrt{ax^2+bx^3+cx^4}}{4x^5} - \frac{b\sqrt{ax^2+bx^3+cx^4}}{24ax^4} - \frac{\int \frac{\frac{1}{2}(5b^2-12ac)+2bcx}{x^2\sqrt{ax^2+bx^3+cx^4}}\,dx}{24a}$$

$$= -\frac{\sqrt{ax^2+bx^3+cx^4}}{4x^5} - \frac{b\sqrt{ax^2+bx^3+cx^4}}{24ax^4} + \frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{96a^2x^3} + \int \frac{\frac{1}{4}b(1$$

$$= -\frac{\sqrt{ax^2+bx^3+cx^4}}{4x^5} - \frac{b\sqrt{ax^2+bx^3+cx^4}}{24ax^4} + \frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{96a^2x^3} - \frac{b(15b$$

$$= -\frac{\sqrt{ax^2+bx^3+cx^4}}{4x^5} - \frac{b\sqrt{ax^2+bx^3+cx^4}}{24ax^4} + \frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{96a^2x^3} - \frac{b(15b$$

$$= -\frac{\sqrt{ax^2+bx^3+cx^4}}{4x^5} - \frac{b\sqrt{ax^2+bx^3+cx^4}}{24ax^4} + \frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{96a^2x^3} - \frac{b(15b$$

$$= -\frac{\sqrt{ax^2+bx^3+cx^4}}{4x^5} - \frac{b\sqrt{ax^2+bx^3+cx^4}}{24ax^4} + \frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{96a^2x^3} - \frac{b(15b$$

Mathematica [A] time = 0.17, size = 160, normalized size = 0.78

$$\frac{\sqrt{x^2(a+x(b+cx))}\left(3x^4\left(16a^2c^2-24ab^2c+5b^4\right)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right)-2\sqrt{a}\sqrt{a+x(b+cx)}\left(48a^3+8a^2x(b+\right.\right.}{384a^{7/2}x^5\sqrt{a+x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^2 + b*x^3 + c*x^4]/x^6,x]
```

```
[Out] (Sqrt[x^2*(a + x*(b + c*x))]*(-2*Sqrt[a]*Sqrt[a + x*(b + c*x)]*(48*a^3 + 15
*b^3*x^3 + 8*a^2*x*(b + 3*c*x) - 2*a*b*x^2*(5*b + 26*c*x)) + 3*(5*b^4 - 24*
a*b^2*c + 16*a^2*c^2)*x^4*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*
x)])]))/(384*a^(7/2)*x^5*Sqrt[a + x*(b + c*x)])
```

fricas [A] time = 0.99, size = 336, normalized size = 1.64

$$\left[\frac{3\left(5b^4-24ab^2c+16a^2c^2\right)\sqrt{a}\,x^5\log\left(-\frac{8abx^2+\left(b^2+4ac\right)x^3+8a^2x+4\sqrt{cx^4+bx^3+ax^2}\,(bx+2a)\sqrt{a}}{x^3}\right)-4\left(8a^3bx+48a^4+\left(15a\right.\right.}{768a^4x^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^6,x, algorithm="fricas")
```

```
[Out] [1/768*(3*(5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*sqrt(a)*x^5*log(-(8*a*b*x^2 + (
b^2 + 4*a*c)*x^3 + 8*a^2*x + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt
(a))/x^3) - 4*(8*a^3*b*x + 48*a^4 + (15*a*b^3 - 52*a^2*b*c)*x^3 - 2*(5*a^2*
b^2 - 12*a^3*c)*x^2)*sqrt(c*x^4 + b*x^3 + a*x^2))/(a^4*x^5), -1/384*(3*(5*b
^4 - 24*a*b^2*c + 16*a^2*c^2)*sqrt(-a)*x^5*arctan(1/2*sqrt(c*x^4 + b*x^3 +
a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) + 2*(8*a^3*b*x + 4
8*a^4 + (15*a*b^3 - 52*a^2*b*c)*x^3 - 2*(5*a^2*b^2 - 12*a^3*c)*x^2)*sqrt(c*
x^4 + b*x^3 + a*x^2))/(a^4*x^5)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^6,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.01, size = 387, normalized size = 1.89

$$\frac{\sqrt{c\,x^4+b\,x^3+a\,x^2}\,\left(48a^{\frac{5}{2}}c^2x^4\ln\left(\frac{bx+2a+2\sqrt{c\,x^2+bx+a}\,\sqrt{a}}{x}\right)-72a^{\frac{3}{2}}b^2c\,x^4\ln\left(\frac{bx+2a+2\sqrt{c\,x^2+bx+a}\,\sqrt{a}}{x}\right)+15\sqrt{a}\,b^4x^4\ln\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^3+a*x^2)^(1/2)/x^6,x)
```

```
[Out] 1/384*(c*x^4+b*x^3+a*x^2)^(1/2)*(48*c^2*a^(5/2)*ln((b*x+2*a+2*(c*x^2+b*x+a)
^(1/2)*a^(1/2))/x)*x^4+24*c^2*(c*x^2+b*x+a)^(1/2)*x^5*a*b-72*c*a^(3/2)*ln((
b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)*x^4*b^2-48*c^2*(c*x^2+b*x+a)^(1/2
)*x^4*a^2-30*c*(c*x^2+b*x+a)^(1/2)*x^5*b^3-24*c*(c*x^2+b*x+a)^(3/2)*x^3*a*b
+84*c*(c*x^2+b*x+a)^(1/2)*x^4*a*b^2+15*a^(1/2)*ln((b*x+2*a+2*(c*x^2+b*x+a)^
(1/2)*a^(1/2))/x)*x^4*b^4+48*c*(c*x^2+b*x+a)^(3/2)*x^2*a^2+30*(c*x^2+b*x+a)
^(3/2)*x^3*b^3-30*(c*x^2+b*x+a)^(1/2)*x^4*b^4-60*(c*x^2+b*x+a)^(3/2)*x^2*a*
b^2+80*(c*x^2+b*x+a)^(3/2)*x*a^2*b-96*(c*x^2+b*x+a)^(3/2)*a^3)/x^5/(c*x^2+b
*x+a)^(1/2)/a^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^4+bx^3+ax^2}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^6,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^3 + a*x^2)/x^6, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^4+b\,x^3+a\,x^2}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3 + c*x^4)^(1/2)/x^6,x)
```

```
[Out] int((a*x^2 + b*x^3 + c*x^4)^(1/2)/x^6, x)
```

<u>**sympy**</u> **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2\left(a+bx+cx^2\right)}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**3+a*x**2)**(1/2)/x**6,x)
```

```
[Out] Integral(sqrt(x**2*(a + b*x + c*x**2))/x**6, x)
```

3.38 $\int x \left(ax^2+bx^3+cx^4\right)^{3/2} dx$

Optimal. Leaf size=422

$$\frac{3x\left(b^2-4ac\right)^2\left(16a^2c^2-72ab^2c+33b^4\right)\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{32768c^{13/2}\sqrt{ax^2+bx^3+cx^4}}-\frac{bx\left(2416a^2c^2-1560ab^2c+2\ldots\right.}{71680c\ldots}$$

```
[Out] 1/112*x*(14*c*x+3*b)*(c*x^4+b*x^3+a*x^2)^(3/2)/c+3/32768*(-4*a*c+b^2)^2*(16
*a^2*c^2-72*a*b^2*c+33*b^4)*x*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(
1/2))*(c*x^2+b*x+a)^(1/2)/c^(13/2)/(c*x^4+b*x^3+a*x^2)^(1/2)+1/286720*(-672
0*a^3*c^3+18896*a^2*b^2*c^2-8988*a*b^4*c+1155*b^6)*(c*x^4+b*x^3+a*x^2)^(1/2
)/c^5-1/573440*b*(-58816*a^3*c^3+81648*a^2*b^2*c^2-30660*a*b^4*c+3465*b^6)*
(c*x^4+b*x^3+a*x^2)^(1/2)/c^6/x-1/71680*b*(2416*a^2*c^2-1560*a*b^2*c+231*b^
4)*x*(c*x^4+b*x^3+a*x^2)^(1/2)/c^4+1/35840*(560*a^2*c^2-568*a*b^2*c+99*b^4)
*x^2*(c*x^4+b*x^3+a*x^2)^(1/2)/c^3-1/4480*x^3*(b*(68*a*c+11*b^2)+10*c*(-28*
a*c+11*b^2)*x)*(c*x^4+b*x^3+a*x^2)^(1/2)/c^2
```

Rubi [A] time = 1.20, antiderivative size = 422, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {1919, 1945, 1949, 12, 1914, 621, 206}

$$\frac{x^2\left(560a^2c^2-568ab^2c+99b^4\right)\sqrt{ax^2+bx^3+cx^4}}{35840c^3}-\frac{bx\left(2416a^2c^2-1560ab^2c+231b^4\right)\sqrt{ax^2+bx^3+cx^4}}{71680c^4}+\frac{(18\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

```
[Out] ((1155*b^6 - 8988*a*b^4*c + 18896*a^2*b^2*c^2 - 6720*a^3*c^3)*Sqrt[a*x^2 +
b*x^3 + c*x^4])/(286720*c^5) - (b*(3465*b^6 - 30660*a*b^4*c + 81648*a^2*b^2
*c^2 - 58816*a^3*c^3)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(573440*c^6*x) - (b*(231
*b^4 - 1560*a*b^2*c + 2416*a^2*c^2)*x*Sqrt[a*x^2 + b*x^3 + c*x^4])/(71680*c
^4) + ((99*b^4 - 568*a*b^2*c + 560*a^2*c^2)*x^2*Sqrt[a*x^2 + b*x^3 + c*x^4]
)/(35840*c^3) - (x^3*(b*(11*b^2 + 68*a*c) + 10*c*(11*b^2 - 28*a*c)*x)*Sqrt[
a*x^2 + b*x^3 + c*x^4])/(4480*c^2) + (x*(3*b + 14*c*x)*(a*x^2 + b*x^3 + c*x
^4)^(3/2))/(112*c) + (3*(b^2 - 4*a*c)^2*(33*b^4 - 72*a*b^2*c + 16*a^2*c^2)*
x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2
])])/(32768*c^(13/2)*Sqrt[a*x^2 + b*x^3 + c*x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1914

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))])/Sqrt[
a*x^q + b*x^n + c*x^(2*n - q)], Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

Rule 1919

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(x^(m - n + q + 1)*(b*(n - q)*p + c*(m + p*q + (n - q)
*(2*p - 1) + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(c*(m + p*(2*
n - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1)), x] + Dist[((n - q)*p)/(c*(m
 + p*(2*n - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1)), Int[x^(m - (n - 2*q
))*Simp[-(a*b*(m + p*q - n + q + 1)) + (2*a*c*(m + p*q + (n - q)*(2*p - 1)
+ 1) - b^2*(m + p*q + (n - q)*(p - 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n +
c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] &&
 PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p
, 0] && RationalQ[m, q] && GtQ[m + p*q + 1, n - q] && NeQ[m + p*(2*n - q) +
 1, 0] && NeQ[m + p*q + (n - q)*(2*p - 1) + 1, 0]
```

Rule 1945

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[(x^(m + 1)*(b*B*(n - q)*p +
 A*c*(m + p*q + (n - q)*(2*p + 1) + 1) + B*c*(m + p*q + 2*(n - q)*p + 1)*x^
(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(c*(m + p*(2*n - q) + 1)*(m + p
*q + (n - q)*(2*p + 1) + 1)), x] + Dist[((n - q)*p)/(c*(m + p*(2*n - q) + 1
)*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(m + q)*Simp[2*a*A*c*(m + p*q +
 (n - q)*(2*p + 1) + 1) - a*b*B*(m + p*q + 1) + (2*a*B*c*(m + p*q + 2*(n -
q)*p + 1) + A*b*c*(m + p*q + (n - q)*(2*p + 1) + 1) - b^2*B*(m + p*q + (n -
 q)*p + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /
; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Intege
rQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q]
&& GtQ[m + p*q, -(n - q) - 1] && NeQ[m + p*(2*n - q) + 1, 0] && NeQ[m + p*q
 + (n - q)*(2*p + 1) + 1, 0]
```

Rule 1949

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[(B*x^(m - n + 1)*(a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1))/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), x] -
Dist[1/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(m - n + q)*Simp[a*B*(m
 + p*q - n + q + 1) + (b*B*(m + p*q + (n - q)*p + 1) - A*c*(m + p*q + (n -
q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x]
/; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Integ
erQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[p, -1] && LtQ[p, 0] && R
ationalQ[m, q] && GeQ[m + p*q, n - q - 1] && NeQ[m + p*q + (n - q)*(2*p + 1
) + 1, 0]
```

Rubi steps

$$\int x\left(ax^2+bx^3+cx^4\right)^{3/2}dx=\frac{x(3b+14cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{112c}+\frac{3\int x^2\left(-4ab-\frac{1}{2}\left(11b^2-28ac\right)x\right)\sqrt{ax^2+}}{112c}$$

$$=-\frac{x^3\left(b\left(11b^2+68ac\right)+10c\left(11b^2-28ac\right)x\right)\sqrt{ax^2+bx^3+cx^4}}{4480c^2}+\frac{x(3b+14cx)}{}$$

$$=\frac{\left(99b^4-568ab^2c+560a^2c^2\right)x^2\sqrt{ax^2+bx^3+cx^4}}{35840c^3}-\frac{x^3\left(b\left(11b^2+68ac\right)+10c\left(\right.\right.}{}$$

$$=-\frac{b\left(231b^4-1560ab^2c+2416a^2c^2\right)x\sqrt{ax^2+bx^3+cx^4}}{71680c^4}+\frac{\left(99b^4-568ab^2c+56\right.}{3}$$

$$=\frac{\left(1155b^6-8988ab^4c+18896a^2b^2c^2-6720a^3c^3\right)\sqrt{ax^2+bx^3+cx^4}}{286720c^5}-\frac{b\left(231b^4\right.}{}$$

$$=\frac{\left(1155b^6-8988ab^4c+18896a^2b^2c^2-6720a^3c^3\right)\sqrt{ax^2+bx^3+cx^4}}{286720c^5}-\frac{b\left(3465b^6\right.}{}$$

$$=\frac{\left(1155b^6-8988ab^4c+18896a^2b^2c^2-6720a^3c^3\right)\sqrt{ax^2+bx^3+cx^4}}{286720c^5}-\frac{b\left(3465b^6\right.}{}$$

$$=\frac{\left(1155b^6-8988ab^4c+18896a^2b^2c^2-6720a^3c^3\right)\sqrt{ax^2+bx^3+cx^4}}{286720c^5}-\frac{b\left(3465b^6\right.}{}$$

$$=\frac{\left(1155b^6-8988ab^4c+18896a^2b^2c^2-6720a^3c^3\right)\sqrt{ax^2+bx^3+cx^4}}{286720c^5}-\frac{b\left(3465b^6\right.}{}$$

$$=\frac{\left(1155b^6-8988ab^4c+18896a^2b^2c^2-6720a^3c^3\right)\sqrt{ax^2+bx^3+cx^4}}{286720c^5}-\frac{b\left(3465b^6\right.}{}$$

Mathematica [A] time = 0.38, size = 236, normalized size = 0.56

$$\frac{\left(x^2(a+x(b+cx))\right)^{3/2}\left(\frac{\left(16a^2c^2-72ab^2c+33b^4\right)\left(2\sqrt{c}(b+2cx)\sqrt{a+x(b+cx)}\left(4c\left(5a+2cx^2\right)-3b^2+8bcx\right)+3\left(b^2-4ac\right)^2\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\right)}{4096c^{11/2}x^3(a+x(b+cx))^{3/2}}\right.}{8c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

```
[Out] ((x^2*(a + x*(b + c*x)))^(3/2)*(a + b*x + c*x^2 - (11*b*(a + x*(b + c*x)))/
(14*c*x) + ((-231*b^3 + 372*a*b*c + 330*b^2*c*x - 280*a*c^2*x)*(a + x*(b +
c*x)))/(560*c^3*x^3) + ((33*b^4 - 72*a*b^2*c + 16*a^2*c^2)*(2*Sqrt[c]*(b +
2*c*x)*Sqrt[a + x*(b + c*x)]*(-3*b^2 + 8*b*c*x + 4*c*(5*a + 2*c*x^2)) + 3*(
b^2 - 4*a*c)^2*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))/(40
96*c^(11/2)*x^3*(a + x*(b + c*x))^(3/2))))/(8*c)
```

fricas [A] time = 0.95, size = 664, normalized size = 1.57

$$\left[\frac{105\left(33b^8-336ab^6c+1120a^2b^4c^2-1280a^3b^2c^3+256a^4c^4\right)\sqrt{c}\,x\log\left(-\frac{8c^2x^3+8bcx^2+4\sqrt{cx^4+bx^3+ax^2}\,(2cx+b)\sqrt{c}+(}{x}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2293760*(105*(33*b^8 - 336*a*b^6*c + 1120*a^2*b^4*c^2 - 1280*a^3*b^2*c^3
 + 256*a^4*c^4)*sqrt(c)*x*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^
3 + a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) + 4*(71680*c^8*x^7 + 8
7040*b*c^7*x^6 - 3465*b^7*c + 30660*a*b^5*c^2 - 81648*a^2*b^3*c^3 + 58816*a
^3*b*c^4 + 1280*(b^2*c^6 + 84*a*c^7)*x^5 - 128*(11*b^3*c^5 - 52*a*b*c^6)*x^
4 + 16*(99*b^4*c^4 - 568*a*b^2*c^5 + 560*a^2*c^6)*x^3 - 8*(231*b^5*c^3 - 15
60*a*b^3*c^4 + 2416*a^2*b*c^5)*x^2 + 2*(1155*b^6*c^2 - 8988*a*b^4*c^3 + 188
96*a^2*b^2*c^4 - 6720*a^3*c^5)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^7*x), -1/
1146880*(105*(33*b^8 - 336*a*b^6*c + 1120*a^2*b^4*c^2 - 1280*a^3*b^2*c^3 +
256*a^4*c^4)*sqrt(-c)*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*
sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) - 2*(71680*c^8*x^7 + 87040*b*c^7*x^6
- 3465*b^7*c + 30660*a*b^5*c^2 - 81648*a^2*b^3*c^3 + 58816*a^3*b*c^4 + 1280
*(b^2*c^6 + 84*a*c^7)*x^5 - 128*(11*b^3*c^5 - 52*a*b*c^6)*x^4 + 16*(99*b^4*
c^4 - 568*a*b^2*c^5 + 560*a^2*c^6)*x^3 - 8*(231*b^5*c^3 - 1560*a*b^3*c^4 +
2416*a^2*b*c^5)*x^2 + 2*(1155*b^6*c^2 - 8988*a*b^4*c^3 + 18896*a^2*b^2*c^4
- 6720*a^3*c^5)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^7*x)]
```

giac [A] time = 1.46, size = 521, normalized size = 1.23

$$\frac{1}{573440}\sqrt{cx^2+bx+a}\left(2\left(4\left(2\left(8\left(10\left(4\left(14\,cx\mathrm{sgn}(x)+17\,b\mathrm{sgn}(x)\right)x+\frac{b^2c^6\mathrm{sgn}(x)+84\,ac^7\mathrm{sgn}(x)}{c^7}\right)x-\frac{11\,b^3c^5\mathrm{s}}{}\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/573440*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*(4*(14*c*x*sgn(x) + 17*b*sgn
(x))*x + (b^2*c^6*sgn(x) + 84*a*c^7*sgn(x))/c^7)*x - (11*b^3*c^5*sgn(x) - 5
2*a*b*c^6*sgn(x))/c^7)*x + (99*b^4*c^4*sgn(x) - 568*a*b^2*c^5*sgn(x) + 560*
a^2*c^6*sgn(x))/c^7)*x - (231*b^5*c^3*sgn(x) - 1560*a*b^3*c^4*sgn(x) + 2416
*a^2*b*c^5*sgn(x))/c^7)*x + (1155*b^6*c^2*sgn(x) - 8988*a*b^4*c^3*sgn(x) +
18896*a^2*b^2*c^4*sgn(x) - 6720*a^3*c^5*sgn(x))/c^7)*x - (3465*b^7*c*sgn(x)
 - 30660*a*b^5*c^2*sgn(x) + 81648*a^2*b^3*c^3*sgn(x) - 58816*a^3*b*c^4*sgn(
x))/c^7) - 3/32768*(33*b^8*sgn(x) - 336*a*b^6*c*sgn(x) + 1120*a^2*b^4*c^2*s
gn(x) - 1280*a^3*b^2*c^3*sgn(x) + 256*a^4*c^4*sgn(x))*log(abs(-2*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(13/2) + 1/1146880*(3465*b^8*log(
abs(-b + 2*sqrt(a)*sqrt(c))) - 35280*a*b^6*c*log(abs(-b + 2*sqrt(a)*sqrt(c)
)) + 117600*a^2*b^4*c^2*log(abs(-b + 2*sqrt(a)*sqrt(c))) - 134400*a^3*b^2*c
^3*log(abs(-b + 2*sqrt(a)*sqrt(c))) + 26880*a^4*c^4*log(abs(-b + 2*sqrt(a)*
sqrt(c))) + 6930*sqrt(a)*b^7*sqrt(c) - 61320*a^(3/2)*b^5*c^(3/2) + 163296*a
^(5/2)*b^3*c^(5/2) - 117632*a^(7/2)*b*c^(7/2))*sgn(x)/c^(13/2)
```

maple [A] time = 0.01, size = 649, normalized size = 1.54

$$\frac{\left(c\,x^4+b\,x^3+a\,x^2\right)^{\frac{3}{2}}\left(26880a^4c^5\ln\left(\frac{2cx+b+2\sqrt{c\,x^2+bx+a}\,\sqrt{c}}{2\sqrt{c}}\right)-134400a^3b^2c^4\ln\left(\frac{2cx+b+2\sqrt{c\,x^2+bx+a}\,\sqrt{c}}{2\sqrt{c}}\right)+117600a^2b^4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^4+b*x^3+a*x^2)^(3/2),x)
```

```
[Out] 1/1146880*(c*x^4+b*x^3+a*x^2)^(3/2)*(26880*ln(1/2*(2*c*x+b+2*(c*x^2+b*x+a)^
(1/2)*c^(1/2))/c^(1/2))*a^4*c^5+3465*ln(1/2*(2*c*x+b+2*(c*x^2+b*x+a)^(1/2)*
c^(1/2))/c^(1/2))*b^8*c+143360*x^3*(c*x^2+b*x+a)^(5/2)*c^(13/2)-59136*(c*x^
```

```
2+b*x+a)^(5/2)*c^(7/2)*b^3+18480*(c*x^2+b*x+a)^(3/2)*c^(5/2)*b^5-6930*(c*x^
2+b*x+a)^(1/2)*c^(3/2)*b^7+42840*(c*x^2+b*x+a)^(1/2)*c^(5/2)*a*b^5+8960*(c*
x^2+b*x+a)^(3/2)*c^(9/2)*a^2*b-40320*(c*x^2+b*x+a)^(3/2)*c^(7/2)*a*b^3-1344
00*ln(1/2*(2*c*x+b+2*(c*x^2+b*x+a)^(1/2)*c^(1/2))/c^(1/2))*a^3*b^2*c^4+1176
00*ln(1/2*(2*c*x+b+2*(c*x^2+b*x+a)^(1/2)*c^(1/2))/c^(1/2))*a^2*b^4*c^3-3528
0*ln(1/2*(2*c*x+b+2*(c*x^2+b*x+a)^(1/2)*c^(1/2))/c^(1/2))*a*b^6*c^2-13860*(
c*x^2+b*x+a)^(1/2)*c^(5/2)*x*b^6+13440*(c*x^2+b*x+a)^(1/2)*c^(9/2)*a^3*b-63
840*(c*x^2+b*x+a)^(1/2)*c^(7/2)*a^2*b^3-112640*(c*x^2+b*x+a)^(5/2)*c^(11/2)
*x^2*b-71680*(c*x^2+b*x+a)^(5/2)*c^(11/2)*x*a+84480*(c*x^2+b*x+a)^(5/2)*c^(
9/2)*x*b^2+95232*(c*x^2+b*x+a)^(5/2)*c^(9/2)*a*b+17920*(c*x^2+b*x+a)^(3/2)*
c^(11/2)*x*a^2+36960*(c*x^2+b*x+a)^(3/2)*c^(7/2)*x*b^4+26880*(c*x^2+b*x+a)^
(1/2)*c^(11/2)*x*a^3-80640*(c*x^2+b*x+a)^(3/2)*c^(9/2)*x*a*b^2-127680*(c*x^
2+b*x+a)^(1/2)*c^(9/2)*x*a^2*b^2+85680*(c*x^2+b*x+a)^(1/2)*c^(7/2)*x*a*b^4)
/x^3/(c*x^2+b*x+a)^(3/2)/c^(15/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^4 + bx^3 + ax^2\right)^{\frac{3}{2}} x \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
[Out] integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)*x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x \left(c\,x^4 + b\,x^3 + a\,x^2\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a*x^2 + b*x^3 + c*x^4)^(3/2),x)
[Out] int(x*(a*x^2 + b*x^3 + c*x^4)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \left(x^2 \left(a + bx + cx^2\right)\right)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**4+b*x**3+a*x**2)**(3/2),x)
[Out] Integral(x*(x**2*(a + b*x + c*x**2))**(3/2), x)
```

3.39 $\int \left(ax^2+bx^3+cx^4\right)^{3/2} dx$

Optimal. Leaf size=364

$$-\frac{b\left(1168a^2c^2-728ab^2c+105b^4\right)\sqrt{ax^2+bx^3+cx^4}}{17920c^4}+\frac{\left(-2048a^3c^3+5488a^2b^2c^2-2520ab^4c+315b^6\right)\sqrt{ax^2+bx^3}}{35840c^5x}$$

```
[Out] 1/7*x*(c*x^4+b*x^3+a*x^2)^(3/2)-3/2048*b*(-4*a*c+b^2)^2*(-4*a*c+3*b^2)*x*ar
ctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))*(c*x^2+b*x+a)^(1/2)/c^(11/
2)/(c*x^4+b*x^3+a*x^2)^(1/2)-1/17920*b*(1168*a^2*c^2-728*a*b^2*c+105*b^4)*(
c*x^4+b*x^3+a*x^2)^(1/2)/c^4+1/35840*(-2048*a^3*c^3+5488*a^2*b^2*c^2-2520*a
*b^4*c+315*b^6)*(c*x^4+b*x^3+a*x^2)^(1/2)/c^5/x+1/4480*(-32*a*c+7*b^2)*(-4*
a*c+3*b^2)*x*(c*x^4+b*x^3+a*x^2)^(1/2)/c^3-1/2240*b*(-44*a*c+9*b^2)*x^2*(c*
x^4+b*x^3+a*x^2)^(1/2)/c^2+1/280*x^3*(10*b*c*x+24*a*c+b^2)*(c*x^4+b*x^3+a*x
^2)^(1/2)/c
```

Rubi [A] time = 1.04, antiderivative size = 364, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1906, 1945, 1949, 12, 1914, 621, 206}

$$-\frac{b\left(1168a^2c^2-728ab^2c+105b^4\right)\sqrt{ax^2+bx^3+cx^4}}{17920c^4}+\frac{\left(5488a^2b^2c^2-2048a^3c^3-2520ab^4c+315b^6\right)\sqrt{ax^2+bx^3}}{35840c^5x}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

```
[Out] -(b*(105*b^4 - 728*a*b^2*c + 1168*a^2*c^2)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(17
920*c^4) + ((315*b^6 - 2520*a*b^4*c + 5488*a^2*b^2*c^2 - 2048*a^3*c^3)*Sqrt
[a*x^2 + b*x^3 + c*x^4])/(35840*c^5*x) + ((7*b^2 - 32*a*c)*(3*b^2 - 4*a*c)*
x*Sqrt[a*x^2 + b*x^3 + c*x^4])/(4480*c^3) - (b*(9*b^2 - 44*a*c)*x^2*Sqrt[a*
x^2 + b*x^3 + c*x^4])/(2240*c^2) + (x^3*(b^2 + 24*a*c + 10*b*c*x)*Sqrt[a*x^
2 + b*x^3 + c*x^4])/(280*c) + (x*(a*x^2 + b*x^3 + c*x^4)^(3/2))/7 - (3*b*(b
^2 - 4*a*c)^2*(3*b^2 - 4*a*c)*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(
2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2048*c^(11/2)*Sqrt[a*x^2 + b*x^3 + c*x^
4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1906

```
Int[((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_), x_Symbol
] :> Simp[(x*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(p*(2*n - q) + 1), x] + Dis
```

```
t[((n - q)*p)/(p*(2*n - q) + 1), Int[x^q*(2*a + b*x^(n - q))*(a*x^q + b*x^n
 + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c, n, q}, x] && EqQ[r, 2*
n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0]
&& NeQ[p*(2*n - q) + 1, 0]
```

Rule 1914

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))])/Sqrt[
a*x^q + b*x^n + c*x^(2*n - q)], Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

Rule 1945

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[(x^(m + 1)*(b*B*(n - q)*p +
 A*c*(m + p*q + (n - q)*(2*p + 1) + 1) + B*c*(m + p*q + 2*(n - q)*p + 1)*x^
(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(c*(m + p*(2*n - q) + 1)*(m + p
*q + (n - q)*(2*p + 1) + 1)), x] + Dist[((n - q)*p)/(c*(m + p*(2*n - q) + 1
)*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(m + q)*Simp[2*a*A*c*(m + p*q +
 (n - q)*(2*p + 1) + 1) - a*b*B*(m + p*q + 1) + (2*a*B*c*(m + p*q + 2*(n -
q)*p + 1) + A*b*c*(m + p*q + (n - q)*(2*p + 1) + 1) - b^2*B*(m + p*q + (n -
 q)*p + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /
; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Intege
rQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q]
&& GtQ[m + p*q, -(n - q) - 1] && NeQ[m + p*(2*n - q) + 1, 0] && NeQ[m + p*q
 + (n - q)*(2*p + 1) + 1, 0]
```

Rule 1949

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[(B*x^(m - n + 1)*(a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1))/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), x] -
Dist[1/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(m - n + q)*Simp[a*B*(m
 + p*q - n + q + 1) + (b*B*(m + p*q + (n - q)*p + 1) - A*c*(m + p*q + (n -
q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x]
/; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Integ
erQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[p, -1] && LtQ[p, 0] && R
ationalQ[m, q] && GeQ[m + p*q, n - q - 1] && NeQ[m + p*q + (n - q)*(2*p + 1
) + 1, 0]
```

Rubi steps

$$\int \left(ax^2+bx^3+cx^4\right)^{3/2} dx = \frac{1}{7}x\left(ax^2+bx^3+cx^4\right)^{3/2} + \frac{3}{14}\int x^2(2a+bx)\sqrt{ax^2+bx^3+cx^4}\, dx$$

$$= \frac{x^3\left(b^2+24ac+10bcx\right)\sqrt{ax^2+bx^3+cx^4}}{280c} + \frac{1}{7}x\left(ax^2+bx^3+cx^4\right)^{3/2} + \frac{\int \frac{x^4\left(-4a\left(b^2-6\right.\right.}{\sqrt{ax}}}{}$$

$$= -\frac{b\left(9b^2-44ac\right)x^2\sqrt{ax^2+bx^3+cx^4}}{2240c^2} + \frac{x^3\left(b^2+24ac+10bcx\right)\sqrt{ax^2+bx^3+cx^4}}{280c} +$$

$$= \frac{\left(7b^2-32ac\right)\left(3b^2-4ac\right)x\sqrt{ax^2+bx^3+cx^4}}{4480c^3} - \frac{b\left(9b^2-44ac\right)x^2\sqrt{ax^2+bx^3+cx^4}}{2240c^2}$$

$$= -\frac{b\left(105b^4-728ab^2c+1168a^2c^2\right)\sqrt{ax^2+bx^3+cx^4}}{17920c^4} + \frac{\left(7b^2-32ac\right)\left(3b^2-4ac\right)x\sqrt{}}{4480c^3}$$

$$= -\frac{b\left(105b^4-728ab^2c+1168a^2c^2\right)\sqrt{ax^2+bx^3+cx^4}}{17920c^4} + \frac{\left(315b^6-2520ab^4c+5488a^2\right.}{3}$$

$$= -\frac{b\left(105b^4-728ab^2c+1168a^2c^2\right)\sqrt{ax^2+bx^3+cx^4}}{17920c^4} + \frac{\left(315b^6-2520ab^4c+5488a^2\right.}{3}$$

$$= -\frac{b\left(105b^4-728ab^2c+1168a^2c^2\right)\sqrt{ax^2+bx^3+cx^4}}{17920c^4} + \frac{\left(315b^6-2520ab^4c+5488a^2\right.}{3}$$

$$= -\frac{b\left(105b^4-728ab^2c+1168a^2c^2\right)\sqrt{ax^2+bx^3+cx^4}}{17920c^4} + \frac{\left(315b^6-2520ab^4c+5488a^2\right.}{3}$$

$$= -\frac{b\left(105b^4-728ab^2c+1168a^2c^2\right)\sqrt{ax^2+bx^3+cx^4}}{17920c^4} + \frac{\left(315b^6-2520ab^4c+5488a^2\right.}{3}$$

Mathematica [A] time = 0.25, size = 197, normalized size = 0.54

$$\frac{\left(x^2(a+x(b+cx))\right)^{3/2}\left(\frac{\left(-16ac+21b^2-30bcx\right)(a+x(b+cx))}{40c^2} + \frac{7\left(4abc-3b^3\right)\left(2\sqrt{c}\,(b+2cx)\sqrt{a+x(b+cx)}\left(4c\left(5a+2cx^2\right)-3b^2+8bcx\right)+3\left(b^2-4ac\right)^2\tan\right.}{2048c^{9/2}(a+x(b+cx))^{3/2}}\right.}{7cx^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

```
[Out] ((x^2*(a + x*(b + c*x)))^(3/2)*(x^2*(a + x*(b + c*x)) + ((21*b^2 - 16*a*c -
 30*b*c*x)*(a + x*(b + c*x)))/(40*c^2) + (7*(-3*b^3 + 4*a*b*c)*(2*Sqrt[c]*(
b + 2*c*x)*Sqrt[a + x*(b + c*x)]*(-3*b^2 + 8*b*c*x + 4*c*(5*a + 2*c*x^2)) +
 3*(b^2 - 4*a*c)^2*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))
/(2048*c^(9/2)*(a + x*(b + c*x))^(3/2))))/(7*c*x^3)
```

fricas [A] time = 0.73, size = 558, normalized size = 1.53

$$\left[-\frac{105\left(3b^7-28ab^5c+80a^2b^3c^2-64a^3bc^3\right)\sqrt{c}\,x\log\left(-\frac{8c^2x^3+8bcx^2+4\sqrt{cx^4+bx^3+ax^2}\,(2cx+b)\sqrt{c}+\left(b^2+4ac\right)x}{x}\right)-4\left(5120c\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/143360*(105*(3*b^7 - 28*a*b^5*c + 80*a^2*b^3*c^2 - 64*a^3*b*c^3)*sqrt(c
)*x*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)
*sqrt(c) + (b^2 + 4*a*c)*x)/x) - 4*(5120*c^7*x^6 + 6400*b*c^6*x^5 + 315*b^6
*c - 2520*a*b^4*c^2 + 5488*a^2*b^2*c^3 - 2048*a^3*c^4 + 128*(b^2*c^5 + 64*a
*c^6)*x^4 - 16*(9*b^3*c^4 - 44*a*b*c^5)*x^3 + 8*(21*b^4*c^3 - 124*a*b^2*c^4
 + 128*a^2*c^5)*x^2 - 2*(105*b^5*c^2 - 728*a*b^3*c^3 + 1168*a^2*b*c^4)*x)*s
qrt(c*x^4 + b*x^3 + a*x^2))/(c^6*x), 1/71680*(105*(3*b^7 - 28*a*b^5*c + 80*
a^2*b^3*c^2 - 64*a^3*b*c^3)*sqrt(-c)*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^
2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) + 2*(5120*c^7*x^6 + 64
00*b*c^6*x^5 + 315*b^6*c - 2520*a*b^4*c^2 + 5488*a^2*b^2*c^3 - 2048*a^3*c^4
 + 128*(b^2*c^5 + 64*a*c^6)*x^4 - 16*(9*b^3*c^4 - 44*a*b*c^5)*x^3 + 8*(21*b
^4*c^3 - 124*a*b^2*c^4 + 128*a^2*c^5)*x^2 - 2*(105*b^5*c^2 - 728*a*b^3*c^3
+ 1168*a^2*b*c^4)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^6*x)]
```

giac [A] time = 1.35, size = 429, normalized size = 1.18

$$\frac{1}{35840}\sqrt{cx^2+bx+a}\,\Bigg(2\bigg(4\bigg(2\bigg(8\bigg(10\big(4\,cx\mathrm{sgn}(x)+5\,b\mathrm{sgn}(x)\big)x+\frac{b^2c^5\mathrm{sgn}(x)+64\,ac^6\mathrm{sgn}(x)}{c^6}\bigg)x-\frac{9\,b^3c^4\mathrm{sgn}(x}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/35840*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*(4*c*x*sgn(x) + 5*b*sgn(x))*x
 + (b^2*c^5*sgn(x) + 64*a*c^6*sgn(x))/c^6)*x - (9*b^3*c^4*sgn(x) - 44*a*b*c
^5*sgn(x))/c^6)*x + (21*b^4*c^3*sgn(x) - 124*a*b^2*c^4*sgn(x) + 128*a^2*c^5
*sgn(x))/c^6)*x - (105*b^5*c^2*sgn(x) - 728*a*b^3*c^3*sgn(x) + 1168*a^2*b*c
^4*sgn(x))/c^6)*x + (315*b^6*c*sgn(x) - 2520*a*b^4*c^2*sgn(x) + 5488*a^2*b^
2*c^3*sgn(x) - 2048*a^3*c^4*sgn(x))/c^6) + 3/2048*(3*b^7*sgn(x) - 28*a*b^5*
c*sgn(x) + 80*a^2*b^3*c^2*sgn(x) - 64*a^3*b*c^3*sgn(x))*log(abs(-2*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(11/2) - 1/71680*(315*b^7*log(a
bs(-b + 2*sqrt(a)*sqrt(c))) - 2940*a*b^5*c*log(abs(-b + 2*sqrt(a)*sqrt(c)))
 + 8400*a^2*b^3*c^2*log(abs(-b + 2*sqrt(a)*sqrt(c))) - 6720*a^3*b*c^3*log(a
bs(-b + 2*sqrt(a)*sqrt(c))) + 630*sqrt(a)*b^6*sqrt(c) - 5040*a^(3/2)*b^4*c^
(3/2) + 10976*a^(5/2)*b^2*c^(5/2) - 4096*a^(7/2)*c^(7/2))*sgn(x)/c^(11/2)
```

maple [A] time = 0.01, size = 479, normalized size = 1.32

$$\frac{\left(c\,x^4+b\,x^3+a\,x^2\right)^{\frac{3}{2}}\Bigg(6720a^3b\,c^4\ln\left(\frac{2cx+b+2\sqrt{c\,x^2+bx+a}\,\sqrt{c}}{2\sqrt{c}}\right)-8400a^2b^3c^3\ln\left(\frac{2cx+b+2\sqrt{c\,x^2+bx+a}\,\sqrt{c}}{2\sqrt{c}}\right)+2940a\,b^5c^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^3+a*x^2)^(3/2),x)
```

```
[Out] 1/71680*(c*x^4+b*x^3+a*x^2)^(3/2)*(10240*x^2*(c*x^2+b*x+a)^(5/2)*c^(11/2)-7
680*c^(9/2)*(c*x^2+b*x+a)^(5/2)*x*b-4096*c^(9/2)*(c*x^2+b*x+a)^(5/2)*a+5376
*c^(7/2)*(c*x^2+b*x+a)^(5/2)*b^2+4480*c^(9/2)*(c*x^2+b*x+a)^(3/2)*x*a*b-336
0*c^(7/2)*(c*x^2+b*x+a)^(3/2)*x*b^3+2240*c^(7/2)*(c*x^2+b*x+a)^(3/2)*a*b^2-
1680*c^(5/2)*(c*x^2+b*x+a)^(3/2)*b^4+6720*c^(9/2)*(c*x^2+b*x+a)^(1/2)*x*a^2
*b-6720*c^(7/2)*(c*x^2+b*x+a)^(1/2)*x*a*b^3+1260*c^(5/2)*(c*x^2+b*x+a)^(1/2
)*x*b^5+3360*c^(7/2)*(c*x^2+b*x+a)^(1/2)*a^2*b^2-3360*c^(5/2)*(c*x^2+b*x+a)
^(1/2)*a*b^4+630*c^(3/2)*(c*x^2+b*x+a)^(1/2)*b^6+6720*ln(1/2*(2*c*x+b+2*(c*
x^2+b*x+a)^(1/2)*c^(1/2))/c^(1/2))*a^3*b*c^4-8400*ln(1/2*(2*c*x+b+2*(c*x^2+
b*x+a)^(1/2)*c^(1/2))/c^(1/2))*a^2*b^3*c^3+2940*ln(1/2*(2*c*x+b+2*(c*x^2+b*
x+a)^(1/2)*c^(1/2))/c^(1/2))*a*b^5*c^2-315*ln(1/2*(2*c*x+b+2*(c*x^2+b*x+a)^
(1/2)*c^(1/2))/c^(1/2))*b^7*c)/x^3/(c*x^2+b*x+a)^(3/2)/c^(13/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^4 + bx^3 + ax^2\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
[Out] integrate((c*x^4 + b*x^3 + a*x^2)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(c\,x^4 + b\,x^3 + a\,x^2\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3 + c*x^4)^(3/2),x)
[Out] int((a*x^2 + b*x^3 + c*x^4)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ax^2 + bx^3 + cx^4\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**3+a*x**2)**(3/2),x)
[Out] Integral((a*x**2 + b*x**3 + c*x**4)**(3/2), x)
```

3.40 $\int \frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{x}\,dx$

Optimal. Leaf size=288

$$-\frac{b\left(1296a^2c^2-760ab^2c+105b^4\right)\sqrt{ax^2+bx^3+cx^4}}{7680c^4x}+\frac{\left(240a^2c^2-216ab^2c+35b^4\right)\sqrt{ax^2+bx^3+cx^4}}{3840c^3}+\frac{x\left(7b^2\right.}{}$$

[Out] 1/60*(10*c*x+3*b)*(c*x^4+b*x^3+a*x^2)^(3/2)/c/x+1/1024*(-4*a*c+b^2)^2*(-4*a*c+7*b^2)*x*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))*(c*x^2+b*x+a)^(1/2)/c^(9/2)/(c*x^4+b*x^3+a*x^2)^(1/2)+1/3840*(240*a^2*c^2-216*a*b^2*c+35*b^4)*(c*x^4+b*x^3+a*x^2)^(1/2)/c^3-1/7680*b*(1296*a^2*c^2-760*a*b^2*c+105*b^4)*(c*x^4+b*x^3+a*x^2)^(1/2)/c^4/x-1/960*x*(b*(12*a*c+7*b^2)+6*c*(-20*a*c+7*b^2)*x)*(c*x^4+b*x^3+a*x^2)^(1/2)/c^2

Rubi [A] time = 0.52, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1919, 1934, 1949, 12, 1914, 621, 206}

$$\frac{\left(240a^2c^2-216ab^2c+35b^4\right)\sqrt{ax^2+bx^3+cx^4}}{3840c^3}-\frac{b\left(1296a^2c^2-760ab^2c+105b^4\right)\sqrt{ax^2+bx^3+cx^4}}{7680c^4x}-\frac{x\left(6cx\left(7\right.\right.}{}$$

Antiderivative was successfully verified.

[In] Int[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x,x]

[Out] ((35*b^4 - 216*a*b^2*c + 240*a^2*c^2)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(3840*c^3) - (b*(105*b^4 - 760*a*b^2*c + 1296*a^2*c^2)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(7680*c^4*x) - (x*(b*(7*b^2 + 12*a*c) + 6*c*(7*b^2 - 20*a*c)*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(960*c^2) + ((3*b + 10*c*x)*(a*x^2 + b*x^3 + c*x^4)^(3/2))/(60*c*x) + ((b^2 - 4*a*c)^2*(7*b^2 - 4*a*c)*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(1024*c^(9/2)*Sqrt[a*x^2 + b*x^3 + c*x^4])

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1914

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))])/Sqrt[
a*x^q + b*x^n + c*x^(2*n - q)], Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
```

```
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

Rule 1919

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(x^(m - n + q + 1)*(b*(n - q)*p + c*(m + p*q + (n - q)
*(2*p - 1) + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(c*(m + p*(2*
n - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1)), x] + Dist[((n - q)*p)/(c*(m
 + p*(2*n - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1)), Int[x^(m - (n - 2*q
))*Simp[-(a*b*(m + p*q - n + q + 1)) + (2*a*c*(m + p*q + (n - q)*(2*p - 1)
+ 1) - b^2*(m + p*q + (n - q)*(p - 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n +
c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] &&
 PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p
, 0] && RationalQ[m, q] && GtQ[m + p*q + 1, n - q] && NeQ[m + p*(2*n - q) +
 1, 0] && NeQ[m + p*q + (n - q)*(2*p - 1) + 1, 0]
```

Rule 1934

```
Int[((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_)*((A_) + (
B_.)*(x_)^(r_.)), x_Symbol] :> Simp[(x*(b*B*(n - q)*p + A*c*(p*q + (n - q)*
(2*p + 1) + 1) + B*c*(p*(2*n - q) + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n
 - q))^p)/(c*(p*(2*n - q) + 1)*(p*q + (n - q)*(2*p + 1) + 1)), x] + Dist[((
n - q)*p)/(c*(p*(2*n - q) + 1)*(p*q + (n - q)*(2*p + 1) + 1)), Int[x^q*(2*a
*A*c*(p*q + (n - q)*(2*p + 1) + 1) - a*b*B*(p*q + 1) + (2*a*B*c*(p*(2*n - q
) + 1) + A*b*c*(p*q + (n - q)*(2*p + 1) + 1) - b^2*B*(p*q + (n - q)*p + 1))
*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b,
 c, A, B, n, q}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !IntegerQ[p] &&
 NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && NeQ[p*(2*n - q) + 1, 0] && NeQ[p*q + (
n - q)*(2*p + 1) + 1, 0]
```

Rule 1949

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[(B*x^(m - n + 1)*(a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1))/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), x] -
Dist[1/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(m - n + q)*Simp[a*B*(m
 + p*q - n + q + 1) + (b*B*(m + p*q + (n - q)*p + 1) - A*c*(m + p*q + (n -
q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x]
/; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Integ
erQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[p, -1] && LtQ[p, 0] && R
ationalQ[m, q] && GeQ[m + p*q, n - q - 1] && NeQ[m + p*q + (n - q)*(2*p + 1
) + 1, 0]
```

Rubi steps

$$\int \frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{x}\,dx = \frac{(3b+10cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{60cx} + \frac{\int \left(-2ab+\frac{1}{2}\left(-7b^2+20ac\right)x\right)\sqrt{ax^2+bx^3+}}{20c}$$

$$= -\frac{x\left(b\left(7b^2+12ac\right)+6c\left(7b^2-20ac\right)x\right)\sqrt{ax^2+bx^3+cx^4}}{960c^2} + \frac{(3b+10cx)\left(ax^2+\right.}{60cx}$$

$$= \frac{\left(35b^4-216ab^2c+240a^2c^2\right)\sqrt{ax^2+bx^3+cx^4}}{3840c^3} - \frac{x\left(b\left(7b^2+12ac\right)+6c\left(7b^2-2\right.\right.}{960c^2}$$

$$= \frac{\left(35b^4-216ab^2c+240a^2c^2\right)\sqrt{ax^2+bx^3+cx^4}}{3840c^3} - \frac{b\left(105b^4-760ab^2c+1296a^2c^2\right.}{7680c^4x}$$

$$= \frac{\left(35b^4-216ab^2c+240a^2c^2\right)\sqrt{ax^2+bx^3+cx^4}}{3840c^3} - \frac{b\left(105b^4-760ab^2c+1296a^2c^2\right.}{7680c^4x}$$

$$= \frac{\left(35b^4-216ab^2c+240a^2c^2\right)\sqrt{ax^2+bx^3+cx^4}}{3840c^3} - \frac{b\left(105b^4-760ab^2c+1296a^2c^2\right.}{7680c^4x}$$

$$= \frac{\left(35b^4-216ab^2c+240a^2c^2\right)\sqrt{ax^2+bx^3+cx^4}}{3840c^3} - \frac{b\left(105b^4-760ab^2c+1296a^2c^2\right.}{7680c^4x}$$

$$= \frac{\left(35b^4-216ab^2c+240a^2c^2\right)\sqrt{ax^2+bx^3+cx^4}}{3840c^3} - \frac{b\left(105b^4-760ab^2c+1296a^2c^2\right.}{7680c^4x}$$

Mathematica [A] time = 0.22, size = 180, normalized size = 0.62

$$\frac{\left(x^2(a+x(b+cx))\right)^{3/2}\left(\frac{\left(7b^2-4ac\right)\left(2\sqrt{c}\,(b+2cx)\sqrt{a+x(b+cx)}\left(4c\left(5a+2cx^2\right)-3b^2+8bcx\right)+3\left(b^2-4ac\right)^2\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\,\sqrt{a+x(b+cx)}}\right)\right)}{512c^{7/2}(a+x(b+cx))^{3/2}}+x(a+x\right.}{6cx^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x,x]
```

```
[Out] ((x^2*(a + x*(b + c*x)))^(3/2)*((-7*b*(a + x*(b + c*x)))/(10*c) + x*(a + x*
(b + c*x)) + ((7*b^2 - 4*a*c)*(2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b + c*x)]*
(-3*b^2 + 8*b*c*x + 4*c*(5*a + 2*c*x^2)) + 3*(b^2 - 4*a*c)^2*ArcTanh[(b + 2
*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))/(512*c^(7/2)*(a + x*(b + c*x))^(
3/2))))/(6*c*x^3)
```

fricas [A] time = 0.68, size = 474, normalized size = 1.65

$$\left[-\frac{15\left(7\,b^6-60\,ab^4c+144\,a^2b^2c^2-64\,a^3c^3\right)\sqrt{c}\,x\log\left(-\frac{8\,c^2x^3+8\,bcx^2-4\,\sqrt{cx^4+bx^3+ax^2}\,(2\,cx+b)\sqrt{c}+\left(b^2+4\,ac\right)x}{x}\right)-4\left(1280\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [-1/30720*(15*(7*b^6 - 60*a*b^4*c + 144*a^2*b^2*c^2 - 64*a^3*c^3)*sqrt(c)*x
*log(-(8*c^2*x^3 + 8*b*c*x^2 - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sq
rt(c) + (b^2 + 4*a*c)*x)/x) - 4*(1280*c^6*x^5 + 1664*b*c^5*x^4 - 105*b^5*c
```

```
+ 760*a*b^3*c^2 - 1296*a^2*b*c^3 + 16*(3*b^2*c^4 + 140*a*c^5)*x^3 - 8*(7*b^
3*c^3 - 36*a*b*c^4)*x^2 + 2*(35*b^4*c^2 - 216*a*b^2*c^3 + 240*a^2*c^4)*x)*s
qrt(c*x^4 + b*x^3 + a*x^2))/(c^5*x), -1/15360*(15*(7*b^6 - 60*a*b^4*c + 144
*a^2*b^2*c^2 - 64*a^3*c^3)*sqrt(-c)*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2
)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) - 2*(1280*c^6*x^5 + 166
4*b*c^5*x^4 - 105*b^5*c + 760*a*b^3*c^2 - 1296*a^2*b*c^3 + 16*(3*b^2*c^4 +
140*a*c^5)*x^3 - 8*(7*b^3*c^3 - 36*a*b*c^4)*x^2 + 2*(35*b^4*c^2 - 216*a*b^2
*c^3 + 240*a^2*c^4)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^5*x)]
```

giac [A] time = 0.99, size = 365, normalized size = 1.27

$$\frac{1}{7680}\sqrt{cx^2+bx+a}\,\Bigg(2\Bigg(4\Bigg(2\Bigg(8\big(10\,cx\mathrm{sgn}(x)+13\,b\mathrm{sgn}(x)\big)x+\frac{3\,b^2c^4\mathrm{sgn}(x)+140\,ac^5\mathrm{sgn}(x)}{c^5}\Bigg)x-\frac{7\,b^3c^3\mathrm{sgn}(x)-}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x,x, algorithm="giac")
```

```
[Out] 1/7680*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*c*x*sgn(x) + 13*b*sgn(x))*x +
(3*b^2*c^4*sgn(x) + 140*a*c^5*sgn(x))/c^5)*x - (7*b^3*c^3*sgn(x) - 36*a*b*c
^4*sgn(x))/c^5)*x + (35*b^4*c^2*sgn(x) - 216*a*b^2*c^3*sgn(x) + 240*a^2*c^4
*sgn(x))/c^5)*x - (105*b^5*c*sgn(x) - 760*a*b^3*c^2*sgn(x) + 1296*a^2*b*c^3
*sgn(x))/c^5) - 1/1024*(7*b^6*sgn(x) - 60*a*b^4*c*sgn(x) + 144*a^2*b^2*c^2*
sgn(x) - 64*a^3*c^3*sgn(x))*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*
sqrt(c) - b))/c^(9/2) + 1/15360*(105*b^6*log(abs(-b + 2*sqrt(a)*sqrt(c))) -
 900*a*b^4*c*log(abs(-b + 2*sqrt(a)*sqrt(c))) + 2160*a^2*b^2*c^2*log(abs(-b
 + 2*sqrt(a)*sqrt(c))) - 960*a^3*c^3*log(abs(-b + 2*sqrt(a)*sqrt(c))) + 210
*sqrt(a)*b^5*sqrt(c) - 1520*a^(3/2)*b^3*c^(3/2) + 2592*a^(5/2)*b*c^(5/2))*s
gn(x)/c^(9/2)
```

maple [A] time = 0.01, size = 431, normalized size = 1.50

$$\left(c\,x^4+b\,x^3+a\,x^2\right)^{\frac{3}{2}}\Bigg(-960a^3c^4\ln\left(\frac{2cx+b+2\sqrt{c\,x^2+bx+a}\,\sqrt{c}}{2\sqrt{c}}\right)+2160a^2b^2c^3\ln\left(\frac{2cx+b+2\sqrt{c\,x^2+bx+a}\,\sqrt{c}}{2\sqrt{c}}\right)-900a\,b^4c^2\ln\Bigg(\frac{2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^3+a*x^2)^(3/2)/x,x)
```

```
[Out] 1/15360*(c*x^4+b*x^3+a*x^2)^(3/2)*(2560*x*(c*x^2+b*x+a)^(5/2)*c^(9/2)-1792*
c^(7/2)*(c*x^2+b*x+a)^(5/2)*b-640*c^(9/2)*(c*x^2+b*x+a)^(3/2)*x*a+1120*c^(7
/2)*(c*x^2+b*x+a)^(3/2)*x*b^2-320*c^(7/2)*(c*x^2+b*x+a)^(3/2)*a*b+560*c^(5/
2)*(c*x^2+b*x+a)^(3/2)*b^3-960*c^(9/2)*(c*x^2+b*x+a)^(1/2)*x*a^2+1920*c^(7/
2)*(c*x^2+b*x+a)^(1/2)*x*a*b^2-420*c^(5/2)*(c*x^2+b*x+a)^(1/2)*x*b^4-480*c^
(7/2)*(c*x^2+b*x+a)^(1/2)*a^2*b+960*c^(5/2)*(c*x^2+b*x+a)^(1/2)*a*b^3-210*c
^(3/2)*(c*x^2+b*x+a)^(1/2)*b^5-960*ln(1/2*(2*c*x+b+2*(c*x^2+b*x+a)^(1/2)*c^
(1/2))/c^(1/2))*a^3*c^4+2160*ln(1/2*(2*c*x+b+2*(c*x^2+b*x+a)^(1/2)*c^(1/2))
/c^(1/2))*a^2*b^2*c^3-900*ln(1/2*(2*c*x+b+2*(c*x^2+b*x+a)^(1/2)*c^(1/2))/c^
(1/2))*a*b^4*c^2+105*ln(1/2*(2*c*x+b+2*(c*x^2+b*x+a)^(1/2)*c^(1/2))/c^(1/2)
)*b^6*c)/x^3/(c*x^2+b*x+a)^(3/2)/c^(11/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(cx^4+bx^3+ax^2\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^4+b\,x^3+a\,x^2\right)^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x,x)
```

```
[Out] int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{2}\left(a+bx+cx^{2}\right)\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**3+a*x**2)**(3/2)/x,x)
```

```
[Out] Integral((x**2*(a + b*x + c*x**2))**(3/2)/x, x)
```

3.41 $\int \frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{x^2}\,dx$

Optimal. Leaf size=198

$$-\frac{3bx\left(b^2-4ac\right)^2\sqrt{a+bx+cx^2}\,\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{256c^{7/2}\sqrt{ax^2+bx^3+cx^4}}+\frac{3b\left(b^2-4ac\right)(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{128c^3x}-\frac{b(b+2cx)\left(a\cdots\right.}{1\cdots}$$

```
[Out] -1/16*b*(2*c*x+b)*(c*x^4+b*x^3+a*x^2)^(3/2)/c^2/x^3+1/5*(c*x^4+b*x^3+a*x^2)
^(5/2)/c/x^5-3/256*b*(-4*a*c+b^2)^2*x*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+
b*x+a)^(1/2))*(c*x^2+b*x+a)^(1/2)/c^(7/2)/(c*x^4+b*x^3+a*x^2)^(1/2)+3/128*b
*(-4*a*c+b^2)*(2*c*x+b)*(c*x^4+b*x^3+a*x^2)^(1/2)/c^3/x
```

Rubi [A] time = 0.18, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1917, 1918, 1914, 621, 206}

$$\frac{3b\left(b^2-4ac\right)(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{128c^3x}-\frac{3bx\left(b^2-4ac\right)^2\sqrt{a+bx+cx^2}\,\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{256c^{7/2}\sqrt{ax^2+bx^3+cx^4}}-\frac{b(b+2cx)\left(ax\cdots\right.}{16\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^2,x]
```

```
[Out] (3*b*(b^2 - 4*a*c)*(b + 2*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(128*c^3*x) - (
b*(b + 2*c*x)*(a*x^2 + b*x^3 + c*x^4)^(3/2))/(16*c^2*x^3) + (a*x^2 + b*x^3
+ c*x^4)^(5/2)/(5*c*x^5) - (3*b*(b^2 - 4*a*c)^2*x*Sqrt[a + b*x + c*x^2]*Arc
Tanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(256*c^(7/2)*Sqrt[a*x^
2 + b*x^3 + c*x^4])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1914

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))])/Sqrt[
a*x^q + b*x^n + c*x^(2*n - q)], Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

Rule 1917

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(x^(m - n)*(a*x^(n - 1) + b*x^n + c*x^(n + 1))^(p + 1)
)/(2*c*(p + 1)), x] - Dist[b/(2*c), Int[x^(m - 1)*(a*x^(n - 1) + b*x^n + c*
x^(n + 1))^p, x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n -
q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && RationalQ[m, p,
```

```
q] && EqQ[m + p*(n - 1) - 1, 0]
```

Rule 1918

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(x^(m - n + q + 1)*(b + 2*c*x^(n - q))*(a*x^q + b*x^n
+ c*x^(2*n - q))^p)/(2*c*(n - q)*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2
*c*(2*p + 1)), Int[x^(m + q)*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x
] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p]
 && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && Eq
Q[m + p*q + 1, n - q]
```

Rubi steps

$$\int \frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{x^2}\,dx = \frac{\left(ax^2+bx^3+cx^4\right)^{5/2}}{5cx^5} - \frac{b\int \frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{x^3}\,dx}{2c}$$

$$= -\frac{b(b+2cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{16c^2x^3} + \frac{\left(ax^2+bx^3+cx^4\right)^{5/2}}{5cx^5} + \frac{\left(3b\left(b^2-4ac\right)\right)\int \frac{\sqrt{ax}}{}}{32c^2}$$

$$= \frac{3b\left(b^2-4ac\right)(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{128c^3x} - \frac{b(b+2cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{16c^2x^3} + \frac{(a}{}$$

$$= \frac{3b\left(b^2-4ac\right)(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{128c^3x} - \frac{b(b+2cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{16c^2x^3} + \frac{(a}{}$$

$$= \frac{3b\left(b^2-4ac\right)(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{128c^3x} - \frac{b(b+2cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{16c^2x^3} + \frac{(a}{}$$

$$= \frac{3b\left(b^2-4ac\right)(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{128c^3x} - \frac{b(b+2cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{16c^2x^3} + \frac{(a}{}$$

Mathematica [A] time = 0.17, size = 163, normalized size = 0.82

$$\frac{x\sqrt{a+x(b+cx)}\left(2\sqrt{c}\sqrt{a+x(b+cx)}\left(4b^2c\left(2cx^2-25a\right)+8bc^2x\left(7a+22cx^2\right)+128c^2\left(a+cx^2\right)^2+15b^4-10\right.\right.}{1280c^{7/2}\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^2,x]
```

```
[Out] (x*Sqrt[a + x*(b + c*x)]*(2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(15*b^4 - 10*b^3*
c*x + 128*c^2*(a + c*x^2)^2 + 4*b^2*c*(-25*a + 2*c*x^2) + 8*b*c^2*x*(7*a +
22*c*x^2)) - 15*b*(b^2 - 4*a*c)^2*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x
*(b + c*x)])]))/(1280*c^(7/2)*Sqrt[x^2*(a + x*(b + c*x))])
```

fricas [A] time = 0.83, size = 384, normalized size = 1.94

$$\left[\frac{15\left(b^5-8\,ab^3c+16\,a^2bc^2\right)\sqrt{c}\,x\log\left(-\frac{8\,c^2x^3+8\,bcx^2-4\,\sqrt{cx^4+bx^3+ax^2}\,(2\,cx+b)\sqrt{c}+\left(b^2+4\,ac\right)x}{x}\right)+4\left(128\,c^5x^4+176\,bc^4x^3\right.}{2560\,c^4x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/2560*(15*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*sqrt(c)*x*log(-(8*c^2*x^3 + 8*
b*c*x^2 - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)
*x)/x) + 4*(128*c^5*x^4 + 176*b*c^4*x^3 + 15*b^4*c - 100*a*b^2*c^2 + 128*a^
2*c^3 + 8*(b^2*c^3 + 32*a*c^4)*x^2 - 2*(5*b^3*c^2 - 28*a*b*c^3)*x)*sqrt(c*x
^4 + b*x^3 + a*x^2))/(c^4*x), 1/1280*(15*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*s
qrt(-c)*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*
x^3 + b*c*x^2 + a*c*x)) + 2*(128*c^5*x^4 + 176*b*c^4*x^3 + 15*b^4*c - 100*a
*b^2*c^2 + 128*a^2*c^3 + 8*(b^2*c^3 + 32*a*c^4)*x^2 - 2*(5*b^3*c^2 - 28*a*b
*c^3)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^4*x)]
```

giac [A] time = 0.93, size = 284, normalized size = 1.43

$$\frac{1}{640}\sqrt{cx^2+bx+a}\left(2\left(4\left(2\left(8\,cx\mathrm{sgn}(x)+11\,b\mathrm{sgn}(x)\right)x+\frac{b^2c^3\mathrm{sgn}(x)+32\,ac^4\mathrm{sgn}(x)}{c^4}\right)x-\frac{5\,b^3c^2\mathrm{sgn}(x)-28\,abc^3}{c^4}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/640*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*c*x*sgn(x) + 11*b*sgn(x))*x + (b^2*
c^3*sgn(x) + 32*a*c^4*sgn(x))/c^4)*x - (5*b^3*c^2*sgn(x) - 28*a*b*c^3*sgn(x
))/c^4)*x + (15*b^4*c*sgn(x) - 100*a*b^2*c^2*sgn(x) + 128*a^2*c^3*sgn(x))/c
^4) + 3/256*(b^5*sgn(x) - 8*a*b^3*c*sgn(x) + 16*a^2*b*c^2*sgn(x))*log(abs(-
2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(7/2) - 1/1280*(15*b^
5*log(abs(-b + 2*sqrt(a)*sqrt(c))) - 120*a*b^3*c*log(abs(-b + 2*sqrt(a)*sqr
t(c))) + 240*a^2*b*c^2*log(abs(-b + 2*sqrt(a)*sqrt(c))) + 30*sqrt(a)*b^4*sq
rt(c) - 200*a^(3/2)*b^2*c^(3/2) + 256*a^(5/2)*c^(5/2))*sgn(x)/c^(7/2)
```

maple [A] time = 0.01, size = 289, normalized size = 1.46

$$\frac{\left(c\,x^4+b\,x^3+a\,x^2\right)^{\frac{3}{2}}\left(-240a^2b\,c^3\ln\left(\frac{2cx+b+2\sqrt{c\,x^2+bx+a}\,\sqrt{c}}{2\sqrt{c}}\right)+120a\,b^3c^2\ln\left(\frac{2cx+b+2\sqrt{c\,x^2+bx+a}\,\sqrt{c}}{2\sqrt{c}}\right)-15b^5c\ln\left(\frac{2cx+b-}{}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^3+a*x^2)^(3/2)/x^2,x)
```

```
[Out] 1/1280*(c*x^4+b*x^3+a*x^2)^(3/2)*(256*(c*x^2+b*x+a)^(5/2)*c^(7/2)-160*(c*x^
2+b*x+a)^(3/2)*b*c^(7/2)*x-80*(c*x^2+b*x+a)^(3/2)*b^2*c^(5/2)-240*(c*x^2+b*
x+a)^(1/2)*a*b*c^(7/2)*x+60*(c*x^2+b*x+a)^(1/2)*b^3*c^(5/2)*x-120*(c*x^2+b*
x+a)^(1/2)*a*b^2*c^(5/2)+30*(c*x^2+b*x+a)^(1/2)*b^4*c^(3/2)-240*a^2*b*c^3*l
n(1/2*(2*c*x+b+2*(c*x^2+b*x+a)^(1/2)*c^(1/2))/c^(1/2))+120*a*b^3*c^2*ln(1/2
*(2*c*x+b+2*(c*x^2+b*x+a)^(1/2)*c^(1/2))/c^(1/2))-15*b^5*c*ln(1/2*(2*c*x+b+
2*(c*x^2+b*x+a)^(1/2)*c^(1/2))/c^(1/2)))/x^3/(c*x^2+b*x+a)^(3/2)/c^(9/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(cx^4+bx^3+ax^2\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,x^4+b\,x^3+a\,x^2\right)^{3/2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^2,x)
[Out] int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2\left(a+bx+cx^2\right)\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**3+a*x**2)**(3/2)/x**2,x)
[Out] Integral((x**2*(a + b*x + c*x**2))**(3/2)/x**2, x)
```

3.42 $\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^3} \, dx$

Optimal. Leaf size=165

$$\frac{3x\left(b^2-4ac\right)^2\sqrt{a+bx+cx^2}\,\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{128c^{5/2}\sqrt{ax^2+bx^3+cx^4}}-\frac{3\left(b^2-4ac\right)(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{64c^2x}+\frac{(b+2cx)\left(ax^2+\right.}{8cx}$$

```
[Out] 1/8*(2*c*x+b)*(c*x^4+b*x^3+a*x^2)^(3/2)/c/x^3+3/128*(-4*a*c+b^2)^2*x*arctan
h(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))*(c*x^2+b*x+a)^(1/2)/c^(5/2)/(c
*x^4+b*x^3+a*x^2)^(1/2)-3/64*(-4*a*c+b^2)*(2*c*x+b)*(c*x^4+b*x^3+a*x^2)^(1/
2)/c^2/x
```

Rubi [A] time = 0.13, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1918, 1914, 621, 206}

$$-\frac{3\left(b^2-4ac\right)(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{64c^2x}+\frac{3x\left(b^2-4ac\right)^2\sqrt{a+bx+cx^2}\,\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{128c^{5/2}\sqrt{ax^2+bx^3+cx^4}}+\frac{(b+2cx)\left(ax^2\right.}{8c}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^3,x]
```

```
[Out] (-3*(b^2 - 4*a*c)*(b + 2*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(64*c^2*x) + ((b
 + 2*c*x)*(a*x^2 + b*x^3 + c*x^4)^(3/2))/(8*c*x^3) + (3*(b^2 - 4*a*c)^2*x*S
qrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])]
)/(128*c^(5/2)*Sqrt[a*x^2 + b*x^3 + c*x^4])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1914

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))])/Sqrt[
a*x^q + b*x^n + c*x^(2*n - q)], Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

Rule 1918

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(x^(m - n + q + 1)*(b + 2*c*x^(n - q))*(a*x^q + b*x^n
+ c*x^(2*n - q))^p)/(2*c*(n - q)*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2
*c*(2*p + 1)), Int[x^(m + q)*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x
] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p]
 && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && Eq
```

```
Q[m + p*q + 1, n - q]
```

Rubi steps

$$\int \frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{x^3}\,dx = \frac{(b+2cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{8cx^3} - \frac{\left(3\left(b^2-4ac\right)\right)\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x}\,dx}{16c}$$

$$= -\frac{3\left(b^2-4ac\right)(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{64c^2x} + \frac{(b+2cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{8cx^3} + \frac{(3}{}$$

$$= -\frac{3\left(b^2-4ac\right)(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{64c^2x} + \frac{(b+2cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{8cx^3} + \frac{(3}{}$$

$$= -\frac{3\left(b^2-4ac\right)(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{64c^2x} + \frac{(b+2cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{8cx^3} + \frac{(3}{}$$

$$= -\frac{3\left(b^2-4ac\right)(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{64c^2x} + \frac{(b+2cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{8cx^3} + \frac{3(}{}$$

Mathematica [A] time = 0.06, size = 132, normalized size = 0.80

$$\frac{x\sqrt{a+x(b+cx)}\left(2\sqrt{c}\,(b+2cx)\sqrt{a+x(b+cx)}\left(4c\left(5a+2cx^2\right)-3b^2+8bcx\right)+3\left(b^2-4ac\right)^2\tanh^{-1}\left(\frac{b+2}{2\sqrt{c}\,\sqrt{a+}}\right.\right.}{128c^{5/2}\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^3,x]
```

```
[Out] (x*Sqrt[a + x*(b + c*x)]*(2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b + c*x)]*(-3*b
^2 + 8*b*c*x + 4*c*(5*a + 2*c*x^2)) + 3*(b^2 - 4*a*c)^2*ArcTanh[(b + 2*c*x)
/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))/(128*c^(5/2)*Sqrt[x^2*(a + x*(b + c*x
))])
```

fricas [A] time = 0.74, size = 320, normalized size = 1.94

$$\left[\frac{3\left(b^4-8\,ab^2c+16\,a^2c^2\right)\sqrt{c}\,x\log\left(-\frac{8\,c^2x^3+8\,bcx^2+4\,\sqrt{cx^4+bx^3+ax^2}\,(2\,cx+b)\sqrt{c}+\left(b^2+4\,ac\right)x}{x}\right)+4\left(16\,c^4x^3+24\,bc^3x^2-3\,b\right.}{256\,c^3x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/256*(3*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(c)*x*log(-(8*c^2*x^3 + 8*b*c*
x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/
x) + 4*(16*c^4*x^3 + 24*b*c^3*x^2 - 3*b^3*c + 20*a*b*c^2 + 2*(b^2*c^2 + 20*
a*c^3)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^3*x), -1/128*(3*(b^4 - 8*a*b^2*c
+ 16*a^2*c^2)*sqrt(-c)*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)
*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) - 2*(16*c^4*x^3 + 24*b*c^3*x^2 - 3*b
^3*c + 20*a*b*c^2 + 2*(b^2*c^2 + 20*a*c^3)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/
(c^3*x)]
```

giac [A] time = 1.01, size = 232, normalized size = 1.41

$$\frac{1}{64}\sqrt{cx^2+bx+a}\left(2\left(4\left(2\,cx\mathrm{sgn}(x)+3\,b\mathrm{sgn}(x)\right)x+\frac{b^2c^2\mathrm{sgn}(x)+20\,ac^3\mathrm{sgn}(x)}{c^3}\right)x-\frac{3\,b^3c\mathrm{sgn}(x)-20\,abc^2\mathrm{sg}}{c^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/64*sqrt(c*x^2 + b*x + a)*(2*(4*(2*c*x*sgn(x) + 3*b*sgn(x))*x + (b^2*c^2*s
gn(x) + 20*a*c^3*sgn(x))/c^3)*x - (3*b^3*c*sgn(x) - 20*a*b*c^2*sgn(x))/c^3)
 - 3/128*(b^4*sgn(x) - 8*a*b^2*c*sgn(x) + 16*a^2*c^2*sgn(x))*log(abs(-2*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(5/2) + 1/128*(3*b^4*log(a
bs(-b + 2*sqrt(a)*sqrt(c))) - 24*a*b^2*c*log(abs(-b + 2*sqrt(a)*sqrt(c))) +
 48*a^2*c^2*log(abs(-b + 2*sqrt(a)*sqrt(c))) + 6*sqrt(a)*b^3*sqrt(c) - 40*a
^(3/2)*b*c^(3/2))*sgn(x)/c^(5/2)
```

maple [A] time = 0.00, size = 265, normalized size = 1.61

$$\frac{\left(c\,x^4+b\,x^3+a\,x^2\right)^{\frac{3}{2}}\left(48a^2c^3\ln\left(\frac{2cx+b+2\sqrt{c\,x^2+bx+a}\,\sqrt{c}}{2\sqrt{c}}\right)-24a\,b^2c^2\ln\left(\frac{2cx+b+2\sqrt{c\,x^2+bx+a}\,\sqrt{c}}{2\sqrt{c}}\right)+3b^4c\ln\left(\frac{2cx+b+2\sqrt{c\,x^2+\cdots}}{2\sqrt{c}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^3+a*x^2)^(3/2)/x^3,x)
```

```
[Out] 1/128*(c*x^4+b*x^3+a*x^2)^(3/2)*(32*(c*x^2+b*x+a)^(3/2)*c^(7/2)*x+16*(c*x^2
+b*x+a)^(3/2)*b*c^(5/2)+48*(c*x^2+b*x+a)^(1/2)*a*c^(7/2)*x-12*(c*x^2+b*x+a)
^(1/2)*b^2*c^(5/2)*x+24*(c*x^2+b*x+a)^(1/2)*a*b*c^(5/2)-6*(c*x^2+b*x+a)^(1/
2)*b^3*c^(3/2)+48*a^2*c^3*ln(1/2*(2*c*x+b+2*(c*x^2+b*x+a)^(1/2)*c^(1/2))/c^
(1/2))-24*a*b^2*c^2*ln(1/2*(2*c*x+b+2*(c*x^2+b*x+a)^(1/2)*c^(1/2))/c^(1/2))
+3*b^4*c*ln(1/2*(2*c*x+b+2*(c*x^2+b*x+a)^(1/2)*c^(1/2))/c^(1/2)))/x^3/(c*x^
2+b*x+a)^(3/2)/c^(7/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^4+bx^3+ax^2\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,x^4+b\,x^3+a\,x^2\right)^{3/2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^3,x)
```

```
[Out] int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2\left(a+bx+cx^2\right)\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**3+a*x**2)**(3/2)/x**3,x)
```

```
[Out] Integral((x**2*(a + b*x + c*x**2))**(3/2)/x**3, x)
```

3.43 $\int \frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{x^4}\,dx$

Optimal. Leaf size=227

$$-\frac{a^{3/2}x\sqrt{a+bx+cx^2}\,\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\,\sqrt{a+bx+cx^2}}\right)}{\sqrt{ax^2+bx^3+cx^4}}-\frac{bx\left(b^2-12ac\right)\sqrt{a+bx+cx^2}\,\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\,\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}\sqrt{ax^2+bx^3+cx^4}}+\frac{\left(8ac+b\right.}{}$$

```
[Out] 1/3*(c*x^4+b*x^3+a*x^2)^(3/2)/x^3-a^(3/2)*x*arctanh(1/2*(b*x+2*a)/a^(1/2)/(
c*x^2+b*x+a)^(1/2))*(c*x^2+b*x+a)^(1/2)/(c*x^4+b*x^3+a*x^2)^(1/2)-1/16*b*(-
12*a*c+b^2)*x*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))*(c*x^2+b*x
+a)^(1/2)/c^(3/2)/(c*x^4+b*x^3+a*x^2)^(1/2)+1/8*(2*b*c*x+8*a*c+b^2)*(c*x^4+
b*x^3+a*x^2)^(1/2)/c/x
```

Rubi [A] time = 0.26, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1921, 1945, 1933, 843, 621, 206, 724}

$$-\frac{a^{3/2}x\sqrt{a+bx+cx^2}\,\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\,\sqrt{a+bx+cx^2}}\right)}{\sqrt{ax^2+bx^3+cx^4}}-\frac{bx\left(b^2-12ac\right)\sqrt{a+bx+cx^2}\,\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\,\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}\sqrt{ax^2+bx^3+cx^4}}+\frac{\left(8ac+b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^4,x]
```

```
[Out] ((b^2 + 8*a*c + 2*b*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(8*c*x) + (a*x^2 + b*
x^3 + c*x^4)^(3/2)/(3*x^3) - (a^(3/2)*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(2*a
+ b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/Sqrt[a*x^2 + b*x^3 + c*x^4] - (b
*(b^2 - 12*a*c)*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt
[a + b*x + c*x^2])])/(16*c^(3/2)*Sqrt[a*x^2 + b*x^3 + c*x^4])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1921

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(x^(m + 1)*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(m + p*(
2*n - q) + 1), x] + Dist[((n - q)*p)/(m + p*(2*n - q) + 1), Int[x^(m + q)*(
2*a + b*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ
[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^
2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && GtQ[m + p*q
+ 1, -(n - q)] && NeQ[m + p*(2*n - q) + 1, 0]
```

Rule 1933

```
Int[((A_) + (B_.)*(x_)^(j_.))/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c
_.)*(x_)^(r_.)], x_Symbol] :> Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(
n - q))])/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)], Int[(A + B*x^(n - q))/(x^(q/
2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]), x], x] /; FreeQ[{a, b, c, A, B
, n, q}, x] && EqQ[j, n - q] && EqQ[r, 2*n - q] && PosQ[n - q] && EqQ[n, 3]
 && EqQ[q, 2]
```

Rule 1945

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[(x^(m + 1)*(b*B*(n - q)*p +
 A*c*(m + p*q + (n - q)*(2*p + 1) + 1) + B*c*(m + p*q + 2*(n - q)*p + 1)*x^
(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(c*(m + p*(2*n - q) + 1)*(m + p
*q + (n - q)*(2*p + 1) + 1)), x] + Dist[((n - q)*p)/(c*(m + p*(2*n - q) + 1
)*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(m + q)*Simp[2*a*A*c*(m + p*q +
 (n - q)*(2*p + 1) + 1) - a*b*B*(m + p*q + 1) + (2*a*B*c*(m + p*q + 2*(n -
q)*p + 1) + A*b*c*(m + p*q + (n - q)*(2*p + 1) + 1) - b^2*B*(m + p*q + (n -
 q)*p + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /
; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Intege
rQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q]
&& GtQ[m + p*q, -(n - q) - 1] && NeQ[m + p*(2*n - q) + 1, 0] && NeQ[m + p*q
 + (n - q)*(2*p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{x^4}\,dx = \frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{3x^3}+\frac{1}{2}\int \frac{(2a+bx)\sqrt{ax^2+bx^3+cx^4}}{x^2}\,dx$$

$$= \frac{\left(b^2+8ac+2bcx\right)\sqrt{ax^2+bx^3+cx^4}}{8cx}+\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{3x^3}+\frac{\int \frac{8a^2c-\frac{1}{2}b\left(b^2-12ac\right)x}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{8c}$$

$$= \frac{\left(b^2+8ac+2bcx\right)\sqrt{ax^2+bx^3+cx^4}}{8cx}+\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{3x^3}+\frac{\left(x\sqrt{a+bx+cx^2}\right)\int}{8c\sqrt{ax^2+}}$$

$$= \frac{\left(b^2+8ac+2bcx\right)\sqrt{ax^2+bx^3+cx^4}}{8cx}+\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{3x^3}+\frac{\left(a^2x\sqrt{a+bx+cx^2}\right)}{\sqrt{ax^2+bx^3}}$$

$$= \frac{\left(b^2+8ac+2bcx\right)\sqrt{ax^2+bx^3+cx^4}}{8cx}+\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{3x^3}-\frac{\left(2a^2x\sqrt{a+bx+cx^2}\right.}{\sqrt{a}}$$

$$= \frac{\left(b^2+8ac+2bcx\right)\sqrt{ax^2+bx^3+cx^4}}{8cx}+\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{3x^3}-\frac{a^{3/2}x\sqrt{a+bx+cx^2}}{\sqrt{ax^2+}}$$

Mathematica [A] time = 0.22, size = 166, normalized size = 0.73

$$\frac{x\sqrt{a+x(b+cx)}\left(-48a^{3/2}c^{3/2}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right)+2\sqrt{c}\sqrt{a+x(b+cx)}\left(8c\left(4a+cx^2\right)+3b^2+14bcx\right)-3\cdots\right.}{48c^{3/2}\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^4,x]
```

```
[Out] (x*Sqrt[a + x*(b + c*x)]*(2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(3*b^2 + 14*b*c*x
 + 8*c*(4*a + c*x^2)) - 48*a^(3/2)*c^(3/2)*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*S
qrt[a + x*(b + c*x)])] - 3*b*(b^2 - 12*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*
Sqrt[a + x*(b + c*x)])]))/(48*c^(3/2)*Sqrt[x^2*(a + x*(b + c*x))])
```

fricas [A] time = 0.90, size = 791, normalized size = 3.48

$$\left[\frac{48\,a^{\frac{3}{2}}c^2x\log\left(-\frac{8\,abx^2+\left(b^2+4\,ac\right)x^3+8\,a^2x-4\,\sqrt{cx^4+bx^3+ax^2}\,(bx+2\,a)\sqrt{a}}{x^3}\right)-3\left(b^3-12\,abc\right)\sqrt{c}\,x\log\left(-\frac{8\,c^2x^3+8\,bcx^2+4\,\sqrt{cx^4+bx\cdots}}{\cdots}\right.}{96\,c^2x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/96*(48*a^(3/2)*c^2*x*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*s
qrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) - 3*(b^3 - 12*a*b*c)*s
qrt(c)*x*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x
 + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) + 4*(8*c^3*x^2 + 14*b*c^2*x + 3*b^2*c +
 32*a*c^2)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^2*x), 1/48*(24*a^(3/2)*c^2*x*log
(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*
(b*x + 2*a)*sqrt(a))/x^3) + 3*(b^3 - 12*a*b*c)*sqrt(-c)*x*arctan(1/2*sqrt(c
*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) + 2
*(8*c^3*x^2 + 14*b*c^2*x + 3*b^2*c + 32*a*c^2)*sqrt(c*x^4 + b*x^3 + a*x^2))
/(c^2*x), 1/96*(96*sqrt(-a)*a*c^2*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*
(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) - 3*(b^3 - 12*a*b*c)*sqrt
(c)*x*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x +
b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) + 4*(8*c^3*x^2 + 14*b*c^2*x + 3*b^2*c + 32
*a*c^2)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^2*x), 1/48*(48*sqrt(-a)*a*c^2*x*arc
tan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2
 + a^2*x)) + 3*(b^3 - 12*a*b*c)*sqrt(-c)*x*arctan(1/2*sqrt(c*x^4 + b*x^3 +
a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) + 2*(8*c^3*x^2 + 1
4*b*c^2*x + 3*b^2*c + 32*a*c^2)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^2*x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Warning, replacing 0 by ` u`, a substitution variable should perhaps be pur
ged.Warning, replacing 0 by ` u`, a substitution variable should perhaps be
 purged.Warning, replacing 0 by ` u`, a substitution variable should perhap
s be purged.index.cc index_m operator + Error: Bad Argument Value
```

maple [A] time = 0.01, size = 222, normalized size = 0.98

$$-\frac{\left(c\,x^4+b\,x^3+a\,x^2\right)^{\frac{3}{2}}\left(48a^{\frac{3}{2}}c^{\frac{5}{2}}\ln\left(\frac{bx+2a+2\sqrt{c\,x^2+bx+a}\,\sqrt{a}}{x}\right)-36ab\,c^2\ln\left(\frac{2cx+b+2\sqrt{c\,x^2+bx+a}\,\sqrt{c}}{2\sqrt{c}}\right)+3b^3c\ln\left(\frac{2cx+b+2\sqrt{c\,x}}{2\sqrt{}}\right.\right.}{48\left(c\,x^2+bx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^3+a*x^2)^(3/2)/x^4,x)
```

```
[Out] -1/48*(c*x^4+b*x^3+a*x^2)^(3/2)*(48*c^(5/2)*a^(3/2)*ln((b*x+2*a+2*(c*x^2+b*
x+a)^(1/2)*a^(1/2))/x)-16*(c*x^2+b*x+a)^(3/2)*c^(5/2)-12*(c*x^2+b*x+a)^(1/2
)*b*c^(5/2)*x-48*c^(5/2)*(c*x^2+b*x+a)^(1/2)*a-6*(c*x^2+b*x+a)^(1/2)*b^2*c^
(3/2)-36*a*b*c^2*ln(1/2*(2*c*x+b+2*(c*x^2+b*x+a)^(1/2)*c^(1/2))/c^(1/2))+3*
b^3*c*ln(1/2*(2*c*x+b+2*(c*x^2+b*x+a)^(1/2)*c^(1/2))/c^(1/2)))/x^3/(c*x^2+b
*x+a)^(3/2)/c^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^4+bx^3+ax^2\right)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^4+b\,x^3+a\,x^2\right)^{3/2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^4,x)
```

```
[Out] int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2\left(a+bx+cx^2\right)\right)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**3+a*x**2)**(3/2)/x**4,x)
```

```
[Out] Integral((x**2*(a + b*x + c*x**2))**(3/2)/x**4, x)
```

3.44 $\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^5}\,dx$

Optimal. Leaf size=219

$$\frac{3x\left(4ac+b^2\right)\sqrt{a+bx+cx^2}\,\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}\sqrt{ax^2+bx^3+cx^4}}-\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{x^4}+\frac{3(3b+2cx)\sqrt{ax^2+bx^3+cx^4}}{4x}-3$$

```
[Out] -(c*x^4+b*x^3+a*x^2)^(3/2)/x^4-3/2*b*x*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2
+b*x+a)^(1/2))*a^(1/2)*(c*x^2+b*x+a)^(1/2)/(c*x^4+b*x^3+a*x^2)^(1/2)+3/8*(4
*a*c+b^2)*x*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))*(c*x^2+b*x+a
)^(1/2)/c^(1/2)/(c*x^4+b*x^3+a*x^2)^(1/2)+3/4*(2*c*x+3*b)*(c*x^4+b*x^3+a*x^
2)^(1/2)/x
```

Rubi [A] time = 0.24, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1920, 1945, 1933, 843, 621, 206, 724}

$$\frac{3x\left(4ac+b^2\right)\sqrt{a+bx+cx^2}\,\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}\sqrt{ax^2+bx^3+cx^4}}-\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{x^4}+\frac{3(3b+2cx)\sqrt{ax^2+bx^3+cx^4}}{4x}-3$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^5,x]
```

```
[Out] (3*(3*b + 2*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(4*x) - (a*x^2 + b*x^3 + c*x^
4)^(3/2)/x^4 - (3*Sqrt[a]*b*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(2*a + b*x)/(2*
Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[a*x^2 + b*x^3 + c*x^4]) + (3*(b^2
+ 4*a*c)*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*
x + c*x^2])])/(8*Sqrt[c]*Sqrt[a*x^2 + b*x^3 + c*x^4])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1920

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(x^(m + 1)*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(m + p*q
 + 1), x] - Dist[((n - q)*p)/(m + p*q + 1), Int[x^(m + n)*(b + 2*c*x^(n - q
))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &
& EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] &&
 IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q + 1, -(n - q) +
1] && NeQ[m + p*q + 1, 0]
```

Rule 1933

```
Int[((A_) + (B_.)*(x_)^(j_.))/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c
_.)*(x_)^(r_.)], x_Symbol] :> Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(
n - q))])/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)], Int[(A + B*x^(n - q))/(x^(q/
2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]), x], x] /; FreeQ[{a, b, c, A, B
, n, q}, x] && EqQ[j, n - q] && EqQ[r, 2*n - q] && PosQ[n - q] && EqQ[n, 3]
 && EqQ[q, 2]
```

Rule 1945

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[(x^(m + 1)*(b*B*(n - q)*p +
 A*c*(m + p*q + (n - q)*(2*p + 1) + 1) + B*c*(m + p*q + 2*(n - q)*p + 1)*x^
(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(c*(m + p*(2*n - q) + 1)*(m + p
*q + (n - q)*(2*p + 1) + 1)), x] + Dist[((n - q)*p)/(c*(m + p*(2*n - q) + 1
)*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(m + q)*Simp[2*a*A*c*(m + p*q +
 (n - q)*(2*p + 1) + 1) - a*b*B*(m + p*q + 1) + (2*a*B*c*(m + p*q + 2*(n -
q)*p + 1) + A*b*c*(m + p*q + (n - q)*(2*p + 1) + 1) - b^2*B*(m + p*q + (n -
 q)*p + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /
; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Intege
rQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q]
&& GtQ[m + p*q, -(n - q) - 1] && NeQ[m + p*(2*n - q) + 1, 0] && NeQ[m + p*q
 + (n - q)*(2*p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{x^5}\,dx = -\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{x^4}+\frac{3}{2}\int \frac{(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{x^2}\,dx$$

$$= \frac{3(3b+2cx)\sqrt{ax^2+bx^3+cx^4}}{4x}-\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{x^4}+\frac{3\int \frac{4abc+c\left(b^2+4ac\right)x}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{8c}$$

$$= \frac{3(3b+2cx)\sqrt{ax^2+bx^3+cx^4}}{4x}-\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{x^4}+\frac{\left(3x\sqrt{a+bx+cx^2}\right)\int \frac{4abc+}{x\sqrt{a}}}{8c\sqrt{ax^2+bx^3+c}}$$ ... (cut off at the page edge)

$$= \frac{3(3b+2cx)\sqrt{ax^2+bx^3+cx^4}}{4x}-\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{x^4}+\frac{\left(3abx\sqrt{a+bx+cx^2}\right)\int \frac{}{x\sqrt{}}}{2\sqrt{ax^2+bx^3+cx}}$$ ... (cut off at the page edge)

$$= \frac{3(3b+2cx)\sqrt{ax^2+bx^3+cx^4}}{4x}-\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{x^4}-\frac{\left(3abx\sqrt{a+bx+cx^2}\right)\text{Subs}}{\sqrt{ax^2+b}}$$ ... (cut off at the page edge)

$$= \frac{3(3b+2cx)\sqrt{ax^2+bx^3+cx^4}}{4x}-\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{x^4}-\frac{3\sqrt{a}\,bx\sqrt{a+bx+cx^2}\,\tanh}{2\sqrt{ax^2+bx^3}}$$ ... (cut off at the page edge)

Mathematica [A] time = 0.18, size = 158, normalized size = 0.72

$$\frac{\sqrt{a+x(b+cx)}\left(3x\left(4ac+b^2\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)+2\sqrt{c}\sqrt{a+x(b+cx)}\,(x(5b+2cx)-4a)-12\sqrt{a}\,b\sqrt{c}\,x}{8\sqrt{c}\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^5,x]
```

```
[Out] (Sqrt[a + x*(b + c*x)]*(2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(-4*a + x*(5*b + 2*
c*x)) - 12*Sqrt[a]*b*Sqrt[c]*x*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b
 + c*x)])] + 3*(b^2 + 4*a*c)*x*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b
 + c*x)])]))/(8*Sqrt[c]*Sqrt[x^2*(a + x*(b + c*x))])
```

fricas [A] time = 0.71, size = 757, normalized size = 3.46

$$\left[\frac{12\sqrt{a}\,bcx^2\log\left(-\frac{8\,abx^2+\left(b^2+4\,ac\right)x^3+8\,a^2x-4\,\sqrt{cx^4+bx^3+ax^2}\,(bx+2\,a)\sqrt{a}}{x^3}\right)+3\left(b^2+4\,ac\right)\sqrt{c}\,x^2\log\left(-\frac{8\,c^2x^3+8\,bcx^2+4\,\sqrt{cx^4+b}}{}\right.}{16\,cx^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/16*(12*sqrt(a)*b*c*x^2*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4
*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) + 3*(b^2 + 4*a*c)*sq
rt(c)*x^2*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*
x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c^2
*x^2 + 5*b*c*x - 4*a*c))/(c*x^2), 1/8*(6*sqrt(a)*b*c*x^2*log(-(8*a*b*x^2 +
(b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqr
t(a))/x^3) - 3*(b^2 + 4*a*c)*sqrt(-c)*x^2*arctan(1/2*sqrt(c*x^4 + b*x^3 + a
*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) + 2*sqrt(c*x^4 + b*
x^3 + a*x^2)*(2*c^2*x^2 + 5*b*c*x - 4*a*c))/(c*x^2), 1/16*(24*sqrt(-a)*b*c*
x^2*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 +
a*b*x^2 + a^2*x)) + 3*(b^2 + 4*a*c)*sqrt(c)*x^2*log(-(8*c^2*x^3 + 8*b*c*x^2
 + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x)
+ 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c^2*x^2 + 5*b*c*x - 4*a*c))/(c*x^2), 1/8
*(12*sqrt(-a)*b*c*x^2*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sq
rt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) - 3*(b^2 + 4*a*c)*sqrt(-c)*x^2*arctan(1
/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*
c*x)) + 2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c^2*x^2 + 5*b*c*x - 4*a*c))/(c*x^2
)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Unable to divide, perhaps due to rounding error%%%{%%{[1,0]:[1,0,%%%{-1,[1]
%%%}]%%},[4,0,0,0]%%%}+%%%{%%{[-2,0]:[1,0,%%%{-1,[1]%%%}]%%},[2,0,1,0]%%%}+
%%%{%%{[1,0]:[1,0,%%%{-1,[1]%%%}]%%},[0,0,2,0]%%%} / %%%{%%%{1,[1]%%%},[4,0
,0,0]%%%}+%%%{%%%{-2,[1]%%%},[2,0,1,0]%%%}+%%%{%%%{1,[1]%%%},[0,0,2,0]%%%}
Error: Bad Argument Value
```

maple [A] time = 0.01, size = 254, normalized size = 1.16

$$\frac{\left(c\,x^4+b\,x^3+a\,x^2\right)^{\frac{3}{2}}\left(12a^2c^2x\ln\left(\frac{2cx+b+2\sqrt{c\,x^2+bx+a}\,\sqrt{c}}{2\sqrt{c}}\right)-12a^{\frac{3}{2}}b\,c^{\frac{3}{2}}x\ln\left(\frac{bx+2a+2\sqrt{c\,x^2+bx+a}\,\sqrt{a}}{x}\right)+3a\,b^2cx\ln\left(\frac{2cx+b+}{}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^3+a*x^2)^(3/2)/x^5,x)
```

```
[Out] 1/8*(c*x^4+b*x^3+a*x^2)^(3/2)*(8*c^(5/2)*(c*x^2+b*x+a)^(3/2)*x^2+12*c^(5/2)
*(c*x^2+b*x+a)^(1/2)*x^2*a-12*c^(3/2)*a^(3/2)*ln((b*x+2*a+2*(c*x^2+b*x+a)^(
1/2)*a^(1/2))/x)*x*b-8*(c*x^2+b*x+a)^(5/2)*c^(3/2)+8*c^(3/2)*(c*x^2+b*x+a)^
(3/2)*x*b+18*c^(3/2)*(c*x^2+b*x+a)^(1/2)*x*a*b+12*ln(1/2*(2*c*x+b+2*(c*x^2+
b*x+a)^(1/2)*c^(1/2))/c^(1/2))*x*a^2*c^2+3*c*ln(1/2*(2*c*x+b+2*(c*x^2+b*x+a
)^(1/2)*c^(1/2))/c^(1/2))*x*a*b^2)/x^4/(c*x^2+b*x+a)^(3/2)/a/c^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^4+bx^3+ax^2\right)^{\frac{3}{2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^4+b\,x^3+a\,x^2\right)^{3/2}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^5,x)
```

```
[Out] int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^5, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2\left(a+bx+cx^2\right)\right)^{\frac{3}{2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**3+a*x**2)**(3/2)/x**5,x)
```

```
[Out] Integral((x**2*(a + b*x + c*x**2))**(3/2)/x**5, x)
```

3.45 $\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^6}\,dx$

Optimal. Leaf size=219

$$-\frac{3x\left(4ac+b^2\right)\sqrt{a+bx+cx^2}\,\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}-\frac{3(b-2cx)\sqrt{ax^2+bx^3+cx^4}}{4x^2}+\frac{3b\sqrt{c}\,x\sqrt{a+bx+cx^2}\ \text{ta}\cdots}{2\sqrt{ax^2+b\cdots}}$$

```
[Out] -1/2*(c*x^4+b*x^3+a*x^2)^(3/2)/x^5-3/8*(4*a*c+b^2)*x*arctanh(1/2*(b*x+2*a)/
a^(1/2)/(c*x^2+b*x+a)^(1/2))*(c*x^2+b*x+a)^(1/2)/a^(1/2)/(c*x^4+b*x^3+a*x^2
)^(1/2)+3/2*b*x*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))*c^(1/2)*
(c*x^2+b*x+a)^(1/2)/(c*x^4+b*x^3+a*x^2)^(1/2)-3/4*(-2*c*x+b)*(c*x^4+b*x^3+a
*x^2)^(1/2)/x^2
```

Rubi [A] time = 0.24, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1920, 1941, 1933, 843, 621, 206, 724}

$$-\frac{3x\left(4ac+b^2\right)\sqrt{a+bx+cx^2}\,\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}-\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{2x^5}-\frac{3(b-2cx)\sqrt{ax^2+bx^3+cx^4}}{4x^2}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^6,x]
```

```
[Out] (-3*(b - 2*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(4*x^2) - (a*x^2 + b*x^3 + c*x
^4)^(3/2)/(2*x^5) - (3*(b^2 + 4*a*c)*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(2*a +
 b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(8*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c
*x^4]) + (3*b*Sqrt[c]*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c
]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[a*x^2 + b*x^3 + c*x^4])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1920

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(x^(m + 1)*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(m + p*q
 + 1), x] - Dist[((n - q)*p)/(m + p*q + 1), Int[x^(m + n)*(b + 2*c*x^(n - q
))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &
& EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] &&
 IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q + 1, -(n - q) +
1] && NeQ[m + p*q + 1, 0]
```

Rule 1933

```
Int[((A_) + (B_.)*(x_)^(j_.))/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c
_.)*(x_)^(r_.)], x_Symbol] :> Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(
n - q))])/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)], Int[(A + B*x^(n - q))/(x^(q/
2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]), x], x] /; FreeQ[{a, b, c, A, B
, n, q}, x] && EqQ[j, n - q] && EqQ[r, 2*n - q] && PosQ[n - q] && EqQ[n, 3]
 && EqQ[q, 2]
```

Rule 1941

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[(x^(m + 1)*(A*(m + p*q + (n
 - q)*(2*p + 1) + 1) + B*(m + p*q + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n
 - q))^p)/((m + p*q + 1)*(m + p*q + (n - q)*(2*p + 1) + 1)), x] + Dist[((n
- q)*p)/((m + p*q + 1)*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(n + m)*Si
mp[2*a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(2*p + 1) + 1) + (b*B*(m +
p*q + 1) - 2*A*c*(m + p*q + (n - q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q +
b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[
r, n - q] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGt
Q[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q, -(n - q)] && NeQ[m
+ p*q + 1, 0] && NeQ[m + p*q + (n - q)*(2*p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{x^6}\,dx = -\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{2x^5}+\frac{3}{4}\int \frac{(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{x^3}\,dx$$

$$= -\frac{3(b-2cx)\sqrt{ax^2+bx^3+cx^4}}{4x^2}-\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{2x^5}-\frac{3}{8}\int \frac{-b^2-4ac-4bcx}{\sqrt{ax^2+bx^3+cx^4}}\,dx$$

$$= -\frac{3(b-2cx)\sqrt{ax^2+bx^3+cx^4}}{4x^2}-\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{2x^5}-\frac{\left(3x\sqrt{a+bx+cx^2}\right)\int \frac{-b^2-4}{x\sqrt{a+}}}{8\sqrt{ax^2+bx^3+cx^4}}$$

$$= -\frac{3(b-2cx)\sqrt{ax^2+bx^3+cx^4}}{4x^2}-\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{2x^5}+\frac{\left(3bcx\sqrt{a+bx+cx^2}\right)\int \frac{}{\sqrt{a}}}{2\sqrt{ax^2+bx^3+cx}}$$

$$= -\frac{3(b-2cx)\sqrt{ax^2+bx^3+cx^4}}{4x^2}-\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{2x^5}+\frac{\left(3bcx\sqrt{a+bx+cx^2}\right)\text{Subs}}{\sqrt{ax^2+b}}$$

$$= -\frac{3(b-2cx)\sqrt{ax^2+bx^3+cx^4}}{4x^2}-\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{2x^5}-\frac{3\left(b^2+4ac\right)x\sqrt{a+bx+cx}}{8\sqrt{a}\sqrt{ax^2+}}$$

Mathematica [A] time = 0.19, size = 162, normalized size = 0.74

$$-\frac{\sqrt{x^2(a+x(b+cx))}\left(3x^2\left(4ac+b^2\right)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right)+2\sqrt{a}\left((2a+x(5b-4cx))\sqrt{a+x(b+cx)}-6b\sqrt{\ldots}\right.\right.}{8\sqrt{a}\,x^3\sqrt{a+x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^6,x]
```

```
[Out] -1/8*(Sqrt[x^2*(a + x*(b + c*x))]*(3*(b^2 + 4*a*c)*x^2*ArcTanh[(2*a + b*x)/
(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])] + 2*Sqrt[a]*((2*a + x*(5*b - 4*c*x))*Sqr
t[a + x*(b + c*x)] - 6*b*Sqrt[c]*x^2*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a
+ x*(b + c*x)])])))/(Sqrt[a]*x^3*Sqrt[a + x*(b + c*x)])
```

fricas [A] time = 0.90, size = 757, normalized size = 3.46

$$\left[\frac{12\,ab\sqrt{c}\,x^3\log\left(-\frac{8\,c^2x^3+8\,bcx^2+4\,\sqrt{cx^4+bx^3+ax^2}\,(2\,cx+b)\sqrt{c}+\left(b^2+4\,ac\right)x}{x}\right)+3\left(b^2+4\,ac\right)\sqrt{a}\,x^3\log\left(-\frac{8\,abx^2+\left(b^2+4\,ac\right)x^3+8\,a^2\ldots}{\ldots}\right.}{16\,ax^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^6,x, algorithm="fricas")
```

```
[Out] [1/16*(12*a*b*sqrt(c)*x^3*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^
3 + a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) + 3*(b^2 + 4*a*c)*sqrt
(a)*x^3*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^
3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(4*a*c
*x^2 - 5*a*b*x - 2*a^2))/(a*x^3), -1/16*(24*a*b*sqrt(-c)*x^3*arctan(1/2*sqr
t(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x))
- 3*(b^2 + 4*a*c)*sqrt(a)*x^3*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x
 - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) - 4*sqrt(c*x^4 +
 b*x^3 + a*x^2)*(4*a*c*x^2 - 5*a*b*x - 2*a^2))/(a*x^3), 1/8*(6*a*b*sqrt(c)*
x^3*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)
*sqrt(c) + (b^2 + 4*a*c)*x)/x) + 3*(b^2 + 4*a*c)*sqrt(-a)*x^3*arctan(1/2*sq
rt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x))
 + 2*sqrt(c*x^4 + b*x^3 + a*x^2)*(4*a*c*x^2 - 5*a*b*x - 2*a^2))/(a*x^3), -1
/8*(12*a*b*sqrt(-c)*x^3*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*
sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) - 3*(b^2 + 4*a*c)*sqrt(-a)*x^3*arctan
(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 +
a^2*x)) - 2*sqrt(c*x^4 + b*x^3 + a*x^2)*(4*a*c*x^2 - 5*a*b*x - 2*a^2))/(a*x
^3)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^6,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.01, size = 338, normalized size = 1.54

$$-\frac{\left(c\,x^4+b\,x^3+a\,x^2\right)^{\frac{3}{2}}\left(12a^{\frac{5}{2}}c^{\frac{5}{2}}x^2\ln\left(\frac{bx+2a+2\sqrt{c\,x^2+bx+a}\,\sqrt{a}}{x}\right)-12a^2b\,c^2x^2\ln\left(\frac{2cx+b+2\sqrt{c\,x^2+bx+a}\,\sqrt{c}}{2\sqrt{c}}\right)+3a^{\frac{3}{2}}b^2c^{\frac{3}{2}}x^2\ldots\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^3+a*x^2)^(3/2)/x^6,x)
```

```
[Out] -1/8*(c*x^4+b*x^3+a*x^2)^(3/2)*(12*c^(5/2)*a^(5/2)*ln((b*x+2*a+2*(c*x^2+b*x
+a)^(1/2)*a^(1/2))/x)*x^2-2*c^(5/2)*(c*x^2+b*x+a)^(3/2)*x^3*b-4*c^(5/2)*(c*
x^2+b*x+a)^(3/2)*x^2*a-6*c^(5/2)*(c*x^2+b*x+a)^(1/2)*x^3*a*b+3*c^(3/2)*a^(3
/2)*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)*x^2*b^2-12*c^(5/2)*(c*x^2
+b*x+a)^(1/2)*x^2*a^2+2*c^(3/2)*(c*x^2+b*x+a)^(5/2)*x*b-2*c^(3/2)*(c*x^2+b*
x+a)^(3/2)*x^2*b^2+4*(c*x^2+b*x+a)^(5/2)*a*c^(3/2)-6*c^(3/2)*(c*x^2+b*x+a)^
(1/2)*x^2*a*b^2-12*c^2*ln(1/2*(2*c*x+b+2*(c*x^2+b*x+a)^(1/2)*c^(1/2))/c^(1/
2))*x^2*a^2*b)/x^5/(c*x^2+b*x+a)^(3/2)/a^2/c^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^4 + bx^3 + ax^2\right)^{\frac{3}{2}}}{x^6} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^6,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x^6, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^4 + b\,x^3 + a\,x^2\right)^{3/2}}{x^6} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^6,x)
```

```
[Out] int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^6, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2 \left(a + bx + cx^2\right)\right)^{\frac{3}{2}}}{x^6} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**3+a*x**2)**(3/2)/x**6,x)
```

```
[Out] Integral((x**2*(a + b*x + c*x**2))**(3/2)/x**6, x)
```

3.46 $\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^7}\,dx$

Optimal. Leaf size=257

$$\frac{bx(b^2-12ac)\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{16a^{3/2}\sqrt{ax^2+bx^3+cx^4}}+\frac{(-8ac+b^2+2bcx)\sqrt{ax^2+bx^3+cx^4}}{8ax^2}+\frac{c^{3/2}x\sqrt{a+bx+}}{\sqrt{}}$$

```
[Out] -1/3*(c*x^4+b*x^3+a*x^2)^(3/2)/x^6-1/4*b*(c*x^4+b*x^3+a*x^2)^(3/2)/a/x^5+1/
16*b*(-12*a*c+b^2)*x*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))*(c*
x^2+b*x+a)^(1/2)/a^(3/2)/(c*x^4+b*x^3+a*x^2)^(1/2)+c^(3/2)*x*arctanh(1/2*(2
*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))*(c*x^2+b*x+a)^(1/2)/(c*x^4+b*x^3+a*x^2
)^(1/2)+1/8*(2*b*c*x-8*a*c+b^2)*(c*x^4+b*x^3+a*x^2)^(1/2)/a/x^2
```

Rubi [A] time = 0.35, antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1920, 1951, 1941, 1933, 843, 621, 206, 724}

$$\frac{bx(b^2-12ac)\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{16a^{3/2}\sqrt{ax^2+bx^3+cx^4}}+\frac{(-8ac+b^2+2bcx)\sqrt{ax^2+bx^3+cx^4}}{8ax^2}+\frac{c^{3/2}x\sqrt{a+bx+}}{\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^7,x]
```

```
[Out] ((b^2 - 8*a*c + 2*b*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(8*a*x^2) - (a*x^2 +
b*x^3 + c*x^4)^(3/2)/(3*x^6) - (b*(a*x^2 + b*x^3 + c*x^4)^(3/2))/(4*a*x^5)
+ (b*(b^2 - 12*a*c)*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*
Sqrt[a + b*x + c*x^2])])/(16*a^(3/2)*Sqrt[a*x^2 + b*x^3 + c*x^4]) + (c^(3/2
)*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x
^2])])/Sqrt[a*x^2 + b*x^3 + c*x^4]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
```

```
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1920

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(x^(m + 1)*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(m + p*q
 + 1), x] - Dist[((n - q)*p)/(m + p*q + 1), Int[x^(m + n)*(b + 2*c*x^(n - q
))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &
& EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] &&
 IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q + 1, -(n - q) +
1] && NeQ[m + p*q + 1, 0]
```

Rule 1933

```
Int[((A_) + (B_.)*(x_)^(j_.))/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c
_.)*(x_)^(r_.)], x_Symbol] :> Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(
n - q))])/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)], Int[(A + B*x^(n - q))/(x^(q/
2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]), x], x] /; FreeQ[{a, b, c, A, B
, n, q}, x] && EqQ[j, n - q] && EqQ[r, 2*n - q] && PosQ[n - q] && EqQ[n, 3]
 && EqQ[q, 2]
```

Rule 1941

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[(x^(m + 1)*(A*(m + p*q + (n
 - q)*(2*p + 1) + 1) + B*(m + p*q + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n
 - q))^p)/((m + p*q + 1)*(m + p*q + (n - q)*(2*p + 1) + 1)), x] + Dist[((n
- q)*p)/((m + p*q + 1)*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(n + m)*Si
mp[2*a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(2*p + 1) + 1) + (b*B*(m +
p*q + 1) - 2*A*c*(m + p*q + (n - q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q +
b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[
r, n - q] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGt
Q[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q, -(n - q)] && NeQ[m
+ p*q + 1, 0] && NeQ[m + p*q + (n - q)*(2*p + 1) + 1, 0]
```

Rule 1951

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[(A*x^(m - q + 1)*(a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1))/(a*(m + p*q + 1)), x] + Dist[1/(a*(m + p*q +
 1)), Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p
+ 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b*
x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q
] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
&& RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n - q
)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]
```

Rubi steps

$$\int \frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{x^7}\,dx = -\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{3x^6} + \frac{1}{2}\int \frac{(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{x^4}\,dx$$

$$= -\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{3x^6} - \frac{b\left(ax^2+bx^3+cx^4\right)^{3/2}}{4ax^5} - \frac{\int \frac{\left(\frac{1}{2}\left(b^2-8ac\right)-bcx\right)\sqrt{ax^2+bx^3+cx^4}}{x^3}\,dx}{4a}$$

$$= \frac{\left(b^2-8ac+2bcx\right)\sqrt{ax^2+bx^3+cx^4}}{8ax^2} - \frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{3x^6} - \frac{b\left(ax^2+bx^3+cx^4\right)}{4ax^5}$$

$$= \frac{\left(b^2-8ac+2bcx\right)\sqrt{ax^2+bx^3+cx^4}}{8ax^2} - \frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{3x^6} - \frac{b\left(ax^2+bx^3+cx^4\right)}{4ax^5}$$

$$= \frac{\left(b^2-8ac+2bcx\right)\sqrt{ax^2+bx^3+cx^4}}{8ax^2} - \frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{3x^6} - \frac{b\left(ax^2+bx^3+cx^4\right)}{4ax^5}$$

$$= \frac{\left(b^2-8ac+2bcx\right)\sqrt{ax^2+bx^3+cx^4}}{8ax^2} - \frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{3x^6} - \frac{b\left(ax^2+bx^3+cx^4\right)}{4ax^5}$$

$$= \frac{\left(b^2-8ac+2bcx\right)\sqrt{ax^2+bx^3+cx^4}}{8ax^2} - \frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{3x^6} - \frac{b\left(ax^2+bx^3+cx^4\right)}{4ax^5}$$

Mathematica [A] time = 0.29, size = 175, normalized size = 0.68

$$\frac{\sqrt{x^2(a+x(b+cx))}\left(3bx^3\left(b^2-12ac\right)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right)-2\sqrt{a}\left(\sqrt{a+x(b+cx)}\left(8a^2+2ax(7b+16cx)+\right.\right.\right.}{48a^{3/2}x^4\sqrt{a+x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^7,x]
```

```
[Out] (Sqrt[x^2*(a + x*(b + c*x))]*(3*b*(b^2 - 12*a*c)*x^3*ArcTanh[(2*a + b*x)/(2
*Sqrt[a]*Sqrt[a + x*(b + c*x)])] - 2*Sqrt[a]*(Sqrt[a + x*(b + c*x)]*(8*a^2
+ 3*b^2*x^2 + 2*a*x*(7*b + 16*c*x)) - 24*a*c^(3/2)*x^3*ArcTanh[(b + 2*c*x)/
(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])))/(48*a^(3/2)*x^4*Sqrt[a + x*(b + c*x)]
)
```

fricas [A] time = 0.81, size = 815, normalized size = 3.17

$$\left[\frac{48\,a^2c^{\frac{3}{2}}x^4\log\left(-\frac{8\,c^2x^3+8\,bcx^2+4\,\sqrt{cx^4+bx^3+ax^2}\,(2\,cx+b)\sqrt{c}+\left(b^2+4\,ac\right)x}{x}\right)-3\left(b^3-12\,abc\right)\sqrt{a}\,x^4\log\left(-\frac{8\,abx^2+\left(b^2+4\,ac\right)x^3+8\,a}{}\right.}{96\,a^2x^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^7,x, algorithm="fricas")
```

```
[Out] [1/96*(48*a^2*c^(3/2)*x^4*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^
3 + a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) - 3*(b^3 - 12*a*b*c)*s
qrt(a)*x^4*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b
*x^3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(14
*a^2*b*x + 8*a^3 + (3*a*b^2 + 32*a^2*c)*x^2))/(a^2*x^4), -1/96*(96*a^2*sqrt
(-c)*c*x^4*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2
*x^3 + b*c*x^2 + a*c*x)) + 3*(b^3 - 12*a*b*c)*sqrt(a)*x^4*log(-(8*a*b*x^2 +
```

```
(b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sq
rt(a))/x^3) + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(14*a^2*b*x + 8*a^3 + (3*a*b^2
+ 32*a^2*c)*x^2))/(a^2*x^4), 1/48*(24*a^2*c^(3/2)*x^4*log(-(8*c^2*x^3 + 8*b
*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*
x)/x) - 3*(b^3 - 12*a*b*c)*sqrt(-a)*x^4*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x
^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) - 2*sqrt(c*x^4 + b*x^
3 + a*x^2)*(14*a^2*b*x + 8*a^3 + (3*a*b^2 + 32*a^2*c)*x^2))/(a^2*x^4), -1/4
8*(48*a^2*sqrt(-c)*c*x^4*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)
*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) + 3*(b^3 - 12*a*b*c)*sqrt(-a)*x^4*ar
ctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^
2 + a^2*x)) + 2*sqrt(c*x^4 + b*x^3 + a*x^2)*(14*a^2*b*x + 8*a^3 + (3*a*b^2
+ 32*a^2*c)*x^2))/(a^2*x^4)]
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^7,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Warning, choosing root of [1,0,%%%{-2,[1,0,0,2]%%%}+%%%{-2,[0,1,0,1]%%%}+%%
%{-4,[0,0,1,0]%%%},0,%%%{1,[2,0,0,4]%%%}+%%%{2,[1,1,0,3]%%%}+%%%{1,[0,2,0,2
]%%%}] at parameters values [-97,-82,63.4443001123,-27]Warning, choosing ro
ot of [1,0,%%%{-2,[1,0,0,2]%%%}+%%%{-2,[0,1,0,1]%%%}+%%%{-4,[0,0,1,0]%%%},0
,%%%{1,[2,0,0,4]%%%}+%%%{2,[1,1,0,3]%%%}+%%%{1,[0,2,0,2]%%%}] at parameters
 values [63,-49,35.2935628123,-64]Warning, choosing root of [1,0,%%%{-2,[2,
1,0,0]%%%}+%%%{-2,[1,0,1,0]%%%}+%%%{-4,[0,0,0,1]%%%},0,%%%{1,[4,2,0,0]%%%}+
%%%{2,[3,1,1,0]%%%}+%%%{1,[2,0,2,0]%%%}] at parameters values [22,42,56,43.
9628838282]Sign error (%%%{b-2*sqrt(a)*sqrt(c),0%%%}+%%%{-(-2*a*c+b*sqrt(a)
*sqrt(c))/a,1%%%}+%%%{-(4*a*c*sqrt(a)*sqrt(c)-b^2*sqrt(a)*sqrt(c))/(4*a^2),
2%%%}+%%%{undef,3%%%})Evaluation time: 0.45Limit: Max order reached or unab
le to make series expansion Error: Bad Argument Value
```

maple [**A**] time = 0.01, size = 435, normalized size = 1.69

$$\left(c\,x^{4}+b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}\left(48a^{3}c^{3}x^{3}\ln\left(\frac{2cx+b+2\sqrt{c\,x^{2}+bx+a}\,\sqrt{c}}{2\sqrt{c}}\right)-36a^{\frac{5}{2}}b\,c^{\frac{5}{2}}x^{3}\ln\left(\frac{bx+2a+2\sqrt{c\,x^{2}+bx+a}\,\sqrt{a}}{x}\right)+3a^{\frac{3}{2}}b^{3}c^{\frac{3}{2}}x^{3}\ln\left(\cdots\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^3+a*x^2)^(3/2)/x^7,x)
```

```
[Out] 1/48*(c*x^4+b*x^3+a*x^2)^(3/2)*(32*c^(7/2)*(c*x^2+b*x+a)^(3/2)*x^4*a-36*c^(
5/2)*a^(5/2)*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)*x^3*b+48*c^(7/2)
*(c*x^2+b*x+a)^(1/2)*x^4*a^2-2*c^(5/2)*(c*x^2+b*x+a)^(3/2)*x^4*b^2-32*c^(5/
2)*(c*x^2+b*x+a)^(5/2)*x^2*a+28*c^(5/2)*(c*x^2+b*x+a)^(3/2)*x^3*a*b-6*c^(5/
2)*(c*x^2+b*x+a)^(1/2)*x^4*a*b^2+3*c^(3/2)*a^(3/2)*ln((b*x+2*a+2*(c*x^2+b*x
+a)^(1/2)*a^(1/2))/x)*x^3*b^3+60*c^(5/2)*(c*x^2+b*x+a)^(1/2)*x^3*a^2*b+2*c^
(3/2)*(c*x^2+b*x+a)^(5/2)*x^2*b^2-2*c^(3/2)*(c*x^2+b*x+a)^(3/2)*x^3*b^3+4*c
^(3/2)*(c*x^2+b*x+a)^(5/2)*x*a*b-6*c^(3/2)*(c*x^2+b*x+a)^(1/2)*x^3*a*b^3-16
*(c*x^2+b*x+a)^(5/2)*a^2*c^(3/2)+48*ln(1/2*(2*c*x+b+2*(c*x^2+b*x+a)^(1/2)*c
^(1/2))/c^(1/2))*x^3*a^3*c^3)/x^6/(c*x^2+b*x+a)^(3/2)/a^3/c^(3/2)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^{4}+bx^{3}+ax^{2}\right)^{\frac{3}{2}}}{x^{7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^7,x, algorithm="maxima")
[Out] integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x^7, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^4+b\,x^3+a\,x^2\right)^{3/2}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^7,x)
[Out] int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^7, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2\left(a+bx+cx^2\right)\right)^{\frac{3}{2}}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**3+a*x**2)**(3/2)/x**7,x)
[Out] Integral((x**2*(a + b*x + c*x**2))**(3/2)/x**7, x)
```

3.47 $\int \frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{x^8}\,dx$

Optimal. Leaf size=197

$$-\frac{3\left(b^2-4ac\right)^2\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{128a^{5/2}}+\frac{b\left(3b^2-20ac\right)\sqrt{ax^2+bx^3+cx^4}}{64a^2x^2}-\frac{\left(b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{32ax^3}-\frac{(b+}{}$$

```
[Out] -1/4*(c*x^4+b*x^3+a*x^2)^(3/2)/x^7-3/128*(-4*a*c+b^2)^2*arctanh(1/2*x*(b*x+
2*a)/a^(1/2)/(c*x^4+b*x^3+a*x^2)^(1/2))/a^(5/2)-1/32*(-12*a*c+b^2)*(c*x^4+b
*x^3+a*x^2)^(1/2)/a/x^3+1/64*b*(-20*a*c+3*b^2)*(c*x^4+b*x^3+a*x^2)^(1/2)/a^
2/x^2-1/8*(6*c*x+b)*(c*x^4+b*x^3+a*x^2)^(1/2)/x^4
```

Rubi [A] time = 0.36, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1920, 1941, 1951, 12, 1904, 206}

$$\frac{b\left(3b^2-20ac\right)\sqrt{ax^2+bx^3+cx^4}}{64a^2x^2}-\frac{3\left(b^2-4ac\right)^2\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{128a^{5/2}}-\frac{\left(b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{32ax^3}-\frac{(ax^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^8,x]
```

```
[Out] -((b^2 - 12*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(32*a*x^3) + (b*(3*b^2 - 20*a
*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(64*a^2*x^2) - ((b + 6*c*x)*Sqrt[a*x^2 + b
*x^3 + c*x^4])/(8*x^4) - (a*x^2 + b*x^3 + c*x^4)^(3/2)/(4*x^7) - (3*(b^2 -
4*a*c)^2*ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])])/
(128*a^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1904

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, (x*(2*a + b*x^(n - 2)))/
Sqrt[a*x^2 + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n
 - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1920

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(x^(m + 1)*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(m + p*q
 + 1), x] - Dist[((n - q)*p)/(m + p*q + 1), Int[x^(m + n)*(b + 2*c*x^(n - q
))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &
& EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] &&
 IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q + 1, -(n - q) +
1] && NeQ[m + p*q + 1, 0]
```

Rule 1941

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[(x^(m + 1)*(A*(m + p*q + (n
 - q)*(2*p + 1) + 1) + B*(m + p*q + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n
 - q))^p)/((m + p*q + 1)*(m + p*q + (n - q)*(2*p + 1) + 1)), x] + Dist[((n
- q)*p)/((m + p*q + 1)*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(n + m)*Si
mp[2*a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(2*p + 1) + 1) + (b*B*(m +
p*q + 1) - 2*A*c*(m + p*q + (n - q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q +
b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[
r, n - q] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGt
Q[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q, -(n - q)] && NeQ[m
+ p*q + 1, 0] && NeQ[m + p*q + (n - q)*(2*p + 1) + 1, 0]
```

Rule 1951

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[(A*x^(m - q + 1)*(a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1))/(a*(m + p*q + 1)), x] + Dist[1/(a*(m + p*q +
 1)), Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p
+ 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b*
x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q
] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
&& RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n - q
)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]
```

Rubi steps

$$\int \frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{x^8}\,dx = -\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{4x^7}+\frac{3}{8}\int \frac{(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{x^5}\,dx$$

$$= -\frac{(b+6cx)\sqrt{ax^2+bx^3+cx^4}}{8x^4}-\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{4x^7}+\frac{1}{16}\int \frac{b^2-12ac-4bcx}{x^2\sqrt{ax^2+bx^3+cx}}$$

$$= -\frac{\left(b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{32ax^3}-\frac{(b+6cx)\sqrt{ax^2+bx^3+cx^4}}{8x^4}-\frac{\left(ax^2+bx^3+cx\right.}{4x^7}$$

$$= -\frac{\left(b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{32ax^3}+\frac{b\left(3b^2-20ac\right)\sqrt{ax^2+bx^3+cx^4}}{64a^2x^2}-\frac{(b+6cx)}{}$$

$$= -\frac{\left(b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{32ax^3}+\frac{b\left(3b^2-20ac\right)\sqrt{ax^2+bx^3+cx^4}}{64a^2x^2}-\frac{(b+6cx)}{}$$

$$= -\frac{\left(b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{32ax^3}+\frac{b\left(3b^2-20ac\right)\sqrt{ax^2+bx^3+cx^4}}{64a^2x^2}-\frac{(b+6cx)}{}$$

$$= -\frac{\left(b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{32ax^3}+\frac{b\left(3b^2-20ac\right)\sqrt{ax^2+bx^3+cx^4}}{64a^2x^2}-\frac{(b+6cx)}{}$$

Mathematica [A] time = 0.12, size = 141, normalized size = 0.72

$$-\frac{\sqrt{x^2(a+x(b+cx))}\left(2\sqrt{a}\,(2a+bx)\sqrt{a+x(b+cx)}\left(8a^2+4ax(2b+5cx)-3b^2x^2\right)+3x^4\left(b^2-4ac\right)^2\tanh^{-1}\right(}{128a^{5/2}x^5\sqrt{a+x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^8,x]
```

```
[Out] -1/128*(Sqrt[x^2*(a + x*(b + c*x))]*(2*Sqrt[a]*(2*a + b*x)*Sqrt[a + x*(b +
c*x)]*(8*a^2 - 3*b^2*x^2 + 4*a*x*(2*b + 5*c*x)) + 3*(b^2 - 4*a*c)^2*x^4*Arc
Tanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])]))/(a^(5/2)*x^5*Sqrt[a +
 x*(b + c*x)])
```

fricas [A] time = 1.00, size = 332, normalized size = 1.69

$$\left[\frac{3\left(b^4-8\,ab^2c+16\,a^2c^2\right)\sqrt{a}\,x^5\log\left(-\frac{8\,abx^2+\left(b^2+4\,ac\right)x^3+8\,a^2x-4\,\sqrt{cx^4+bx^3+ax^2}\,(bx+2\,a)\sqrt{a}}{x^3}\right)-4\left(24\,a^3bx+16\,a^4-\left(3\,ab^3\right.\right.}{256\,a^3x^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^8,x, algorithm="fricas")
```

```
[Out] [1/256*(3*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(a)*x^5*log(-(8*a*b*x^2 + (b^2
 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(a)
)/x^3) - 4*(24*a^3*b*x + 16*a^4 - (3*a*b^3 - 20*a^2*b*c)*x^3 + 2*(a^2*b^2 +
 20*a^3*c)*x^2)*sqrt(c*x^4 + b*x^3 + a*x^2))/(a^3*x^5), 1/128*(3*(b^4 - 8*a
*b^2*c + 16*a^2*c^2)*sqrt(-a)*x^5*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b
*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) - 2*(24*a^3*b*x + 16*a^4 -
(3*a*b^3 - 20*a^2*b*c)*x^3 + 2*(a^2*b^2 + 20*a^3*c)*x^2)*sqrt(c*x^4 + b*x^3
 + a*x^2))/(a^3*x^5)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^8,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.01, size = 501, normalized size = 2.54

$$-\frac{\left(c\,x^4+b\,x^3+a\,x^2\right)^{\frac{3}{2}}\left(48a^{\frac{7}{2}}c^2x^4\ln\left(\frac{bx+2a+2\sqrt{c\,x^2+bx+a}\,\sqrt{a}}{x}\right)-24a^{\frac{5}{2}}b^2c\,x^4\ln\left(\frac{bx+2a+2\sqrt{c\,x^2+bx+a}\,\sqrt{a}}{x}\right)+3a^{\frac{3}{2}}b^4x^4\ln\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^3+a*x^2)^(3/2)/x^8,x)
```

```
[Out] -1/128*(c*x^4+b*x^3+a*x^2)^(3/2)*(48*c^2*a^(7/2)*ln((b*x+2*a+2*(c*x^2+b*x+a
)^(1/2)*a^(1/2))/x)*x^4+24*c^2*(c*x^2+b*x+a)^(3/2)*x^5*a*b-24*c*a^(5/2)*ln(
(b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)*x^4*b^2-16*c^2*(c*x^2+b*x+a)^(3/
2)*x^4*a^2+24*c^2*(c*x^2+b*x+a)^(1/2)*x^5*a^2*b-2*c*(c*x^2+b*x+a)^(3/2)*x^5
*b^3-48*c^2*(c*x^2+b*x+a)^(1/2)*x^4*a^3-24*c*(c*x^2+b*x+a)^(5/2)*x^3*a*b+20
*c*(c*x^2+b*x+a)^(3/2)*x^4*a*b^2-6*c*(c*x^2+b*x+a)^(1/2)*x^5*a*b^3+3*a^(3/2
)*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)*x^4*b^4+16*c*(c*x^2+b*x+a)^
(5/2)*x^2*a^2+36*c*(c*x^2+b*x+a)^(1/2)*x^4*a^2*b^2+2*(c*x^2+b*x+a)^(5/2)*x^
3*b^3-2*(c*x^2+b*x+a)^(3/2)*x^4*b^4+4*(c*x^2+b*x+a)^(5/2)*x^2*a*b^2-6*(c*x^
2+b*x+a)^(1/2)*x^4*a*b^4-16*(c*x^2+b*x+a)^(5/2)*x*a^2*b+32*(c*x^2+b*x+a)^(5
/2)*a^3)/x^7/(c*x^2+b*x+a)^(3/2)/a^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(cx^4+bx^3+ax^2\right)^{\frac{3}{2}}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^8,x, algorithm="maxima")
[Out] integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x^8, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,x^4+b\,x^3+a\,x^2\right)^{3/2}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^8,x)
[Out] int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^8, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2\left(a+bx+cx^2\right)\right)^{\frac{3}{2}}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**3+a*x**2)**(3/2)/x**8,x)
[Out] Integral((x**2*(a + b*x + c*x**2))**(3/2)/x**8, x)
```

3.48 $\int \frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{x^9}\,dx$

Optimal. Leaf size=249

$$\frac{3b\left(b^2-4ac\right)^2\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{256a^{7/2}}+\frac{b\left(5b^2-28ac\right)\sqrt{ax^2+bx^3+cx^4}}{320a^2x^3}-\frac{\left(128a^2c^2-100ab^2c+15b^4\right)\sqrt{ax^2+}}{640a^3x^2}$$

```
[Out] -1/5*(c*x^4+b*x^3+a*x^2)^(3/2)/x^8+3/256*b*(-4*a*c+b^2)^2*arctanh(1/2*x*(b*
x+2*a)/a^(1/2)/(c*x^4+b*x^3+a*x^2)^(1/2))/a^(7/2)-1/80*(-8*a*c+b^2)*(c*x^4+
b*x^3+a*x^2)^(1/2)/a/x^4+1/320*b*(-28*a*c+5*b^2)*(c*x^4+b*x^3+a*x^2)^(1/2)/
a^2/x^3-1/640*(128*a^2*c^2-100*a*b^2*c+15*b^4)*(c*x^4+b*x^3+a*x^2)^(1/2)/a^
3/x^2-3/40*(4*c*x+b)*(c*x^4+b*x^3+a*x^2)^(1/2)/x^5
```

Rubi [A] time = 0.50, antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1920, 1941, 1951, 12, 1904, 206}

$$-\frac{\left(128a^2c^2-100ab^2c+15b^4\right)\sqrt{ax^2+bx^3+cx^4}}{640a^3x^2}+\frac{b\left(5b^2-28ac\right)\sqrt{ax^2+bx^3+cx^4}}{320a^2x^3}+\frac{3b\left(b^2-4ac\right)^2\tanh^{-1}\left(\frac{}{2\sqrt{a}\sqrt{}}\right)}{256a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^9,x]
```

```
[Out] -((b^2 - 8*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(80*a*x^4) + (b*(5*b^2 - 28*a*
c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(320*a^2*x^3) - ((15*b^4 - 100*a*b^2*c + 12
8*a^2*c^2)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(640*a^3*x^2) - (3*(b + 4*c*x)*Sqrt
[a*x^2 + b*x^3 + c*x^4])/(40*x^5) - (a*x^2 + b*x^3 + c*x^4)^(3/2)/(5*x^8) +
 (3*b*(b^2 - 4*a*c)^2*ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3
 + c*x^4])])/(256*a^(7/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1904

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, (x*(2*a + b*x^(n - 2)))/
Sqrt[a*x^2 + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n
 - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1920

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(x^(m + 1)*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(m + p*q
 + 1), x] - Dist[((n - q)*p)/(m + p*q + 1), Int[x^(m + n)*(b + 2*c*x^(n - q
))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &
& EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] &&
 IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q + 1, -(n - q) +
```

```
1] && NeQ[m + p*q + 1, 0]
```

Rule 1941

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[(x^(m + 1)*(A*(m + p*q + (n
 - q)*(2*p + 1) + 1) + B*(m + p*q + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n
 - q))^p)/((m + p*q + 1)*(m + p*q + (n - q)*(2*p + 1) + 1)), x] + Dist[((n
- q)*p)/((m + p*q + 1)*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(n + m)*Si
mp[2*a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(2*p + 1) + 1) + (b*B*(m +
p*q + 1) - 2*A*c*(m + p*q + (n - q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q +
b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[
r, n - q] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGt
Q[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q, -(n - q)] && NeQ[m
+ p*q + 1, 0] && NeQ[m + p*q + (n - q)*(2*p + 1) + 1, 0]
```

Rule 1951

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[(A*x^(m - q + 1)*(a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1))/(a*(m + p*q + 1)), x] + Dist[1/(a*(m + p*q +
 1)), Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p
+ 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b*
x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q
] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
&& RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n - q
)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]
```

Rubi steps

$$\int \frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{x^9}\,dx = -\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{5x^8}+\frac{3}{10}\int \frac{(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{x^6}\,dx$$

$$= -\frac{3(b+4cx)\sqrt{ax^2+bx^3+cx^4}}{40x^5}-\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{5x^8}+\frac{3}{160}\int \frac{2\left(b^2-8ac\right)-4}{x^3\sqrt{ax^2+bx^3+}}$$

$$= -\frac{\left(b^2-8ac\right)\sqrt{ax^2+bx^3+cx^4}}{80ax^4}-\frac{3(b+4cx)\sqrt{ax^2+bx^3+cx^4}}{40x^5}-\frac{\left(ax^2+bx^3+cx\right.}{5x^8}$$

$$= -\frac{\left(b^2-8ac\right)\sqrt{ax^2+bx^3+cx^4}}{80ax^4}+\frac{b\left(5b^2-28ac\right)\sqrt{ax^2+bx^3+cx^4}}{320a^2x^3}-\frac{3(b+4cx)}{}$$

$$= -\frac{\left(b^2-8ac\right)\sqrt{ax^2+bx^3+cx^4}}{80ax^4}+\frac{b\left(5b^2-28ac\right)\sqrt{ax^2+bx^3+cx^4}}{320a^2x^3}-\frac{\left(15b^4-10\right.}{}$$

$$= -\frac{\left(b^2-8ac\right)\sqrt{ax^2+bx^3+cx^4}}{80ax^4}+\frac{b\left(5b^2-28ac\right)\sqrt{ax^2+bx^3+cx^4}}{320a^2x^3}-\frac{\left(15b^4-10\right.}{}$$

$$= -\frac{\left(b^2-8ac\right)\sqrt{ax^2+bx^3+cx^4}}{80ax^4}+\frac{b\left(5b^2-28ac\right)\sqrt{ax^2+bx^3+cx^4}}{320a^2x^3}-\frac{\left(15b^4-10\right.}{}$$

$$= -\frac{\left(b^2-8ac\right)\sqrt{ax^2+bx^3+cx^4}}{80ax^4}+\frac{b\left(5b^2-28ac\right)\sqrt{ax^2+bx^3+cx^4}}{320a^2x^3}-\frac{\left(15b^4-10\right.}{}$$

Mathematica [A] time = 0.17, size = 177, normalized size = 0.71

$$\frac{\sqrt{x^2(a+x(b+cx))}\left(15bx^5\left(b^2-4ac\right)^2\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right)-2\sqrt{a}\sqrt{a+x(b+cx)}\left(128a^4+16a^3x(11b+16cx\right.\right.}{1280a^{7/2}x^6\sqrt{a+x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^9,x]
```

```
[Out] (Sqrt[x^2*(a + x*(b + c*x))]*(-2*Sqrt[a]*Sqrt[a + x*(b + c*x)]*(128*a^4 + 1
5*b^4*x^4 - 10*a*b^2*x^3*(b + 10*c*x) + 16*a^3*x*(11*b + 16*c*x) + 8*a^2*x^
2*(b^2 + 7*b*c*x + 16*c^2*x^2)) + 15*b*(b^2 - 4*a*c)^2*x^5*ArcTanh[(2*a + b
*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])]))/(1280*a^(7/2)*x^6*Sqrt[a + x*(b +
c*x)])
```

fricas [A] time = 0.99, size = 394, normalized size = 1.58

$$\left[\frac{15\left(b^5-8ab^3c+16a^2bc^2\right)\sqrt{a}x^6\log\left(-\frac{8abx^2+\left(b^2+4ac\right)x^3+8a^2x+4\sqrt{cx^4+bx^3+ax^2}(bx+2a)\sqrt{a}}{x^3}\right)-4\left(176a^4bx+128a^5+\left(1\right.\right.}{2560a^4x^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^9,x, algorithm="fricas")
```

```
[Out] [1/2560*(15*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*sqrt(a)*x^6*log(-(8*a*b*x^2 +
(b^2 + 4*a*c)*x^3 + 8*a^2*x + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqr
t(a))/x^3) - 4*(176*a^4*b*x + 128*a^5 + (15*a*b^4 - 100*a^2*b^2*c + 128*a^3
*c^2)*x^4 - 2*(5*a^2*b^3 - 28*a^3*b*c)*x^3 + 8*(a^3*b^2 + 32*a^4*c)*x^2)*sq
rt(c*x^4 + b*x^3 + a*x^2))/(a^4*x^6), -1/1280*(15*(b^5 - 8*a*b^3*c + 16*a^2
*b*c^2)*sqrt(-a)*x^6*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqr
t(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) + 2*(176*a^4*b*x + 128*a^5 + (15*a*b^4 -
 100*a^2*b^2*c + 128*a^3*c^2)*x^4 - 2*(5*a^2*b^3 - 28*a^3*b*c)*x^3 + 8*(a^3
*b^2 + 32*a^4*c)*x^2)*sqrt(c*x^4 + b*x^3 + a*x^2))/(a^4*x^6)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^9,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.01, size = 534, normalized size = 2.14

$$\frac{\left(cx^4+bx^3+ax^2\right)^{\frac{3}{2}}\left(240a^{\frac{7}{2}}bc^2x^5\ln\left(\frac{bx+2a+2\sqrt{cx^2+bx+a}\,\sqrt{a}}{x}\right)-120a^{\frac{5}{2}}b^3cx^5\ln\left(\frac{bx+2a+2\sqrt{cx^2+bx+a}\,\sqrt{a}}{x}\right)+15a^{\frac{3}{2}}b^5x^5\ln\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^3+a*x^2)^(3/2)/x^9,x)
```

```
[Out] 1/1280*(c*x^4+b*x^3+a*x^2)^(3/2)*(240*c^2*a^(7/2)*ln((b*x+2*a+2*(c*x^2+b*x+
a)^(1/2)*a^(1/2))/x)*x^5*b+120*c^2*(c*x^2+b*x+a)^(3/2)*x^6*a*b^2-120*c*a^(5
/2)*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)*x^5*b^3-80*c^2*(c*x^2+b*x
+a)^(3/2)*x^5*a^2*b+120*c^2*(c*x^2+b*x+a)^(1/2)*x^6*a^2*b^2-10*c*(c*x^2+b*x
```

```
+a)^(3/2)*x^6*b^4-240*c^2*(c*x^2+b*x+a)^(1/2)*x^5*a^3*b-120*c*(c*x^2+b*x+a)
^(5/2)*x^4*a*b^2+100*c*(c*x^2+b*x+a)^(3/2)*x^5*a*b^3-30*c*(c*x^2+b*x+a)^(1/
2)*x^6*a*b^4+15*a^(3/2)*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)*x^5*b
^5+80*c*(c*x^2+b*x+a)^(5/2)*x^3*a^2*b+180*c*(c*x^2+b*x+a)^(1/2)*x^5*a^2*b^3
+10*(c*x^2+b*x+a)^(5/2)*x^4*b^4-10*(c*x^2+b*x+a)^(3/2)*x^5*b^5+20*(c*x^2+b*
x+a)^(5/2)*x^3*a*b^3-30*(c*x^2+b*x+a)^(1/2)*x^5*a*b^5-80*(c*x^2+b*x+a)^(5/2
)*x^2*a^2*b^2+160*(c*x^2+b*x+a)^(5/2)*x*a^3*b-256*(c*x^2+b*x+a)^(5/2)*a^4)/
x^8/(c*x^2+b*x+a)^(3/2)/a^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^4 + bx^3 + ax^2\right)^{\frac{3}{2}}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^9,x, algorithm="maxima")
[Out] integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x^9, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^4 + b\,x^3 + a\,x^2\right)^{3/2}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^9,x)
[Out] int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^9, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2\left(a + bx + cx^2\right)\right)^{\frac{3}{2}}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**3+a*x**2)**(3/2)/x**9,x)
[Out] Integral((x**2*(a + b*x + c*x**2))**(3/2)/x**9, x)
```

3.49 $\int \frac{x^3}{\sqrt{ax^2+bx^3+cx^4}} \, dx$

Optimal. Leaf size=143

$$\frac{x\left(3b^2-4ac\right)\sqrt{a+bx+cx^2}\,\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\,\sqrt{a+bx+cx^2}}\right)}{8c^{5/2}\sqrt{ax^2+bx^3+cx^4}}-\frac{3b\sqrt{ax^2+bx^3+cx^4}}{4c^2x}+\frac{\sqrt{ax^2+bx^3+cx^4}}{2c}$$

```
[Out] 1/8*(-4*a*c+3*b^2)*x*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))*(c*
x^2+b*x+a)^(1/2)/c^(5/2)/(c*x^4+b*x^3+a*x^2)^(1/2)+1/2*(c*x^4+b*x^3+a*x^2)^
(1/2)/c-3/4*b*(c*x^4+b*x^3+a*x^2)^(1/2)/c^2/x
```

Rubi [A] time = 0.17, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1928, 1949, 12, 1914, 621, 206}

$$\frac{x\left(3b^2-4ac\right)\sqrt{a+bx+cx^2}\,\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\,\sqrt{a+bx+cx^2}}\right)}{8c^{5/2}\sqrt{ax^2+bx^3+cx^4}}-\frac{3b\sqrt{ax^2+bx^3+cx^4}}{4c^2x}+\frac{\sqrt{ax^2+bx^3+cx^4}}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a*x^2 + b*x^3 + c*x^4],x]
```

```
[Out] Sqrt[a*x^2 + b*x^3 + c*x^4]/(2*c) - (3*b*Sqrt[a*x^2 + b*x^3 + c*x^4])/(4*c^
2*x) + ((3*b^2 - 4*a*c)*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt
[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(5/2)*Sqrt[a*x^2 + b*x^3 + c*x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1914

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))])/Sqrt[
a*x^q + b*x^n + c*x^(2*n - q)], Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

Rule 1928

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(x^(m - 2*n + q + 1)*(a*x^q + b*x^n + c*x^(2*n - q))^(
p + 1))/(c*(m + p*q + 2*(n - q)*p + 1)), x] - Dist[1/(c*(m + p*q + 2*(n - q
```

```
)*p + 1)), Int[x^(m - 2*(n - q))*(a*(m + p*q - 2*(n - q) + 1) + b*(m + p*q
+ (n - q)*(p - 1) + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x]
 /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p]
&& NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[p, -1] && LtQ[p, 0] && Rational
Q[m, q] && GtQ[m + p*q + 1, 2*(n - q)]
```

Rule 1949

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[(B*x^(m - n + 1)*(a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1))/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), x] -
Dist[1/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(m - n + q)*Simp[a*B*(m
 + p*q - n + q + 1) + (b*B*(m + p*q + (n - q)*p + 1) - A*c*(m + p*q + (n -
q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x]
/; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Integ
erQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[p, -1] && LtQ[p, 0] && R
ationalQ[m, q] && GeQ[m + p*q, n - q - 1] && NeQ[m + p*q + (n - q)*(2*p + 1
) + 1, 0]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{ax^2+bx^3+cx^4}}\,dx = \frac{\sqrt{ax^2+bx^3+cx^4}}{2c} - \frac{\int \frac{x\left(a+\frac{3bx}{2}\right)}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{2c}$$

$$= \frac{\sqrt{ax^2+bx^3+cx^4}}{2c} - \frac{3b\sqrt{ax^2+bx^3+cx^4}}{4c^2x} + \frac{\int \frac{\left(3b^2-4ac\right)x}{4\sqrt{ax^2+bx^3+cx^4}}\,dx}{2c^2}$$

$$= \frac{\sqrt{ax^2+bx^3+cx^4}}{2c} - \frac{3b\sqrt{ax^2+bx^3+cx^4}}{4c^2x} + \frac{\left(3b^2-4ac\right)\int \frac{x}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{8c^2}$$

$$= \frac{\sqrt{ax^2+bx^3+cx^4}}{2c} - \frac{3b\sqrt{ax^2+bx^3+cx^4}}{4c^2x} + \frac{\left(\left(3b^2-4ac\right)x\sqrt{a+bx+cx^2}\right)\int \frac{1}{\sqrt{a+b\ldots}}}{8c^2\sqrt{ax^2+bx^3+cx^4}}$$

$$= \frac{\sqrt{ax^2+bx^3+cx^4}}{2c} - \frac{3b\sqrt{ax^2+bx^3+cx^4}}{4c^2x} + \frac{\left(\left(3b^2-4ac\right)x\sqrt{a+bx+cx^2}\right)\text{Subst}\left(\ldots\right)}{4c^2\sqrt{ax^2+bx^3+\ldots}}$$

$$= \frac{\sqrt{ax^2+bx^3+cx^4}}{2c} - \frac{3b\sqrt{ax^2+bx^3+cx^4}}{4c^2x} + \frac{\left(3b^2-4ac\right)x\sqrt{a+bx+cx^2}\,\tanh^{-1}\left(\ldots\right)}{8c^{5/2}\sqrt{ax^2+bx^3+cx^4}}$$

Mathematica [A] time = 0.09, size = 105, normalized size = 0.73

$$\frac{x\left(\left(3b^2-4ac\right)\sqrt{a+x(b+cx)}\,\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)+2\sqrt{c}(2cx-3b)(a+x(b+cx))\right)}{8c^{5/2}\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[a*x^2 + b*x^3 + c*x^4],x]
```

```
[Out] (x*(2*Sqrt[c]*(-3*b + 2*c*x)*(a + x*(b + c*x)) + (3*b^2 - 4*a*c)*Sqrt[a + x
*(b + c*x)]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))/(8*c^(
5/2)*Sqrt[x^2*(a + x*(b + c*x))])
```

fricas [A] time = 0.50, size = 226, normalized size = 1.58

$$\left[-\frac{\left(3\,b^2-4\,ac\right)\sqrt{c}\,x\log\left(-\frac{8\,c^2x^3+8\,bcx^2-4\,\sqrt{cx^4+bx^3+ax^2}\,(2\,cx+b)\sqrt{c}+\left(b^2+4\,ac\right)x}{x}\right)-4\,\sqrt{cx^4+bx^3+ax^2}\left(2\,c^2x-3\,bc\right)}{16\,c^3x},-\frac{\left(3\,b\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*((3*b^2 - 4*a*c)*sqrt(c)*x*log(-(8*c^2*x^3 + 8*b*c*x^2 - 4*sqrt(c*x^
4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) - 4*sqrt(c*x^4
 + b*x^3 + a*x^2)*(2*c^2*x - 3*b*c))/(c^3*x), -1/8*((3*b^2 - 4*a*c)*sqrt(-c
)*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 +
b*c*x^2 + a*c*x)) - 2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c^2*x - 3*b*c))/(c^3*x
)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^3}{\sqrt{cx^4+bx^3+ax^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/sqrt(c*x^4 + b*x^3 + a*x^2), x)
```

maple [A] time = 0.01, size = 144, normalized size = 1.01

$$\frac{\sqrt{c\,x^2+bx+a}\,\left(-4a\,c^2\ln\left(\frac{2cx+b+2\sqrt{c\,x^2+bx+a}\,\sqrt{c}}{2\sqrt{c}}\right)+3b^2c\ln\left(\frac{2cx+b+2\sqrt{c\,x^2+bx+a}\,\sqrt{c}}{2\sqrt{c}}\right)+4\sqrt{c\,x^2+bx+a}\,c^{\frac{5}{2}}x-6\sqrt{c\,x^2}\right.}{8\sqrt{c\,x^4+b\,x^3+a\,x^2}\,c^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^4+b*x^3+a*x^2)^(1/2),x)
```

```
[Out] 1/8*x*(c*x^2+b*x+a)^(1/2)*(4*(c*x^2+b*x+a)^(1/2)*c^(5/2)*x-6*(c*x^2+b*x+a)^
(1/2)*b*c^(3/2)-4*a*c^2*ln(1/2*(2*c*x+b+2*(c*x^2+b*x+a)^(1/2)*c^(1/2))/c^(1
/2))+3*b^2*c*ln(1/2*(2*c*x+b+2*(c*x^2+b*x+a)^(1/2)*c^(1/2))/c^(1/2)))/(c*x^
4+b*x^3+a*x^2)^(1/2)/c^(7/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^3}{\sqrt{cx^4+bx^3+ax^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/sqrt(c*x^4 + b*x^3 + a*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{x^3}{\sqrt{c\,x^4+b\,x^3+a\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*x^2 + b*x^3 + c*x^4)^(1/2),x)
[Out] int(x^3/(a*x^2 + b*x^3 + c*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{x^2\left(a+bx+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**4+b*x**3+a*x**2)**(1/2),x)
[Out] Integral(x**3/sqrt(x**2*(a + b*x + c*x**2)), x)
```

3.50 $\int \frac{x^2}{\sqrt{ax^2+bx^3+cx^4}} \, dx$

Optimal. Leaf size=103

$$\frac{\sqrt{ax^2+bx^3+cx^4}}{cx}-\frac{bx\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}\sqrt{ax^2+bx^3+cx^4}}$$

```
[Out] -1/2*b*x*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))*(c*x^2+b*x+a)^(
1/2)/c^(3/2)/(c*x^4+b*x^3+a*x^2)^(1/2)+(c*x^4+b*x^3+a*x^2)^(1/2)/c/x
```

Rubi [A] time = 0.08, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1917, 1914, 621, 206}

$$\frac{\sqrt{ax^2+bx^3+cx^4}}{cx}-\frac{bx\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}\sqrt{ax^2+bx^3+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a*x^2 + b*x^3 + c*x^4],x]
```

```
[Out] Sqrt[a*x^2 + b*x^3 + c*x^4]/(c*x) - (b*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b +
 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*c^(3/2)*Sqrt[a*x^2 + b*x^3 +
 c*x^4])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1914

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))])/Sqrt[
a*x^q + b*x^n + c*x^(2*n - q)], Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

Rule 1917

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(x^(m - n)*(a*x^(n - 1) + b*x^n + c*x^(n + 1))^(p + 1)
)/(2*c*(p + 1)), x] - Dist[b/(2*c), Int[x^(m - 1)*(a*x^(n - 1) + b*x^n + c*
x^(n + 1))^p, x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n -
q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && RationalQ[m, p,
 q] && EqQ[m + p*(n - 1) - 1, 0]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{ax^2+bx^3+cx^4}}\,dx = \frac{\sqrt{ax^2+bx^3+cx^4}}{cx} - \frac{b\int \frac{x}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{2c}$$
$$= \frac{\sqrt{ax^2+bx^3+cx^4}}{cx} - \frac{\left(bx\sqrt{a+bx+cx^2}\right)\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{2c\sqrt{ax^2+bx^3+cx^4}}$$
$$= \frac{\sqrt{ax^2+bx^3+cx^4}}{cx} - \frac{\left(bx\sqrt{a+bx+cx^2}\right)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{c\sqrt{ax^2+bx^3+cx^4}}$$
$$= \frac{\sqrt{ax^2+bx^3+cx^4}}{cx} - \frac{bx\sqrt{a+bx+cx^2}\,\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}\sqrt{ax^2+bx^3+cx^4}}$$

Mathematica [A] time = 0.05, size = 89, normalized size = 0.86

$$\frac{x\left(2\sqrt{c}\,(a+x(b+cx)) - b\sqrt{a+x(b+cx)}\,\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\right)}{2c^{3/2}\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a*x^2 + b*x^3 + c*x^4],x]
```

```
[Out] (x*(2*Sqrt[c]*(a + x*(b + c*x)) - b*Sqrt[a + x*(b + c*x)]*ArcTanh[(b + 2*c*
x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))/(2*c^(3/2)*Sqrt[x^2*(a + x*(b + c*x
))])
```

fricas [A] time = 0.78, size = 188, normalized size = 1.83

$$\left[\frac{b\sqrt{c}\,x\log\left(-\frac{8\,c^2x^3+8\,bcx^2-4\,\sqrt{cx^4+bx^3+ax^2}\,(2\,cx+b)\sqrt{c}+\left(b^2+4\,ac\right)x}{x}\right)+4\,\sqrt{cx^4+bx^3+ax^2}\,c}{4\,c^2x}, \frac{b\sqrt{-c}\,x\arctan\left(\frac{\sqrt{cx^4+bx^3+ax}}{2\left(c^2x^3+l\right.}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(b*sqrt(c)*x*log(-(8*c^2*x^3 + 8*b*c*x^2 - 4*sqrt(c*x^4 + b*x^3 + a*x^
2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) + 4*sqrt(c*x^4 + b*x^3 + a*x^2
)*c)/(c^2*x), 1/2*(b*sqrt(-c)*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c
*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) + 2*sqrt(c*x^4 + b*x^3 + a*x^
2)*c)/(c^2*x)]
```

giac [A] time = 0.91, size = 108, normalized size = 1.05

$$\frac{b\arctan\left(\frac{\sqrt{c+\frac{b}{x}+\frac{a}{x^2}}-\frac{\sqrt{a}}{x}}{\sqrt{-c}}\right)}{\sqrt{-c}\,c} + \frac{b\left(\sqrt{c+\frac{b}{x}+\frac{a}{x^2}}-\frac{\sqrt{a}}{x}\right)-2\,\sqrt{a}\,c}{\left(\left(\sqrt{c+\frac{b}{x}+\frac{a}{x^2}}-\frac{\sqrt{a}}{x}\right)^2-c\right)c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] b*arctan((sqrt(c + b/x + a/x^2) - sqrt(a)/x)/sqrt(-c))/(sqrt(-c)*c) + (b*(s
qrt(c + b/x + a/x^2) - sqrt(a)/x) - 2*sqrt(a)*c)/((((sqrt(c + b/x + a/x^2) -
 sqrt(a)/x)^2 - c)*c)
```

maple [A] time = 0.01, size = 88, normalized size = 0.85

$$\frac{\sqrt{c\,x^2+bx+a}\,\left(-bc\ln\left(\frac{2cx+b+2\sqrt{c\,x^2+bx+a}\,\sqrt{c}}{2\sqrt{c}}\right)+2\sqrt{c\,x^2+bx+a}\,c^{\frac{3}{2}}\right)x}{2\sqrt{c\,x^4+b\,x^3+a\,x^2}\,c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^4+b*x^3+a*x^2)^(1/2),x)
```

```
[Out] 1/2*x*(c*x^2+b*x+a)^(1/2)*(2*(c*x^2+b*x+a)^(1/2)*c^(3/2)-b*ln(1/2*(2*c*x+b+
2*(c*x^2+b*x+a)^(1/2)*c^(1/2))/c^(1/2))*c)/(c*x^4+b*x^3+a*x^2)^(1/2)/c^(5/2
)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{cx^4+bx^3+ax^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/sqrt(c*x^4 + b*x^3 + a*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\sqrt{c\,x^4+b\,x^3+a\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x^2 + b*x^3 + c*x^4)^(1/2),x)
```

```
[Out] int(x^2/(a*x^2 + b*x^3 + c*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{x^2\left(a+bx+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**4+b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(x**2*(a + b*x + c*x**2)), x)
```

3.51 $\int \frac{x}{\sqrt{ax^2+bx^3+cx^4}}\,dx$

Optimal. Leaf size=71

$$\frac{x\sqrt{a+bx+cx^2}\,\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\,\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}\,\sqrt{ax^2+bx^3+cx^4}}$$

```
[Out] x*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))*(c*x^2+b*x+a)^(1/2)/c^
(1/2)/(c*x^4+b*x^3+a*x^2)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1914, 621, 206}

$$\frac{x\sqrt{a+bx+cx^2}\,\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\,\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}\,\sqrt{ax^2+bx^3+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a*x^2 + b*x^3 + c*x^4],x]
```

```
[Out] (x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^
2])])/(Sqrt[c]*Sqrt[a*x^2 + b*x^3 + c*x^4])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1914

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))])/Sqrt[
a*x^q + b*x^n + c*x^(2*n - q)], Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

Rubi steps

$$\int \frac{x}{\sqrt{ax^2+bx^3+cx^4}}\,dx = \frac{\left(x\sqrt{a+bx+cx^2}\right)\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{\sqrt{ax^2+bx^3+cx^4}}$$
$$= \frac{\left(2x\sqrt{a+bx+cx^2}\right)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{\sqrt{ax^2+bx^3+cx^4}}$$
$$= \frac{x\sqrt{a+bx+cx^2}\,\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}\sqrt{ax^2+bx^3+cx^4}}$$

Mathematica [A] time = 0.04, size = 66, normalized size = 0.93

$$\frac{x\sqrt{a+bx+cx^2}\,\log\left(2\sqrt{c}\sqrt{a+bx+cx^2}+b+2cx\right)}{\sqrt{c}\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a*x^2 + b*x^3 + c*x^4],x]
```

```
[Out] (x*Sqrt[a + b*x + c*x^2]*Log[b + 2*c*x + 2*Sqrt[c]*Sqrt[a + b*x + c*x^2]])/
(Sqrt[c]*Sqrt[x^2*(a + x*(b + c*x))])
```

fricas [A] time = 0.66, size = 129, normalized size = 1.82

$$\left[\frac{\log\left(-\frac{8c^2x^3+8bcx^2+4\sqrt{cx^4+bx^3+ax^2}(2cx+b)\sqrt{c}+(b^2+4ac)x}{x}\right)}{2\sqrt{c}}, -\frac{\sqrt{-c}\arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}(2cx+b)\sqrt{-c}}{2(c^2x^3+bcx^2+acx)}\right)}{c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b
)*sqrt(c) + (b^2 + 4*a*c)*x)/x)/sqrt(c), -sqrt(-c)*arctan(1/2*sqrt(c*x^4 +
b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x))/c]
```

giac [A] time = 0.91, size = 37, normalized size = 0.52

$$-\frac{2\arctan\left(\frac{\sqrt{c+\frac{b}{x}+\frac{a}{x^2}}-\frac{\sqrt{a}}{x}}{\sqrt{-c}}\right)}{\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -2*arctan((sqrt(c + b/x + a/x^2) - sqrt(a)/x)/sqrt(-c))/sqrt(-c)
```

maple [A] time = 0.01, size = 65, normalized size = 0.92

$$\frac{\sqrt{c\,x^2+bx+a}\;x\ln\left(\frac{2cx+b+2\sqrt{c\,x^2+bx+a}\,\sqrt{c}}{2\sqrt{c}}\right)}{\sqrt{c\,x^4+b\,x^3+a\,x^2}\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^4+b*x^3+a*x^2)^(1/2),x)
```

```
[Out] 1/(c*x^4+b*x^3+a*x^2)^(1/2)*x*(c*x^2+b*x+a)^(1/2)*ln(1/2*(2*c*x+b+2*(c*x^2+
b*x+a)^(1/2)*c^(1/2))/c^(1/2))/c^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{cx^4 + bx^3 + ax^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/sqrt(c*x^4 + b*x^3 + a*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\sqrt{c\,x^4 + b\,x^3 + a\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x^2 + b*x^3 + c*x^4)^(1/2),x)
```

```
[Out] int(x/(a*x^2 + b*x^3 + c*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{x^2\left(a + bx + cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**4+b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(x/sqrt(x**2*(a + b*x + c*x**2)), x)
```

3.52 $\int \frac{1}{\sqrt{ax^2+bx^3+cx^4}}\,dx$

Optimal. Leaf size=45

$$-\frac{\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{\sqrt{a}}$$

```
[Out] -arctanh(1/2*x*(b*x+2*a)/a^(1/2)/(c*x^4+b*x^3+a*x^2)^(1/2))/a^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1904, 206}

$$-\frac{\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*x^2 + b*x^3 + c*x^4],x]
```

```
[Out] -(ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])]/Sqrt[a])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1904

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, (x*(2*a + b*x^(n - 2)))/
Sqrt[a*x^2 + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n
 - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{ax^2+bx^3+cx^4}}\,dx &= -\left(2\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{x(2a+bx)}{\sqrt{ax^2+bx^3+cx^4}}\right)\right)\\ &= -\frac{\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{\sqrt{a}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 70, normalized size = 1.56

$$-\frac{x\sqrt{a+bx+cx^2}\,\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*x^2 + b*x^3 + c*x^4],x]
```

```
[Out] -((x*Sqrt[a + b*x + c*x^2]*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*
x^2])])/(Sqrt[a]*Sqrt[x^2*(a + x*(b + c*x))]))
```

fricas [A] time = 0.79, size = 130, normalized size = 2.89

$$\left[\frac{\log\left(-\frac{8\,abx^2+\left(b^2+4\,ac\right)x^3+8\,a^2x-4\,\sqrt{cx^4+bx^3+ax^2}\,(bx+2\,a)\sqrt{a}}{x^3}\right)}{2\,\sqrt{a}},\ \frac{\sqrt{-a}\,\arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}\,(bx+2\,a)\sqrt{-a}}{2\left(acx^3+abx^2+a^2x\right)}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3 +
 a*x^2)*(b*x + 2*a)*sqrt(a))/x^3)/sqrt(a), sqrt(-a)*arctan(1/2*sqrt(c*x^4 +
 b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x))/a]
```

giac [A] time = 0.92, size = 59, normalized size = 1.31

$$-\frac{2\,\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)\mathrm{sgn}(x)}{\sqrt{-a}}+\frac{2\,\arctan\left(-\frac{\sqrt{c}\,x-\sqrt{cx^2+bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,\mathrm{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -2*arctan(sqrt(a)/sqrt(-a))*sgn(x)/sqrt(-a) + 2*arctan(-(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))/sqrt(-a))/(sqrt(-a)*sgn(x))
```

maple [A] time = 0.00, size = 66, normalized size = 1.47

$$-\frac{\sqrt{c\,x^2+bx+a}\;x\ln\left(\frac{bx+2a+2\sqrt{c\,x^2+bx+a}\;\sqrt{a}}{x}\right)}{\sqrt{c\,x^4+b\,x^3+a\,x^2}\;\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+b*x^3+a*x^2)^(1/2),x)
```

```
[Out] -1/(c*x^4+b*x^3+a*x^2)^(1/2)*x*(c*x^2+b*x+a)^(1/2)/a^(1/2)*ln((b*x+2*a+2*(c
*x^2+b*x+a)^(1/2)*a^(1/2))/x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^4+bx^3+ax^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(c*x^4 + b*x^3 + a*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{c\,x^4+b\,x^3+a\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^2 + b*x^3 + c*x^4)^(1/2),x)
```

```
[Out] int(1/(a*x^2 + b*x^3 + c*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax^2+bx^3+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*x**2 + b*x**3 + c*x**4), x)
```

3.53 $\int \frac{1}{x\sqrt{ax^2+bx^3+cx^4}}\,dx$

Optimal. Leaf size=77

$$\frac{b\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{2a^{3/2}}-\frac{\sqrt{ax^2+bx^3+cx^4}}{ax^2}$$

```
[Out] 1/2*b*arctanh(1/2*x*(b*x+2*a)/a^(1/2)/(c*x^4+b*x^3+a*x^2)^(1/2))/a^(3/2)-(c
*x^4+b*x^3+a*x^2)^(1/2)/a/x^2
```

Rubi [A] time = 0.05, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1927, 1904, 206}

$$\frac{b\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{2a^{3/2}}-\frac{\sqrt{ax^2+bx^3+cx^4}}{ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a*x^2 + b*x^3 + c*x^4]),x]
```

```
[Out] -(Sqrt[a*x^2 + b*x^3 + c*x^4]/(a*x^2)) + (b*ArcTanh[(x*(2*a + b*x))/(2*Sqrt
[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])])/(2*a^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1904

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, (x*(2*a + b*x^(n - 2)))/
Sqrt[a*x^2 + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n
 - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1927

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> -Simp[(x^(m - q + 1)*(a*x^q + b*x^n + c*x^(2*n - q))^(p + 1
))/(2*a*(n - q)*(p + 1)), x] - Dist[b/(2*a), Int[x^(m + n - q)*(a*x^q + b*x
^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[p,
 -1] && LtQ[p, 0] && RationalQ[m, q] && EqQ[m + p*q + 1, -2*(n - q)*(p + 1)
]
```

Rubi steps

$$\int \frac{1}{x\sqrt{ax^2+bx^3+cx^4}}\,dx = -\frac{\sqrt{ax^2+bx^3+cx^4}}{ax^2} - \frac{b\int \frac{1}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{2a}$$
$$= -\frac{\sqrt{ax^2+bx^3+cx^4}}{ax^2} + \frac{b\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{x(2a+bx)}{\sqrt{ax^2+bx^3+cx^4}}\right)}{a}$$
$$= -\frac{\sqrt{ax^2+bx^3+cx^4}}{ax^2} + \frac{b\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{2a^{3/2}}$$

Mathematica [A] time = 0.05, size = 89, normalized size = 1.16

$$\frac{bx\sqrt{a+x(b+cx)}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right) - 2\sqrt{a}\,(a+x(b+cx))}{2a^{3/2}\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a*x^2 + b*x^3 + c*x^4]),x]
```

```
[Out] (-2*Sqrt[a]*(a + x*(b + c*x)) + b*x*Sqrt[a + x*(b + c*x)]*ArcTanh[(2*a + b*
x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])])/(2*a^(3/2)*Sqrt[x^2*(a + x*(b + c*x)
)])
```

fricas [A] time = 0.68, size = 194, normalized size = 2.52

$$\left[\frac{\sqrt{a}\,bx^2\log\left(-\frac{8\,abx^2+\left(b^2+4\,ac\right)x^3+8\,a^2x+4\,\sqrt{cx^4+bx^3+ax^2}\,(bx+2\,a)\sqrt{a}}{x^3}\right) - 4\,\sqrt{cx^4+bx^3+ax^2}\,a}{4\,a^2x^2}, -\frac{\sqrt{-a}\,bx^2\arctan\left(\frac{\sqrt{cx^4+bx^3+}}{2\left(acx^3\right.}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(a)*b*x^2*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x + 4*sqrt(
c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) - 4*sqrt(c*x^4 + b*x^3 + a
*x^2)*a)/(a^2*x^2), -1/2*(sqrt(-a)*b*x^2*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*
x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) + 2*sqrt(c*x^4 + b*x
^3 + a*x^2)*a)/(a^2*x^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,0,0,0]%%%}+%%%{-2,[0,1,0,1]%%%}+%%%{-2,
[0,0,1,2]%%%},0,%%%{1,[0,2,0,2]%%%}+%%%{2,[0,1,1,3]%%%}+%%%{1,[0,0,2,4]%%%}
] at parameters values [54.7579903365,-49,-33,-70]-1/a*sqrt(a*(1/x)^2+b/x+c
)-2*b/4/a/sqrt(a)*ln(abs(2*sqrt(a)*(sqrt(a*(1/x)^2+b/x+c)-sqrt(a)/x)-b))
```

maple [A] time = 0.01, size = 88, normalized size = 1.14

$$-\frac{\sqrt{c\,x^2+bx+a}\,\left(-abx\ln\left(\frac{bx+2a+2\sqrt{c\,x^2+bx+a}\,\sqrt{a}}{x}\right)+2\sqrt{c\,x^2+bx+a}\,a^{\frac{3}{2}}\right)}{2\sqrt{c\,x^4+b\,x^3+a\,x^2}\,a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^4+b*x^3+a*x^2)^(1/2),x)
```

```
[Out] -1/2*(c*x^2+b*x+a)^(1/2)*(2*(c*x^2+b*x+a)^(1/2)*a^(3/2)-b*ln((b*x+2*a+2*(c*
x^2+b*x+a)^(1/2)*a^(1/2))/x)*a*x)/(c*x^4+b*x^3+a*x^2)^(1/2)/a^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^4 + bx^3 + ax^2}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^3 + a*x^2)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\,\sqrt{c\,x^4 + b\,x^3 + a\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a*x^2 + b*x^3 + c*x^4)^(1/2)),x)
```

```
[Out] int(1/(x*(a*x^2 + b*x^3 + c*x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{x^2\left(a + bx + cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**4+b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(x**2*(a + b*x + c*x**2))), x)
```

3.54 $\int \frac{1}{x^2\sqrt{ax^2+bx^3+cx^4}}\,dx$

Optimal. Leaf size=119

$$-\frac{\left(3b^2-4ac\right)\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{8a^{5/2}}+\frac{3b\sqrt{ax^2+bx^3+cx^4}}{4a^2x^2}-\frac{\sqrt{ax^2+bx^3+cx^4}}{2ax^3}$$

```
[Out] -1/8*(-4*a*c+3*b^2)*arctanh(1/2*x*(b*x+2*a)/a^(1/2)/(c*x^4+b*x^3+a*x^2)^(1/
2))/a^(5/2)-1/2*(c*x^4+b*x^3+a*x^2)^(1/2)/a/x^3+3/4*b*(c*x^4+b*x^3+a*x^2)^(
1/2)/a^2/x^2
```

Rubi [A] time = 0.15, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1929, 1951, 12, 1904, 206}

$$-\frac{\left(3b^2-4ac\right)\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{8a^{5/2}}+\frac{3b\sqrt{ax^2+bx^3+cx^4}}{4a^2x^2}-\frac{\sqrt{ax^2+bx^3+cx^4}}{2ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[a*x^2 + b*x^3 + c*x^4]),x]
```

```
[Out] -Sqrt[a*x^2 + b*x^3 + c*x^4]/(2*a*x^3) + (3*b*Sqrt[a*x^2 + b*x^3 + c*x^4])/
(4*a^2*x^2) - ((3*b^2 - 4*a*c)*ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^
2 + b*x^3 + c*x^4])])/(8*a^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1904

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, (x*(2*a + b*x^(n - 2)))/
Sqrt[a*x^2 + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n
 - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1929

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(x^(m - q + 1)*(a*x^q + b*x^n + c*x^(2*n - q))^(p + 1)
)/(a*(m + p*q + 1)), x] - Dist[1/(a*(m + p*q + 1)), Int[x^(m + n - q)*(b*(m
 + p*q + (n - q)*(p + 1) + 1) + c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n -
q))*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c}, x] && EqQ
[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ
[n, 0] && GeQ[p, -1] && LtQ[p, 0] && RationalQ[m, q] && LtQ[m + p*q + 1, 0]
```

Rule 1951

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[(A*x^(m - q + 1)*(a*x^q + b
```

```
*x^n + c*x^(2*n - q))^(p + 1))/(a*(m + p*q + 1)), x] + Dist[1/(a*(m + p*q +
 1)), Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p
+ 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b*
x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q
] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
&& RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n - q
)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{ax^2+bx^3+cx^4}}\,dx = -\frac{\sqrt{ax^2+bx^3+cx^4}}{2ax^3}+\frac{\int \frac{-\frac{3b}{2}-cx}{x\sqrt{ax^2+bx^3+cx^4}}\,dx}{2a}$$

$$= -\frac{\sqrt{ax^2+bx^3+cx^4}}{2ax^3}+\frac{3b\sqrt{ax^2+bx^3+cx^4}}{4a^2x^2}-\frac{\int \frac{-\frac{3b^2}{4}+ac}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{2a^2}$$

$$= -\frac{\sqrt{ax^2+bx^3+cx^4}}{2ax^3}+\frac{3b\sqrt{ax^2+bx^3+cx^4}}{4a^2x^2}+\frac{\left(3b^2-4ac\right)\int \frac{1}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{8a^2}$$

$$= -\frac{\sqrt{ax^2+bx^3+cx^4}}{2ax^3}+\frac{3b\sqrt{ax^2+bx^3+cx^4}}{4a^2x^2}-\frac{\left(3b^2-4ac\right)\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx,x,\cdots\right.}{4a^2}$$

$$= -\frac{\sqrt{ax^2+bx^3+cx^4}}{2ax^3}+\frac{3b\sqrt{ax^2+bx^3+cx^4}}{4a^2x^2}-\frac{\left(3b^2-4ac\right)\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+\cdots}}\right.}{8a^{5/2}}$$

Mathematica [A] time = 0.08, size = 112, normalized size = 0.94

$$\frac{-\left(x^2\left(3b^2-4ac\right)\sqrt{a+x(b+cx)}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right)\right)-2\sqrt{a}\,(2a-3bx)(a+x(b+cx))}{8a^{5/2}x\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[a*x^2 + b*x^3 + c*x^4]),x]
```

```
[Out] (-2*Sqrt[a]*(2*a - 3*b*x)*(a + x*(b + c*x)) - (3*b^2 - 4*a*c)*x^2*Sqrt[a +
x*(b + c*x)]*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])])/(8*a^(
5/2)*x*Sqrt[x^2*(a + x*(b + c*x))])
```

fricas [A] time = 0.76, size = 232, normalized size = 1.95

$$\left[-\frac{\left(3\,b^2-4\,ac\right)\sqrt{a}\,x^3\log\left(-\frac{8\,abx^2+\left(b^2+4\,ac\right)x^3+8\,a^2x+4\,\sqrt{cx^4+bx^3+ax^2}\,(bx+2\,a)\sqrt{a}}{x^3}\right)-4\,\sqrt{cx^4+bx^3+ax^2}\left(3\,abx-2\,a^2\right)}{16\,a^3x^3},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*((3*b^2 - 4*a*c)*sqrt(a)*x^3*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8
*a^2*x + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) - 4*sqrt(c
*x^4 + b*x^3 + a*x^2)*(3*a*b*x - 2*a^2))/(a^3*x^3), 1/8*((3*b^2 - 4*a*c)*sq
rt(-a)*x^3*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c
*x^3 + a*b*x^2 + a^2*x)) + 2*sqrt(c*x^4 + b*x^3 + a*x^2)*(3*a*b*x - 2*a^2))
/(a^3*x^3)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,0,0,0]%%%}+%%%{-2,[0,1,0,1]%%%}+%%%{-2,
[0,0,1,2]%%%},0,%%%{1,[0,2,0,2]%%%}+%%%{2,[0,1,1,3]%%%}+%%%{1,[0,0,2,4]%%%}
] at parameters values [54.7579903365,-49,-33,-70]2*(-4*a/16/a^2/x+6*b/16/a
^2)*sqrt(a*(1/x)^2+b/x+c)+2*(-4*a*c+3*b^2)/16/a^2/sqrt(a)*ln(abs(2*sqrt(a)*
(sqrt(a*(1/x)^2+b/x+c)-sqrt(a)/x)-b))
```

maple [A] time = 0.01, size = 152, normalized size = 1.28

$$-\frac{\sqrt{c\,x^{2}+bx+a}\,\left(-4a^{2}c\,x^{2}\ln\left(\frac{bx+2a+2\sqrt{c\,x^{2}+bx+a}\,\sqrt{a}}{x}\right)+3a\,b^{2}x^{2}\ln\left(\frac{bx+2a+2\sqrt{c\,x^{2}+bx+a}\,\sqrt{a}}{x}\right)-6\sqrt{c\,x^{2}+bx+a}\,a^{\frac{3}{2}}bx\right.}{8\sqrt{c\,x^{4}+b\,x^{3}+a\,x^{2}}\,a^{\frac{7}{2}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^4+b*x^3+a*x^2)^(1/2),x)
```

```
[Out] -1/8*(c*x^2+b*x+a)^(1/2)*(4*(c*x^2+b*x+a)^(1/2)*a^(5/2)-6*(c*x^2+b*x+a)^(1/
2)*a^(3/2)*x*b-4*c*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)*x^2*a^2+3*
ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)*x^2*a*b^2)/x/(c*x^4+b*x^3+a*x
^2)^(1/2)/a^(7/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^{4}+bx^{3}+ax^{2}}\,x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^3 + a*x^2)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{2}\,\sqrt{c\,x^{4}+b\,x^{3}+a\,x^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a*x^2 + b*x^3 + c*x^4)^(1/2)),x)
```

```
[Out] int(1/(x^2*(a*x^2 + b*x^3 + c*x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{2}\sqrt{x^{2}\left(a+bx+cx^{2}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**4+b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(x**2*(a + b*x + c*x**2))), x)
```

3.55 $\int \frac{x^7}{\left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=262

$$\frac{3x\left(5b^2-4ac\right)\sqrt{a+bx+cx^2}\,\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\,\sqrt{a+bx+cx^2}}\right)}{8c^{7/2}\sqrt{ax^2+bx^3+cx^4}}-\frac{b\left(15b^2-52ac\right)\sqrt{ax^2+bx^3+cx^4}}{4c^3x\left(b^2-4ac\right)}+\frac{\left(5b^2-12ac\right)\sqrt{ax^2}}{2c^2\left(b^2-4\right.}$$

```
[Out] 2*x^4*(b*x+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^3+a*x^2)^(1/2)+3/8*(-4*a*c+5*b^2)*x
*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))*(c*x^2+b*x+a)^(1/2)/c^(
7/2)/(c*x^4+b*x^3+a*x^2)^(1/2)+1/2*(-12*a*c+5*b^2)*(c*x^4+b*x^3+a*x^2)^(1/2
)/c^2/(-4*a*c+b^2)-1/4*b*(-52*a*c+15*b^2)*(c*x^4+b*x^3+a*x^2)^(1/2)/c^3/(-4
*a*c+b^2)/x-2*b*x*(c*x^4+b*x^3+a*x^2)^(1/2)/c/(-4*a*c+b^2)
```

Rubi [A] time = 0.51, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1923, 1949, 12, 1914, 621, 206}

$$\frac{\left(5b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{2c^2\left(b^2-4ac\right)}-\frac{b\left(15b^2-52ac\right)\sqrt{ax^2+bx^3+cx^4}}{4c^3x\left(b^2-4ac\right)}+\frac{3x\left(5b^2-4ac\right)\sqrt{a+bx+cx^2}\,\tanh^{-1}\left(\frac{}{2\sqrt{}}\right.}{8c^{7/2}\sqrt{ax^2+bx^3+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

```
[Out] (2*x^4*(2*a + b*x))/((b^2 - 4*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4]) + ((5*b^2 -
 12*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(2*c^2*(b^2 - 4*a*c)) - (b*(15*b^2 -
52*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(4*c^3*(b^2 - 4*a*c)*x) - (2*b*x*Sqrt[
a*x^2 + b*x^3 + c*x^4])/(c*(b^2 - 4*a*c)) + (3*(5*b^2 - 4*a*c)*x*Sqrt[a + b
*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(7
/2)*Sqrt[a*x^2 + b*x^3 + c*x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1914

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))])/Sqrt[
a*x^q + b*x^n + c*x^(2*n - q)], Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
```

```
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

Rule 1923

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> -Simp[(x^(m - 2*n + q + 1)*(2*a + b*x^(n - q))*(a*x^q + b*x
^n + c*x^(2*n - q))^(p + 1))/((n - q)*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(
(n - q)*(p + 1)*(b^2 - 4*a*c)), Int[x^(m - 2*n + q)*(2*a*(m + p*q - 2*(n -
q) + 1) + b*(m + p*q + (n - q)*(2*p + 1) + 1)*x^(n - q))*(a*x^q + b*x^n + c
*x^(2*n - q))^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p,
 -1] && RationalQ[m, q] && GtQ[m + p*q + 1, 2*(n - q)]
```

Rule 1949

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[(B*x^(m - n + 1)*(a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1))/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), x] -
Dist[1/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(m - n + q)*Simp[a*B*(m
 + p*q - n + q + 1) + (b*B*(m + p*q + (n - q)*p + 1) - A*c*(m + p*q + (n -
q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x]
/; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Integ
erQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[p, -1] && LtQ[p, 0] && R
ationalQ[m, q] && GeQ[m + p*q, n - q - 1] && NeQ[m + p*q + (n - q)*(2*p + 1
) + 1, 0]
```

Rubi steps

$$\int \frac{x^7}{\left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx = \frac{2x^4(2a+bx)}{\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}} - \frac{2\int \frac{x^3(6a+3bx)}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{b^2-4ac}$$

$$= \frac{2x^4(2a+bx)}{\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}} - \frac{2bx\sqrt{ax^2+bx^3+cx^4}}{c\left(b^2-4ac\right)} + \frac{2\int \frac{x^2\left(6ab+\frac{3}{2}\left(5b^2-12ac\right)x\right)}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{3c\left(b^2-4ac\right)}$$

$$= \frac{2x^4(2a+bx)}{\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}} + \frac{\left(5b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{2c^2\left(b^2-4ac\right)} - \frac{2bx\sqrt{ax^2+bx^3+}}{c\left(b^2-4ac\right)}$$

$$= \frac{2x^4(2a+bx)}{\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}} + \frac{\left(5b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{2c^2\left(b^2-4ac\right)} - \frac{b\left(15b^2-52ac\right)}{4c^3\left(b^2\right.}$$

$$= \frac{2x^4(2a+bx)}{\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}} + \frac{\left(5b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{2c^2\left(b^2-4ac\right)} - \frac{b\left(15b^2-52ac\right)}{4c^3\left(b^2\right.}$$

$$= \frac{2x^4(2a+bx)}{\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}} + \frac{\left(5b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{2c^2\left(b^2-4ac\right)} - \frac{b\left(15b^2-52ac\right)}{4c^3\left(b^2\right.}$$

$$= \frac{2x^4(2a+bx)}{\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}} + \frac{\left(5b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{2c^2\left(b^2-4ac\right)} - \frac{b\left(15b^2-52ac\right)}{4c^3\left(b^2\right.}$$

$$= \frac{2x^4(2a+bx)}{\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}} + \frac{\left(5b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{2c^2\left(b^2-4ac\right)} - \frac{b\left(15b^2-52ac\right)}{4c^3\left(b^2\right.}$$

Mathematica [A] time = 0.23, size = 183, normalized size = 0.70

$$\frac{x\left(2\sqrt{c}\left(4a^2c(6cx-13b)+a\left(15b^3-62b^2cx-20bc^2x^2+8c^3x^3\right)+b^2x\left(15b^2+5bcx-2c^2x^2\right)\right)-3\left(16a^2c^2-2\right.\right.}{8c^{7/2}\left(4ac-b^2\right)\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

```
[Out] (x*(2*Sqrt[c]*(4*a^2*c*(-13*b + 6*c*x) + b^2*x*(15*b^2 + 5*b*c*x - 2*c^2*x^
2) + a*(15*b^3 - 62*b^2*c*x - 20*b*c^2*x^2 + 8*c^3*x^3)) - 3*(5*b^4 - 24*a*
b^2*c + 16*a^2*c^2)*Sqrt[a + x*(b + c*x)]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sq
rt[a + x*(b + c*x)])]))/(8*c^(7/2)*(-b^2 + 4*a*c)*Sqrt[x^2*(a + x*(b + c*x)
)])
```

fricas [A] time = 0.82, size = 616, normalized size = 2.35

$$\left[-\frac{3\left(\left(5\,b^4c-24\,ab^2c^2+16\,a^2c^3\right)x^3+\left(5\,b^5-24\,ab^3c+16\,a^2bc^2\right)x^2+\left(5\,ab^4-24\,a^2b^2c+16\,a^3c^2\right)x\right)\sqrt{c}\,\log\left(-\right.}{16\left(\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(3*((5*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*x^3 + (5*b^5 - 24*a*b^3*c
+ 16*a^2*b*c^2)*x^2 + (5*a*b^4 - 24*a^2*b^2*c + 16*a^3*c^2)*x)*sqrt(c)*log(
-(8*c^2*x^3 + 8*b*c*x^2 - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(c)
 + (b^2 + 4*a*c)*x)/x) + 4*(15*a*b^3*c - 52*a^2*b*c^2 - 2*(b^2*c^3 - 4*a*c^
4)*x^3 + 5*(b^3*c^2 - 4*a*b*c^3)*x^2 + (15*b^4*c - 62*a*b^2*c^2 + 24*a^2*c^
3)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/((b^2*c^5 - 4*a*c^6)*x^3 + (b^3*c^4 - 4*
a*b*c^5)*x^2 + (a*b^2*c^4 - 4*a^2*c^5)*x), -1/8*(3*((5*b^4*c - 24*a*b^2*c^2
 + 16*a^2*c^3)*x^3 + (5*b^5 - 24*a*b^3*c + 16*a^2*b*c^2)*x^2 + (5*a*b^4 - 2
4*a^2*b^2*c + 16*a^3*c^2)*x)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2
)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) + 2*(15*a*b^3*c - 52*a^
2*b*c^2 - 2*(b^2*c^3 - 4*a*c^4)*x^3 + 5*(b^3*c^2 - 4*a*b*c^3)*x^2 + (15*b^4
*c - 62*a*b^2*c^2 + 24*a^2*c^3)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/((b^2*c^5 -
 4*a*c^6)*x^3 + (b^3*c^4 - 4*a*b*c^5)*x^2 + (a*b^2*c^4 - 4*a^2*c^5)*x)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\left(cx^4+bx^3+ax^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^7/(c*x^4 + b*x^3 + a*x^2)^(3/2), x)
```

maple [A] time = 0.01, size = 283, normalized size = 1.08

$$\frac{\left(c\,x^2+bx+a\right)\left(16a\,c^{\frac{9}{2}}x^3-4b^2c^{\frac{7}{2}}x^3-40ab\,c^{\frac{7}{2}}x^2+10b^3c^{\frac{5}{2}}x^2+48a^2c^{\frac{7}{2}}x-124a\,b^2c^{\frac{5}{2}}x+30b^4c^{\frac{3}{2}}x-48\sqrt{c\,x^2+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(c*x^4+b*x^3+a*x^2)^(3/2),x)
```

```
[Out] 1/8*x^3*(c*x^2+b*x+a)/c^(9/2)*(16*c^(9/2)*x^3*a-4*c^(7/2)*x^3*b^2-40*c^(7/2
)*x^2*a*b+10*c^(5/2)*x^2*b^3+48*c^(7/2)*x*a^2-124*c^(5/2)*x*a*b^2+30*c^(3/2
)*x*b^4-104*c^(5/2)*a^2*b+30*c^(3/2)*a*b^3-48*ln(1/2*(2*c*x+b+2*(c*x^2+b*x+
a)^(1/2)*c^(1/2))/c^(1/2))*(c*x^2+b*x+a)^(1/2)*a^2*c^3+72*ln(1/2*(2*c*x+b+2
*(c*x^2+b*x+a)^(1/2)*c^(1/2))/c^(1/2))*(c*x^2+b*x+a)^(1/2)*a*b^2*c^2-15*ln(
1/2*(2*c*x+b+2*(c*x^2+b*x+a)^(1/2)*c^(1/2))/c^(1/2))*(c*x^2+b*x+a)^(1/2)*b^
4*c)/(c*x^4+b*x^3+a*x^2)^(3/2)/(4*a*c-b^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\left(cx^4 + bx^3 + ax^2\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^7/(c*x^4 + b*x^3 + a*x^2)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^7}{\left(c\,x^4 + b\,x^3 + a\,x^2\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a*x^2 + b*x^3 + c*x^4)^(3/2),x)
```

```
[Out] int(x^7/(a*x^2 + b*x^3 + c*x^4)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\left(x^2\left(a + bx + cx^2\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(c*x**4+b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral(x**7/(x**2*(a + b*x + c*x**2))**(3/2), x)
```

3.56 $\int \frac{x^6}{\left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=201

$$\frac{\left(3b^2-8ac\right)\sqrt{ax^2+bx^3+cx^4}}{c^2x\left(b^2-4ac\right)}+\frac{2x^3(2a+bx)}{\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}-\frac{2b\sqrt{ax^2+bx^3+cx^4}}{c\left(b^2-4ac\right)}-\frac{3bx\sqrt{a+bx+cx^2}\,\tanh^{-1}}{2c^{5/2}\sqrt{ax^2+bx}}$$

```
[Out] 2*x^3*(b*x+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^3+a*x^2)^(1/2)-3/2*b*x*arctanh(1/2*
(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))*(c*x^2+b*x+a)^(1/2)/c^(5/2)/(c*x^4+b
*x^3+a*x^2)^(1/2)-2*b*(c*x^4+b*x^3+a*x^2)^(1/2)/c/(-4*a*c+b^2)+(-8*a*c+3*b^
2)*(c*x^4+b*x^3+a*x^2)^(1/2)/c^2/(-4*a*c+b^2)/x
```

Rubi [A] time = 0.30, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1923, 1949, 12, 1914, 621, 206}

$$\frac{\left(3b^2-8ac\right)\sqrt{ax^2+bx^3+cx^4}}{c^2x\left(b^2-4ac\right)}+\frac{2x^3(2a+bx)}{\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}-\frac{2b\sqrt{ax^2+bx^3+cx^4}}{c\left(b^2-4ac\right)}-\frac{3bx\sqrt{a+bx+cx^2}\,\tanh^{-1}}{2c^{5/2}\sqrt{ax^2+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

```
[Out] (2*x^3*(2*a + b*x))/((b^2 - 4*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4]) - (2*b*Sqrt
[a*x^2 + b*x^3 + c*x^4])/(c*(b^2 - 4*a*c)) + ((3*b^2 - 8*a*c)*Sqrt[a*x^2 +
b*x^3 + c*x^4])/(c^2*(b^2 - 4*a*c)*x) - (3*b*x*Sqrt[a + b*x + c*x^2]*ArcTan
h[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*c^(5/2)*Sqrt[a*x^2 + b
*x^3 + c*x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1914

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))])/Sqrt[
a*x^q + b*x^n + c*x^(2*n - q)], Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

Rule 1923

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> -Simp[(x^(m - 2*n + q + 1)*(2*a + b*x^(n - q))*(a*x^q + b*x
^n + c*x^(2*n - q))^(p + 1))/((n - q)*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(
(n - q)*(p + 1)*(b^2 - 4*a*c)), Int[x^(m - 2*n + q)*(2*a*(m + p*q - 2*(n -
q) + 1) + b*(m + p*q + (n - q)*(2*p + 1) + 1)*x^(n - q))*(a*x^q + b*x^n + c
*x^(2*n - q))^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p,
 -1] && RationalQ[m, q] && GtQ[m + p*q + 1, 2*(n - q)]
```

Rule 1949

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[(B*x^(m - n + 1)*(a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1))/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), x] -
Dist[1/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(m - n + q)*Simp[a*B*(m
 + p*q - n + q + 1) + (b*B*(m + p*q + (n - q)*p + 1) - A*c*(m + p*q + (n -
q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x]
/; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Integ
erQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[p, -1] && LtQ[p, 0] && R
ationalQ[m, q] && GeQ[m + p*q, n - q - 1] && NeQ[m + p*q + (n - q)*(2*p + 1
) + 1, 0]
```

Rubi steps

$$\int \frac{x^6}{\left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx = \frac{2x^3(2a+bx)}{\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}} - \frac{2\int \frac{x^2(4a+2bx)}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{b^2-4ac}$$

$$= \frac{2x^3(2a+bx)}{\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}} - \frac{2b\sqrt{ax^2+bx^3+cx^4}}{c\left(b^2-4ac\right)} + \frac{\int \frac{x\left(2ab+\left(3b^2-8ac\right)x\right)}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{c\left(b^2-4ac\right)}$$

$$= \frac{2x^3(2a+bx)}{\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}} - \frac{2b\sqrt{ax^2+bx^3+cx^4}}{c\left(b^2-4ac\right)} + \frac{\left(3b^2-8ac\right)\sqrt{ax^2+bx^3+c}}{c^2\left(b^2-4ac\right)x}$$

$$= \frac{2x^3(2a+bx)}{\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}} - \frac{2b\sqrt{ax^2+bx^3+cx^4}}{c\left(b^2-4ac\right)} + \frac{\left(3b^2-8ac\right)\sqrt{ax^2+bx^3+c}}{c^2\left(b^2-4ac\right)x}$$

$$= \frac{2x^3(2a+bx)}{\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}} - \frac{2b\sqrt{ax^2+bx^3+cx^4}}{c\left(b^2-4ac\right)} + \frac{\left(3b^2-8ac\right)\sqrt{ax^2+bx^3+c}}{c^2\left(b^2-4ac\right)x}$$

$$= \frac{2x^3(2a+bx)}{\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}} - \frac{2b\sqrt{ax^2+bx^3+cx^4}}{c\left(b^2-4ac\right)} + \frac{\left(3b^2-8ac\right)\sqrt{ax^2+bx^3+c}}{c^2\left(b^2-4ac\right)x}$$

$$= \frac{2x^3(2a+bx)}{\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}} - \frac{2b\sqrt{ax^2+bx^3+cx^4}}{c\left(b^2-4ac\right)} + \frac{\left(3b^2-8ac\right)\sqrt{ax^2+bx^3+c}}{c^2\left(b^2-4ac\right)x}$$

Mathematica [A] time = 0.15, size = 141, normalized size = 0.70

$$\frac{x\left(2\sqrt{c}\left(8a^2c+a\left(-3b^2+10bcx+4c^2x^2\right)-b^2x(3b+cx)\right)+3b\left(b^2-4ac\right)\sqrt{a+x(b+cx)}\,\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\,\sqrt{a+x(b+cx)}}\right)}{2c^{5/2}\left(4ac-b^2\right)\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

```
[Out] (x*(2*Sqrt[c]*(8*a^2*c - b^2*x*(3*b + c*x) + a*(-3*b^2 + 10*b*c*x + 4*c^2*x
^2)) + 3*b*(b^2 - 4*a*c)*Sqrt[a + x*(b + c*x)]*ArcTanh[(b + 2*c*x)/(2*Sqrt[
c]*Sqrt[a + x*(b + c*x)])]))/(2*c^(5/2)*(-b^2 + 4*a*c)*Sqrt[x^2*(a + x*(b +
 c*x))])
```

fricas [A] time = 0.88, size = 486, normalized size = 2.42

$$\left[\frac{3\left(\left(b^3c - 4\,abc^2\right)x^3 + \left(b^4 - 4\,ab^2c\right)x^2 + \left(ab^3 - 4\,a^2bc\right)x\right)\sqrt{c}\,\log\left(-\frac{8\,c^2x^3 + 8\,bcx^2 - 4\,\sqrt{cx^4+bx^3+ax^2}\,(2\,cx+b)\sqrt{c} + \left(b^2+4\,ac\right)x}{x}\right.}{4\left(\left(b^2c^4 - 4\,ac^5\right)x^3 + \left(b^3c^3 - 4\,abc^4\right)x^2 + \left(ab\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(3*((b^3*c - 4*a*b*c^2)*x^3 + (b^4 - 4*a*b^2*c)*x^2 + (a*b^3 - 4*a^2*b
*c)*x)*sqrt(c)*log(-(8*c^2*x^3 + 8*b*c*x^2 - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*
(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(
3*a*b^2*c - 8*a^2*c^2 + (b^2*c^2 - 4*a*c^3)*x^2 + (3*b^3*c - 10*a*b*c^2)*x)
)/((b^2*c^4 - 4*a*c^5)*x^3 + (b^3*c^3 - 4*a*b*c^4)*x^2 + (a*b^2*c^3 - 4*a^2
*c^4)*x), 1/2*(3*((b^3*c - 4*a*b*c^2)*x^3 + (b^4 - 4*a*b^2*c)*x^2 + (a*b^3
- 4*a^2*b*c)*x)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)
*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) + 2*sqrt(c*x^4 + b*x^3 + a*x^2)*(3*a
*b^2*c - 8*a^2*c^2 + (b^2*c^2 - 4*a*c^3)*x^2 + (3*b^3*c - 10*a*b*c^2)*x))/(
(b^2*c^4 - 4*a*c^5)*x^3 + (b^3*c^3 - 4*a*b*c^4)*x^2 + (a*b^2*c^3 - 4*a^2*c^
4)*x)]
```

giac [A] time = 0.89, size = 195, normalized size = 0.97

$$\frac{2\left(\frac{b^3c^2 - 3\,abc^3}{b^2c^4 - 4\,ac^5} + \frac{ab^2c^2 - 2\,a^2c^3}{\left(b^2c^4 - 4\,ac^5\right)x}\right)}{\sqrt{c + \frac{b}{x} + \frac{a}{x^2}}} + \frac{3\,b\arctan\left(\frac{\sqrt{c + \frac{b}{x} + \frac{a}{x^2}} - \frac{\sqrt{a}}{x}}{\sqrt{-c}}\right)}{\sqrt{-c}\,c^2} + \frac{b\left(\sqrt{c + \frac{b}{x} + \frac{a}{x^2}} - \frac{\sqrt{a}}{x}\right) - 2\,\sqrt{a}\,c}{\left(\left(\sqrt{c + \frac{b}{x} + \frac{a}{x^2}} - \frac{\sqrt{a}}{x}\right)^2 - c\right)c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 2*((b^3*c^2 - 3*a*b*c^3)/(b^2*c^4 - 4*a*c^5) + (a*b^2*c^2 - 2*a^2*c^3)/((b^
2*c^4 - 4*a*c^5)*x))/sqrt(c + b/x + a/x^2) + 3*b*arctan((sqrt(c + b/x + a/x
^2) - sqrt(a)/x)/sqrt(-c))/(sqrt(-c)*c^2) + (b*(sqrt(c + b/x + a/x^2) - sqr
t(a)/x) - 2*sqrt(a)*c)/(((sqrt(c + b/x + a/x^2) - sqrt(a)/x)^2 - c)*c^2)
```

maple [A] time = 0.01, size = 199, normalized size = 0.99

$$\frac{\left(c\,x^2 + bx + a\right)\left(8a\,c^{\frac{7}{2}}x^2 - 2b^2c^{\frac{5}{2}}x^2 + 20ab\,c^{\frac{5}{2}}x - 6b^3c^{\frac{3}{2}}x - 12\sqrt{c\,x^2 + bx + a}\;abc^2\ln\left(\frac{2cx+b+2\sqrt{c\,x^2+bx+a}\;\sqrt{c}}{2\sqrt{c}}\right) + 3\right.}{2\left(c\,x^4 + b\,x^3 + a\,x^2\right)^{\frac{3}{2}}\left(4ac - b^2\right)c^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(c*x^4+b*x^3+a*x^2)^(3/2),x)
```

```
[Out] 1/2*x^3*(c*x^2+b*x+a)/c^(7/2)*(8*c^(7/2)*x^2*a-2*c^(5/2)*x^2*b^2+20*c^(5/2)
*x*a*b-6*c^(3/2)*x*b^3+16*c^(5/2)*a^2-6*c^(3/2)*a*b^2-12*ln(1/2*(2*c*x+b+2*
(c*x^2+b*x+a)^(1/2)*c^(1/2))/c^(1/2))*(c*x^2+b*x+a)^(1/2)*a*b*c^2+3*ln(1/2*
```

```
(2*c*x+b+2*(c*x^2+b*x+a)^(1/2)*c^(1/2))/c^(1/2))*(c*x^2+b*x+a)^(1/2)*b^3*c)
/(c*x^4+b*x^3+a*x^2)^(3/2)/(4*a*c-b^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(cx^4+bx^3+ax^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^6/(c*x^4 + b*x^3 + a*x^2)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^6}{{\left(c\,x^4+b\,x^3+a\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a*x^2 + b*x^3 + c*x^4)^(3/2),x)
```

```
[Out] int(x^6/(a*x^2 + b*x^3 + c*x^4)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(x^2\left(a+bx+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(c*x**4+b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral(x**6/(x**2*(a + b*x + c*x**2))**(3/2), x)
```

3.57 $\int \frac{x^5}{\left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=153

$$\frac{2x^2(2a+bx)}{\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}-\frac{2b\sqrt{ax^2+bx^3+cx^4}}{cx\left(b^2-4ac\right)}+\frac{x\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}\sqrt{ax^2+bx^3+cx^4}}$$

```
[Out] 2*x^2*(b*x+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^3+a*x^2)^(1/2)+x*arctanh(1/2*(2*c*x
+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))*(c*x^2+b*x+a)^(1/2)/c^(3/2)/(c*x^4+b*x^3+a
*x^2)^(1/2)-2*b*(c*x^4+b*x^3+a*x^2)^(1/2)/c/(-4*a*c+b^2)/x
```

Rubi [A] time = 0.18, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1923, 1949, 12, 1914, 621, 206}

$$\frac{2x^2(2a+bx)}{\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}-\frac{2b\sqrt{ax^2+bx^3+cx^4}}{cx\left(b^2-4ac\right)}+\frac{x\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}\sqrt{ax^2+bx^3+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

```
[Out] (2*x^2*(2*a + b*x))/((b^2 - 4*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4]) - (2*b*Sqrt
[a*x^2 + b*x^3 + c*x^4])/(c*(b^2 - 4*a*c)*x) + (x*Sqrt[a + b*x + c*x^2]*Arc
Tanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(c^(3/2)*Sqrt[a*x^2 +
b*x^3 + c*x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1914

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))])/Sqrt[
a*x^q + b*x^n + c*x^(2*n - q)], Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

Rule 1923

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> -Simp[(x^(m - 2*n + q + 1)*(2*a + b*x^(n - q))*(a*x^q + b*x
```

```
^n + c*x^(2*n - q))^(p + 1))/((n - q)*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(
(n - q)*(p + 1)*(b^2 - 4*a*c)), Int[x^(m - 2*n + q)*(2*a*(m + p*q - 2*(n -
q) + 1) + b*(m + p*q + (n - q)*(2*p + 1) + 1)*x^(n - q))*(a*x^q + b*x^n + c
*x^(2*n - q))^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p,
 -1] && RationalQ[m, q] && GtQ[m + p*q + 1, 2*(n - q)]
```

Rule 1949

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[(B*x^(m - n + 1)*(a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1))/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), x] -
Dist[1/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(m - n + q)*Simp[a*B*(m
 + p*q - n + q + 1) + (b*B*(m + p*q + (n - q)*p + 1) - A*c*(m + p*q + (n -
q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x]
/; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Integ
erQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[p, -1] && LtQ[p, 0] && R
ationalQ[m, q] && GeQ[m + p*q, n - q - 1] && NeQ[m + p*q + (n - q)*(2*p + 1
) + 1, 0]
```

Rubi steps

$$\int \frac{x^5}{\left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx = \frac{2x^2(2a+bx)}{\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}} - \frac{2\int \frac{x(2a+bx)}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{b^2-4ac}$$

$$= \frac{2x^2(2a+bx)}{\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}} - \frac{2b\sqrt{ax^2+bx^3+cx^4}}{c\left(b^2-4ac\right)x} + \frac{2\int \frac{\left(b^2-4ac\right)x}{2\sqrt{ax^2+bx^3+cx^4}}\,dx}{c\left(b^2-4ac\right)}$$

$$= \frac{2x^2(2a+bx)}{\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}} - \frac{2b\sqrt{ax^2+bx^3+cx^4}}{c\left(b^2-4ac\right)x} + \frac{\int \frac{x}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{c}$$

$$= \frac{2x^2(2a+bx)}{\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}} - \frac{2b\sqrt{ax^2+bx^3+cx^4}}{c\left(b^2-4ac\right)x} + \frac{\left(x\sqrt{a+bx+cx^2}\right)\int \frac{1}{\sqrt{a+bx+\ldots}}}{c\sqrt{ax^2+bx^3+cx^4}}$$

$$= \frac{2x^2(2a+bx)}{\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}} - \frac{2b\sqrt{ax^2+bx^3+cx^4}}{c\left(b^2-4ac\right)x} + \frac{\left(2x\sqrt{a+bx+cx^2}\right)\text{Subst}\Big(\ldots}{c\sqrt{ax^2+bx\ldots}}$$

$$= \frac{2x^2(2a+bx)}{\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}} - \frac{2b\sqrt{ax^2+bx^3+cx^4}}{c\left(b^2-4ac\right)x} + \frac{x\sqrt{a+bx+cx^2}\,\tanh^{-1}\Big(\frac{\ldots}{2\sqrt{\ldots}}}{c^{3/2}\sqrt{ax^2+bx^3+c\ldots}}$$

Mathematica [A] time = 0.12, size = 112, normalized size = 0.73

$$-\frac{x\left(2\sqrt{c}\,\left(-ab+2acx+b^2(-x)\right)+\left(b^2-4ac\right)\sqrt{a+x(b+cx)}\,\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\,\sqrt{a+x(b+cx)}}\right)\right)}{c^{3/2}\left(4ac-b^2\right)\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

```
[Out] -((x*(2*Sqrt[c]*(-(a*b) - b^2*x + 2*a*c*x) + (b^2 - 4*a*c)*Sqrt[a + x*(b +
c*x)]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))/(c^(3/2)*(-b
^2 + 4*a*c)*Sqrt[x^2*(a + x*(b + c*x))]))
```

fricas [A] time = 0.77, size = 414, normalized size = 2.71

$$\left[\frac{\left(\left(b^2c-4\,ac^2\right)x^3+\left(b^3-4\,abc\right)x^2+\left(ab^2-4\,a^2c\right)x\right)\sqrt{c}\,\log\left(-\frac{8\,c^2x^3+8\,bcx^2+4\,\sqrt{cx^4+bx^3+ax^2}\,(2\,cx+b)\sqrt{c}+\left(b^2+4\,ac\right)x}{x}\right)-4}{2\left(\left(b^2c^3-4\,ac^4\right)x^3+\left(b^3c^2-4\,abc^3\right)x^2+\left(ab^2c^2-4\,a^2c^3\right)x\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(((b^2*c - 4*a*c^2)*x^3 + (b^3 - 4*a*b*c)*x^2 + (a*b^2 - 4*a^2*c)*x)*s
qrt(c)*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x +
 b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(a*b*c +
(b^2*c - 2*a*c^2)*x))/((b^2*c^3 - 4*a*c^4)*x^3 + (b^3*c^2 - 4*a*b*c^3)*x^2
+ (a*b^2*c^2 - 4*a^2*c^3)*x), -(((b^2*c - 4*a*c^2)*x^3 + (b^3 - 4*a*b*c)*x^
2 + (a*b^2 - 4*a^2*c)*x)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2
*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) + 2*sqrt(c*x^4 + b*x^3 + a*
x^2)*(a*b*c + (b^2*c - 2*a*c^2)*x))/((b^2*c^3 - 4*a*c^4)*x^3 + (b^3*c^2 - 4
*a*b*c^3)*x^2 + (a*b^2*c^2 - 4*a^2*c^3)*x)]
```

giac [A] time = 0.98, size = 110, normalized size = 0.72

$$-\frac{2\left(\frac{abc}{\left(b^2c^2-4\,ac^3\right)x}+\frac{b^2c-2\,ac^2}{b^2c^2-4\,ac^3}\right)}{\sqrt{c+\frac{b}{x}+\frac{a}{x^2}}}-\frac{2\,\arctan\left(\frac{\sqrt{c+\frac{b}{x}+\frac{a}{x^2}}-\frac{\sqrt{a}}{x}}{\sqrt{-c}}\right)}{\sqrt{-c}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] -2*(a*b*c/((b^2*c^2 - 4*a*c^3)*x) + (b^2*c - 2*a*c^2)/(b^2*c^2 - 4*a*c^3))/
sqrt(c + b/x + a/x^2) - 2*arctan((sqrt(c + b/x + a/x^2) - sqrt(a)/x)/sqrt(-
c))/(sqrt(-c)*c)
```

maple [A] time = 0.01, size = 166, normalized size = 1.08

$$\frac{\left(c\,x^2+bx+a\right)\left(-4a\,c^{\frac{5}{2}}x+2b^2c^{\frac{3}{2}}x+4\sqrt{c\,x^2+bx+a}\,\,a\,c^2\ln\left(\frac{2cx+b+2\sqrt{c\,x^2+bx+a}\,\,\sqrt{c}}{2\sqrt{c}}\right)-\sqrt{c\,x^2+bx+a}\,\,b^2c\ln\left(\frac{2c}{}\right.\right.}{\left(c\,x^4+b\,x^3+a\,x^2\right)^{\frac{3}{2}}\left(4ac-b^2\right)c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(c*x^4+b*x^3+a*x^2)^(3/2),x)
```

```
[Out] x^3*(c*x^2+b*x+a)/c^(5/2)*(-4*c^(5/2)*x*a+2*c^(3/2)*x*b^2+4*(c*x^2+b*x+a)^(
1/2)*ln(1/2*(2*c*x+b+2*(c*x^2+b*x+a)^(1/2)*c^(1/2))/c^(1/2))*a*c^2-(c*x^2+b
*x+a)^(1/2)*ln(1/2*(2*c*x+b+2*(c*x^2+b*x+a)^(1/2)*c^(1/2))/c^(1/2))*b^2*c+2
*c^(3/2)*a*b)/(c*x^4+b*x^3+a*x^2)^(3/2)/(4*a*c-b^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(cx^4+bx^3+ax^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^5/(c*x^4 + b*x^3 + a*x^2)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5}{\left(c\,x^4+b\,x^3+a\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a*x^2 + b*x^3 + c*x^4)^(3/2),x)
[Out] int(x^5/(a*x^2 + b*x^3 + c*x^4)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(x^2\left(a+bx+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(c*x**4+b*x**3+a*x**2)**(3/2),x)
[Out] Integral(x**5/(x**2*(a + b*x + c*x**2))**(3/2), x)
```

3.58 $\int \frac{x^4}{\left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=40

$$\frac{2x(2a+bx)}{\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}$$

```
[Out] 2*x*(b*x+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^3+a*x^2)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {1916}

$$\frac{2x(2a+bx)}{\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
[Out] (2*x*(2*a + b*x))/((b^2 - 4*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])
```

Rule 1916

```
Int[(x_)^(m_.)/((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(3/
2), x_Symbol] :> Simp[(x^((n - 1)/2)*(4*a + 2*b*x))/((b^2 - 4*a*c)*Sqrt[a*x
^(n - 1) + b*x^n + c*x^(n + 1)]), x] /; FreeQ[{a, b, c, n}, x] && EqQ[m, (3
*n - 1)/2] && EqQ[q, n - 1] && EqQ[r, n + 1] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^4}{\left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx = \frac{2x(2a+bx)}{\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}$$

Mathematica [A] time = 0.07, size = 37, normalized size = 0.92

$$\frac{2x(2a+bx)}{\left(b^2-4ac\right)\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
[Out] (2*x*(2*a + b*x))/((b^2 - 4*a*c)*Sqrt[x^2*(a + x*(b + c*x))])
```

fricas [A] time = 0.62, size = 73, normalized size = 1.82

$$\frac{2\sqrt{cx^4+bx^3+ax^2}\,(bx+2\,a)}{\left(b^2c-4\,ac^2\right)x^3+\left(b^3-4\,abc\right)x^2+\left(ab^2-4\,a^2c\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
[Out] 2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)/((b^2*c - 4*a*c^2)*x^3 + (b^3 - 4
*a*b*c)*x^2 + (a*b^2 - 4*a^2*c)*x)
```

giac [A] time = 0.92, size = 45, normalized size = 1.12

$$\frac{2\left(\frac{b}{b^2-4\,ac}+\frac{2\,a}{(b^2-4\,ac)x}\right)}{\sqrt{c+\frac{b}{x}+\frac{a}{x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 2*(b/(b^2 - 4*a*c) + 2*a/((b^2 - 4*a*c)*x))/sqrt(c + b/x + a/x^2)
```

maple [A] time = 0.00, size = 53, normalized size = 1.32

$$-\frac{2\left(c\,x^2+bx+a\right)(bx+2a)\,x^3}{\left(4ac-b^2\right)\left(c\,x^4+b\,x^3+a\,x^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^4+b*x^3+a*x^2)^(3/2),x)
```

```
[Out] -2*(c*x^2+b*x+a)*(b*x+2*a)*x^3/(4*a*c-b^2)/(c*x^4+b*x^3+a*x^2)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(cx^4+bx^3+ax^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(c*x^4 + b*x^3 + a*x^2)^(3/2), x)
```

mupad [B] time = 2.12, size = 75, normalized size = 1.88

$$-\frac{\left(\frac{4\,a\,c}{4\,a\,c^2-b^2\,c}+\frac{2\,b\,c\,x}{4\,a\,c^2-b^2\,c}\right)\sqrt{c\,x^4+b\,x^3+a\,x^2}}{x\,\left(c\,x^2+b\,x+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a*x^2 + b*x^3 + c*x^4)^(3/2),x)
```

```
[Out] -(((4*a*c)/(4*a*c^2 - b^2*c) + (2*b*c*x)/(4*a*c^2 - b^2*c))*(a*x^2 + b*x^3
+ c*x^4)^(1/2))/(x*(a + b*x + c*x^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(x^2\left(a+bx+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**4+b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral(x**4/(x**2*(a + b*x + c*x**2))**(3/2), x)
```

3.59 $\int \frac{x^3}{\left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=39

$$-\frac{2x(b+2cx)}{\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}$$

```
[Out] -2*x*(2*c*x+b)/(-4*a*c+b^2)/(c*x^4+b*x^3+a*x^2)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {1915}

$$-\frac{2x(b+2cx)}{\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

```
[Out] (-2*x*(b + 2*c*x))/((b^2 - 4*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])
```

Rule 1915

```
Int[(x_)^(m_.)/((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(3/
2), x_Symbol] :> Simp[(-2*x^((n - 1)/2)*(b + 2*c*x))/((b^2 - 4*a*c)*Sqrt[a*
x^(n - 1) + b*x^n + c*x^(n + 1)]), x] /; FreeQ[{a, b, c, n}, x] && EqQ[m, (
3*(n - 1))/2] && EqQ[q, n - 1] && EqQ[r, n + 1] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^3}{\left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx=-\frac{2x(b+2cx)}{\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}$$

Mathematica [A] time = 0.02, size = 36, normalized size = 0.92

$$-\frac{2x(b+2cx)}{\left(b^2-4ac\right)\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

```
[Out] (-2*x*(b + 2*c*x))/((b^2 - 4*a*c)*Sqrt[x^2*(a + x*(b + c*x))])
```

fricas [A] time = 0.66, size = 72, normalized size = 1.85

$$-\frac{2\sqrt{cx^4+bx^3+ax^2}\,(2\,cx+b)}{\left(b^2c-4\,ac^2\right)x^3+\left(b^3-4\,abc\right)x^2+\left(ab^2-4\,a^2c\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)/((b^2*c - 4*a*c^2)*x^3 + (b^3 -
4*a*b*c)*x^2 + (a*b^2 - 4*a^2*c)*x)
```

giac [A] time = 0.76, size = 45, normalized size = 1.15

$$-\frac{2\left(\frac{2c}{b^2-4ac}+\frac{b}{(b^2-4ac)x}\right)}{\sqrt{c+\frac{b}{x}+\frac{a}{x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] -2*(2*c/(b^2 - 4*a*c) + b/((b^2 - 4*a*c)*x))/sqrt(c + b/x + a/x^2)
```

maple [A] time = 0.00, size = 52, normalized size = 1.33

$$\frac{2\left(cx^2+bx+a\right)(2cx+b)x^3}{\left(4ac-b^2\right)\left(cx^4+bx^3+ax^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^4+b*x^3+a*x^2)^(3/2),x)
```

```
[Out] 2*(c*x^2+b*x+a)*(2*c*x+b)*x^3/(4*a*c-b^2)/(c*x^4+b*x^3+a*x^2)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(cx^4+bx^3+ax^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(c*x^4 + b*x^3 + a*x^2)^(3/2), x)
```

mupad [B] time = 2.03, size = 75, normalized size = 1.92

$$\frac{\left(\frac{4c^2x}{4ac^2-b^2c}+\frac{2bc}{4ac^2-b^2c}\right)\sqrt{cx^4+bx^3+ax^2}}{x\left(cx^2+bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*x^2 + b*x^3 + c*x^4)^(3/2),x)
```

```
[Out] (((4*c^2*x)/(4*a*c^2 - b^2*c) + (2*b*c)/(4*a*c^2 - b^2*c))*(a*x^2 + b*x^3 +
 c*x^4)^(1/2))/(x*(a + b*x + c*x^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(x^2\left(a+bx+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**4+b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral(x**3/(x**2*(a + b*x + c*x**2))**(3/2), x)
```

3.60 $\int \frac{x^2}{\left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=94

$$\frac{2x\left(-2ac+b^2+bcx\right)}{a\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}-\frac{\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{a^{3/2}}$$

```
[Out] -arctanh(1/2*x*(b*x+2*a)/a^(1/2)/(c*x^4+b*x^3+a*x^2)^(1/2))/a^(3/2)+2*x*(b*
c*x-2*a*c+b^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^3+a*x^2)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1922, 1904, 206}

$$\frac{2x\left(-2ac+b^2+bcx\right)}{a\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}-\frac{\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

```
[Out] (2*x*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4]) -
 ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])]/a^(3/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1904

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, (x*(2*a + b*x^(n - 2)))/
Sqrt[a*x^2 + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n
 - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1922

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> -Simp[(x^(m - q + 1)*(b^2 - 2*a*c + b*c*x^(n - q))*(a*x^q +
 b*x^n + c*x^(2*n - q))^(p + 1))/(a*(n - q)*(p + 1)*(b^2 - 4*a*c)), x] + Di
st[(2*a*c - b^2*(p + 2))/(a*(p + 1)*(b^2 - 4*a*c)), Int[x^(m - q)*(a*x^q +
b*x^n + c*x^(2*n - q))^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n
 - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
&& LtQ[p, -1] && RationalQ[m, p, q] && EqQ[m + p*q + 1, -((n - q)*(2*p + 3)
)]
```

Rubi steps

$$\int \frac{x^2}{\left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx = \frac{2x\left(b^2-2ac+bcx\right)}{a\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}} + \frac{\int \frac{1}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{a}$$

$$= \frac{2x\left(b^2-2ac+bcx\right)}{a\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}} - \frac{2\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{x(2a+bx)}{\sqrt{ax^2+bx^3+cx^4}}\right)}{a}$$

$$= \frac{2x\left(b^2-2ac+bcx\right)}{a\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}} - \frac{\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{a^{3/2}}$$

Mathematica [A] time = 0.13, size = 109, normalized size = 1.16

$$\frac{x\left(b^2-4ac\right)\sqrt{a+x(b+cx)}\,\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right)-2\sqrt{a}\,x\left(-2ac+b^2+bcx\right)}{a^{3/2}\left(4ac-b^2\right)\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

```
[Out] (-2*Sqrt[a]*x*(b^2 - 2*a*c + b*c*x) + (b^2 - 4*a*c)*x*Sqrt[a + x*(b + c*x)]
*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])])/(a^(3/2)*(-b^2 + 4
*a*c)*Sqrt[x^2*(a + x*(b + c*x))])
```

fricas [B] time = 0.93, size = 411, normalized size = 4.37

$$\left[\frac{\left(\left(b^2c-4\,ac^2\right)x^3+\left(b^3-4\,abc\right)x^2+\left(ab^2-4\,a^2c\right)x\right)\sqrt{a}\,\log\left(-\frac{8\,abx^2+\left(b^2+4\,ac\right)x^3+8\,a^2x-4\,\sqrt{cx^4+bx^3+ax^2}\,(bx+2\,a)\sqrt{a}}{x^3}\right)+4\,\sqrt{}}{2\left(\left(a^2b^2c-4\,a^3c^2\right)x^3+\left(a^2b^3-4\,a^3bc\right)x^2+\left(a^3b^2-4\,a^4c\right)x\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(((b^2*c - 4*a*c^2)*x^3 + (b^3 - 4*a*b*c)*x^2 + (a*b^2 - 4*a^2*c)*x)*s
qrt(a)*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3
 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(a*b*c*
x + a*b^2 - 2*a^2*c))/((a^2*b^2*c - 4*a^3*c^2)*x^3 + (a^2*b^3 - 4*a^3*b*c)*
x^2 + (a^3*b^2 - 4*a^4*c)*x), (((b^2*c - 4*a*c^2)*x^3 + (b^3 - 4*a*b*c)*x^2
 + (a*b^2 - 4*a^2*c)*x)*sqrt(-a)*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*
x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) + 2*sqrt(c*x^4 + b*x^3 + a*x
^2)*(a*b*c*x + a*b^2 - 2*a^2*c))/((a^2*b^2*c - 4*a^3*c^2)*x^3 + (a^2*b^3 -
4*a^3*b*c)*x^2 + (a^3*b^2 - 4*a^4*c)*x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,0,0,0]%%%}+%%%{-2,[0,1,0,1]%%%}+%%%{-2,
[0,0,1,2]%%%},0,%%%{1,[0,2,0,2]%%%}+%%%{2,[0,1,1,3]%%%}+%%%{1,[0,0,2,4]%%%}
] at parameters values [54.7579903365,-49,-33,-70]2*(-(-4*a*c+2*b^2)/(8*a^2
```

```
*c-2*a*b^2)/x-2*b*c/(8*a^2*c-2*a*b^2))*sqrt(a*(1/x)^2+b/x+c)/(a*(1/x)^2+b/x
+c)+1/a/sqrt(a)*ln(abs(2*sqrt(a)*(sqrt(a*(1/x)^2+b/x+c)-sqrt(a)/x)-b))
```

maple [A] time = 0.01, size = 166, normalized size = 1.77

$$-\frac{\left(c\,x^{2}+bx+a\right)\left(2a^{\frac{3}{2}}bcx+4\sqrt{c\,x^{2}+bx+a}\;a^{2}c\ln\left(\frac{bx+2a+2\sqrt{c\,x^{2}+bx+a}\;\sqrt{a}}{x}\right)-\sqrt{c\,x^{2}+bx+a}\;a\,b^{2}\ln\left(\frac{bx+2a+2\sqrt{c\,x}}{x}\right.\right.}{\left(c\,x^{4}+b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}\left(4ac-b^{2}\right)a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^4+b*x^3+a*x^2)^(3/2),x)
```

```
[Out] -x^3*(c*x^2+b*x+a)*(2*a^(3/2)*x*b*c+4*(c*x^2+b*x+a)^(1/2)*ln((b*x+2*a+2*(c*
x^2+b*x+a)^(1/2)*a^(1/2))/x)*a^2*c-(c*x^2+b*x+a)^(1/2)*ln((b*x+2*a+2*(c*x^2
+b*x+a)^(1/2)*a^(1/2))/x)*a*b^2-4*a^(5/2)*c+2*a^(3/2)*b^2)/(c*x^4+b*x^3+a*x
^2)^(3/2)/a^(5/2)/(4*a*c-b^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}}{\left(cx^{4}+bx^{3}+ax^{2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(c*x^4 + b*x^3 + a*x^2)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{2}}{\left(c\,x^{4}+b\,x^{3}+a\,x^{2}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x^2 + b*x^3 + c*x^4)^(3/2),x)
```

```
[Out] int(x^2/(a*x^2 + b*x^3 + c*x^4)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}}{\left(x^{2}\left(a+bx+cx^{2}\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**4+b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral(x**2/(x**2*(a + b*x + c*x**2))**(3/2), x)
```

3.61 $\int \frac{x}{\left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=144

$$\frac{3b\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{2a^{5/2}}-\frac{\left(3b^2-8ac\right)\sqrt{ax^2+bx^3+cx^4}}{a^2x^2\left(b^2-4ac\right)}+\frac{2\left(-2ac+b^2+bcx\right)}{a\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}$$

```
[Out] 3/2*b*arctanh(1/2*x*(b*x+2*a)/a^(1/2)/(c*x^4+b*x^3+a*x^2)^(1/2))/a^(5/2)+2*
(b*c*x-2*a*c+b^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^3+a*x^2)^(1/2)-(-8*a*c+3*b^2)*(
c*x^4+b*x^3+a*x^2)^(1/2)/a^2/(-4*a*c+b^2)/x^2
```

Rubi [A] time = 0.16, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1924, 1951, 12, 1904, 206}

$$-\frac{\left(3b^2-8ac\right)\sqrt{ax^2+bx^3+cx^4}}{a^2x^2\left(b^2-4ac\right)}+\frac{3b\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{2a^{5/2}}+\frac{2\left(-2ac+b^2+bcx\right)}{a\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

```
[Out] (2*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4]) - (
(3*b^2 - 8*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(a^2*(b^2 - 4*a*c)*x^2) + (3*b
*ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])])/(2*a^(5/
2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1904

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, (x*(2*a + b*x^(n - 2)))/
Sqrt[a*x^2 + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n
 - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1924

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> -Simp[(x^(m - q + 1)*(b^2 - 2*a*c + b*c*x^(n - q))*(a*x^q +
 b*x^n + c*x^(2*n - q))^(p + 1))/(a*(n - q)*(p + 1)*(b^2 - 4*a*c)), x] + Di
st[1/(a*(n - q)*(p + 1)*(b^2 - 4*a*c)), Int[x^(m - q)*(b^2*(m + p*q + (n -
q)*(p + 1) + 1) - 2*a*c*(m + p*q + 2*(n - q)*(p + 1) + 1) + b*c*(m + p*q +
(n - q)*(2*p + 3) + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p + 1),
x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] &&  !Intege
rQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p, -1] && RationalQ[m, q]
 && LtQ[m + p*q + 1, n - q]
```

Rule 1951

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[(A*x^(m - q + 1)*(a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1))/(a*(m + p*q + 1)), x] + Dist[1/(a*(m + p*q +
 1)), Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p
+ 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b*
x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q
] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
&& RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n - q
)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]
```

Rubi steps

$$\int \frac{x}{\left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx = \frac{2\left(b^2-2ac+bcx\right)}{a\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}} - \frac{2\int \frac{-\frac{3b^2}{2}+4ac-bcx}{x\sqrt{ax^2+bx^3+cx^4}}\,dx}{a\left(b^2-4ac\right)}$$

$$= \frac{2\left(b^2-2ac+bcx\right)}{a\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}} - \frac{\left(3b^2-8ac\right)\sqrt{ax^2+bx^3+cx^4}}{a^2\left(b^2-4ac\right)x^2} + \frac{2\int -\frac{3b\left(b^2-4\right.}{4\sqrt{ax^2+bx}}}{a^2\left(b^2-4\right.}$$

$$= \frac{2\left(b^2-2ac+bcx\right)}{a\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}} - \frac{\left(3b^2-8ac\right)\sqrt{ax^2+bx^3+cx^4}}{a^2\left(b^2-4ac\right)x^2} - \frac{(3b)\int \frac{1}{\sqrt{ax^2+bx}}}{2a^2}$$

$$= \frac{2\left(b^2-2ac+bcx\right)}{a\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}} - \frac{\left(3b^2-8ac\right)\sqrt{ax^2+bx^3+cx^4}}{a^2\left(b^2-4ac\right)x^2} + \frac{(3b)\,\mathrm{Subst}\left(\int\right.}{}$$

$$= \frac{2\left(b^2-2ac+bcx\right)}{a\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}} - \frac{\left(3b^2-8ac\right)\sqrt{ax^2+bx^3+cx^4}}{a^2\left(b^2-4ac\right)x^2} + \frac{3b\tanh^{-1}\left(\frac{}{2\sqrt{}}\right.}{2a}$$

Mathematica [A] time = 0.10, size = 138, normalized size = 0.96

$$\frac{2\sqrt{a}\left(-4a^2c+a\left(b^2-10bcx-8c^2x^2\right)+3b^2x(b+cx)\right)-3bx\left(b^2-4ac\right)\sqrt{a+x(b+cx)}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right)}{2a^{5/2}\left(4ac-b^2\right)\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

```
[Out] (2*Sqrt[a]*(-4*a^2*c + 3*b^2*x*(b + c*x) + a*(b^2 - 10*b*c*x - 8*c^2*x^2))
- 3*b*(b^2 - 4*a*c)*x*Sqrt[a + x*(b + c*x)]*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*
Sqrt[a + x*(b + c*x)])])/(2*a^(5/2)*(-b^2 + 4*a*c)*Sqrt[x^2*(a + x*(b + c*x
))])
```

fricas [A] time = 0.62, size = 496, normalized size = 3.44

$$\left[\frac{3\left(\left(b^3c-4\,abc^2\right)x^4+\left(b^4-4\,ab^2c\right)x^3+\left(ab^3-4\,a^2bc\right)x^2\right)\sqrt{a}\,\log\left(-\frac{8\,abx^2+\left(b^2+4\,ac\right)x^3+8\,a^2x+4\,\sqrt{cx^4+bx^3+ax^2}\,(bx+2\,a)\sqrt{}}{x^3}\right.}{4\left(\left(a^3b^2c-4\,a^4c^2\right)x^4+\left(a^3b^3-4\,a^4bc\right)x^3+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(3*((b^3*c - 4*a*b*c^2)*x^4 + (b^4 - 4*a*b^2*c)*x^3 + (a*b^3 - 4*a^2*b
*c)*x^2)*sqrt(a)*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x + 4*sqrt(c*x
^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) - 4*sqrt(c*x^4 + b*x^3 + a*x^
2)*(a^2*b^2 - 4*a^3*c + (3*a*b^2*c - 8*a^2*c^2)*x^2 + (3*a*b^3 - 10*a^2*b*c
)*x))/((a^3*b^2*c - 4*a^4*c^2)*x^4 + (a^3*b^3 - 4*a^4*b*c)*x^3 + (a^4*b^2 -
 4*a^5*c)*x^2), -1/2*(3*((b^3*c - 4*a*b*c^2)*x^4 + (b^4 - 4*a*b^2*c)*x^3 +
(a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-a)*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b
*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) + 2*sqrt(c*x^4 + b*x^3 + a*
x^2)*(a^2*b^2 - 4*a^3*c + (3*a*b^2*c - 8*a^2*c^2)*x^2 + (3*a*b^3 - 10*a^2*b
*c)*x))/((a^3*b^2*c - 4*a^4*c^2)*x^4 + (a^3*b^3 - 4*a^4*b*c)*x^3 + (a^4*b^2
 - 4*a^5*c)*x^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,0,0,0]%%%}+%%%{-2,[0,1,0,1]%%%}+%%%{-2,
[0,0,1,2]%%%},0,%%%{1,[0,2,0,2]%%%}+%%%{2,[0,1,1,3]%%%}+%%%{1,[0,0,2,4]%%%}
] at parameters values [54.7579903365,-49,-33,-70]2*((-(8*a^2*c-2*a*b^2)/(1
6*a^3*c-4*a^2*b^2)/x-(20*a*b*c-6*b^3)/(16*a^3*c-4*a^2*b^2))/x-(16*a*c^2-6*b
^2*c)/(16*a^3*c-4*a^2*b^2))*sqrt(a*(1/x)^2+b/x+c)/(a*(1/x)^2+b/x+c)-6*b/4/a
^2/sqrt(a)*ln(abs(2*sqrt(a)*(sqrt(a*(1/x)^2+b/x+c)-sqrt(a)/x)-b))
```

maple [A] time = 0.01, size = 201, normalized size = 1.40

$$\frac{\left(c\,x^{2}+bx+a\right)\left(-16a^{\frac{5}{2}}c^{2}x^{2}+6a^{\frac{3}{2}}b^{2}c\,x^{2}+12\sqrt{c\,x^{2}+bx+a}\;a^{2}bcx\ln\left(\frac{bx+2a+2\sqrt{c\,x^{2}+bx+a}\;\sqrt{a}}{x}\right)-3\sqrt{c\,x^{2}+bx+a}\;a\,b\cdots\right.}{2\left(c\,x^{4}+b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}\left(4ac-b^{2}\right)a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^4+b*x^3+a*x^2)^(3/2),x)
```

```
[Out] 1/2*x^2*(c*x^2+b*x+a)*(-16*a^(5/2)*x^2*c^2+6*a^(3/2)*x^2*b^2*c+12*(c*x^2+b*
x+a)^(1/2)*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)*x*a^2*b*c-3*(c*x^2
+b*x+a)^(1/2)*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)*x*a*b^3-20*a^(5
/2)*x*b*c+6*a^(3/2)*x*b^3-8*a^(7/2)*c+2*a^(5/2)*b^2)/(c*x^4+b*x^3+a*x^2)^(3
/2)/a^(7/2)/(4*a*c-b^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(cx^{4}+bx^{3}+ax^{2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(c*x^4 + b*x^3 + a*x^2)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{{\left(c\,x^4+b\,x^3+a\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x^2 + b*x^3 + c*x^4)^(3/2),x)
```

```
[Out] int(x/(a*x^2 + b*x^3 + c*x^4)^(3/2), x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(x^2 \left(a + bx + cx^2\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**4+b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral(x/(x**2*(a + b*x + c*x**2))**(3/2), x)
```

3.62 $\int \frac{1}{\left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=209

$$-\frac{3\left(5b^2-4ac\right)\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{8a^{7/2}}+\frac{b\left(15b^2-52ac\right)\sqrt{ax^2+bx^3+cx^4}}{4a^3x^2\left(b^2-4ac\right)}-\frac{\left(5b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{2a^2x^3\left(b^2-4ac\right)}+\cdots$$

```
[Out] -3/8*(-4*a*c+5*b^2)*arctanh(1/2*x*(b*x+2*a)/a^(1/2)/(c*x^4+b*x^3+a*x^2)^(1/
2))/a^(7/2)+2*(b*c*x-2*a*c+b^2)/a/(-4*a*c+b^2)/x/(c*x^4+b*x^3+a*x^2)^(1/2)-
1/2*(-12*a*c+5*b^2)*(c*x^4+b*x^3+a*x^2)^(1/2)/a^2/(-4*a*c+b^2)/x^3+1/4*b*(-
52*a*c+15*b^2)*(c*x^4+b*x^3+a*x^2)^(1/2)/a^3/(-4*a*c+b^2)/x^2
```

Rubi [A] time = 0.29, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1907, 1951, 12, 1904, 206}

$$\frac{b\left(15b^2-52ac\right)\sqrt{ax^2+bx^3+cx^4}}{4a^3x^2\left(b^2-4ac\right)}-\frac{\left(5b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{2a^2x^3\left(b^2-4ac\right)}-\frac{3\left(5b^2-4ac\right)\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{8a^{7/2}}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3 + c*x^4)^(-3/2),x]
```

```
[Out] (2*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*x*Sqrt[a*x^2 + b*x^3 + c*x^4]) -
 ((5*b^2 - 12*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(2*a^2*(b^2 - 4*a*c)*x^3) +
 (b*(15*b^2 - 52*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(4*a^3*(b^2 - 4*a*c)*x^2
) - (3*(5*b^2 - 4*a*c)*ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^
3 + c*x^4])])/(8*a^(7/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1904

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, (x*(2*a + b*x^(n - 2)))/
Sqrt[a*x^2 + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n
 - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1907

```
Int[((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_), x_Symbol
] :> -Simp[(x^(-q + 1)*(b^2 - 2*a*c + b*c*x^(n - q))*(a*x^q + b*x^n + c*x^(
2*n - q))^(p + 1))/(a*(n - q)*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(a*(n - q
)*(p + 1)*(b^2 - 4*a*c)), Int[(((p*q + 1)*(b^2 - 2*a*c) + (n - q)*(p + 1)*(
b^2 - 4*a*c) + b*c*(p*q + (n - q)*(2*p + 3) + 1)*x^(n - q))*(a*x^q + b*x^n
+ c*x^(2*n - q))^(p + 1))/x^q, x], x] /; FreeQ[{a, b, c, n, q}, x] && EqQ[r
, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && LtQ[p,
```

```
-1]
```

Rule 1951

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[(A*x^(m - q + 1)*(a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1))/(a*(m + p*q + 1)), x] + Dist[1/(a*(m + p*q +
 1)), Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p
+ 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b*
x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q
] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
&& RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n - q
)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]
```

Rubi steps

$$\int \frac{1}{\left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx = \frac{2\left(b^2-2ac+bcx\right)}{a\left(b^2-4ac\right)x\sqrt{ax^2+bx^3+cx^4}} - \frac{2\int \frac{-2\left(b^2-2ac\right)+\frac{1}{2}\left(-b^2+4ac\right)-2bcx}{x^2\sqrt{ax^2+bx^3+cx^4}}\,dx}{a\left(b^2-4ac\right)}$$

$$= \frac{2\left(b^2-2ac+bcx\right)}{a\left(b^2-4ac\right)x\sqrt{ax^2+bx^3+cx^4}} - \frac{\left(5b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{2a^2\left(b^2-4ac\right)x^3} + \frac{\int \frac{-\frac{1}{4}b\left(15b^2-5\cdots\right.}{x\sqrt{\cdots}}}{a^2\cdots}$$

$$= \frac{2\left(b^2-2ac+bcx\right)}{a\left(b^2-4ac\right)x\sqrt{ax^2+bx^3+cx^4}} - \frac{\left(5b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{2a^2\left(b^2-4ac\right)x^3} + \frac{b\left(15b^2-5\cdots\right.}{4a\cdots}$$

$$= \frac{2\left(b^2-2ac+bcx\right)}{a\left(b^2-4ac\right)x\sqrt{ax^2+bx^3+cx^4}} - \frac{\left(5b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{2a^2\left(b^2-4ac\right)x^3} + \frac{b\left(15b^2-5\cdots\right.}{4a\cdots}$$

$$= \frac{2\left(b^2-2ac+bcx\right)}{a\left(b^2-4ac\right)x\sqrt{ax^2+bx^3+cx^4}} - \frac{\left(5b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{2a^2\left(b^2-4ac\right)x^3} + \frac{b\left(15b^2-5\cdots\right.}{4a\cdots}$$

$$= \frac{2\left(b^2-2ac+bcx\right)}{a\left(b^2-4ac\right)x\sqrt{ax^2+bx^3+cx^4}} - \frac{\left(5b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{2a^2\left(b^2-4ac\right)x^3} + \frac{b\left(15b^2-5\cdots\right.}{4a\cdots}$$

Mathematica [A] time = 0.16, size = 181, normalized size = 0.87

$$-\frac{3x^2\left(16a^2c^2-24ab^2c+5b^4\right)\sqrt{a+x(b+cx)}\,\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right)+2\sqrt{a}\left(-8a^3c+2a^2\left(b^2+10bcx-12c^2\cdots\right.\right.}{8a^{7/2}x\left(b^2-4ac\right)\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3 + c*x^4)^(-3/2),x]
```

```
[Out] -1/8*(2*Sqrt[a]*(-8*a^3*c - 15*b^3*x^2*(b + c*x) + 2*a^2*(b^2 + 10*b*c*x -
12*c^2*x^2) + a*b*x*(-5*b^2 + 62*b*c*x + 52*c^2*x^2)) + 3*(5*b^4 - 24*a*b^2
*c + 16*a^2*c^2)*x^2*Sqrt[a + x*(b + c*x)]*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*S
qrt[a + x*(b + c*x)])])/(a^(7/2)*(b^2 - 4*a*c)*x*Sqrt[x^2*(a + x*(b + c*x))
])
```

fricas [A] time = 0.92, size = 630, normalized size = 3.01

$$\left[-\frac{3\left(\left(5\,b^{4}c-24\,ab^{2}c^{2}+16\,a^{2}c^{3}\right)x^{5}+\left(5\,b^{5}-24\,ab^{3}c+16\,a^{2}bc^{2}\right)x^{4}+\left(5\,ab^{4}-24\,a^{2}b^{2}c+16\,a^{3}c^{2}\right)x^{3}\right)\sqrt{a}\,\log\left(-\frac{8a}{}\right.}{16\left(\left(a\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(3*((5*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*x^5 + (5*b^5 - 24*a*b^3*c
+ 16*a^2*b*c^2)*x^4 + (5*a*b^4 - 24*a^2*b^2*c + 16*a^3*c^2)*x^3)*sqrt(a)*lo
g(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x + 4*sqrt(c*x^4 + b*x^3 + a*x^2)
*(b*x + 2*a)*sqrt(a))/x^3) + 4*(2*a^3*b^2 - 8*a^4*c - (15*a*b^3*c - 52*a^2*
b*c^2)*x^3 - (15*a*b^4 - 62*a^2*b^2*c + 24*a^3*c^2)*x^2 - 5*(a^2*b^3 - 4*a^
3*b*c)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/((a^4*b^2*c - 4*a^5*c^2)*x^5 + (a^4*
b^3 - 4*a^5*b*c)*x^4 + (a^5*b^2 - 4*a^6*c)*x^3), 1/8*(3*((5*b^4*c - 24*a*b^
2*c^2 + 16*a^2*c^3)*x^5 + (5*b^5 - 24*a*b^3*c + 16*a^2*b*c^2)*x^4 + (5*a*b^
4 - 24*a^2*b^2*c + 16*a^3*c^2)*x^3)*sqrt(-a)*arctan(1/2*sqrt(c*x^4 + b*x^3
+ a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) - 2*(2*a^3*b^2 -
 8*a^4*c - (15*a*b^3*c - 52*a^2*b*c^2)*x^3 - (15*a*b^4 - 62*a^2*b^2*c + 24*
a^3*c^2)*x^2 - 5*(a^2*b^3 - 4*a^3*b*c)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/((a^
4*b^2*c - 4*a^5*c^2)*x^5 + (a^4*b^3 - 4*a^5*b*c)*x^4 + (a^5*b^2 - 4*a^6*c)*
x^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.01, size = 292, normalized size = 1.40

$$-\frac{\left(c\,x^{2}+bx+a\right)\left(-104a^{\frac{5}{2}}b\,c^{2}x^{3}+30a^{\frac{3}{2}}b^{3}c\,x^{3}-48\sqrt{c\,x^{2}+bx+a}\;a^{3}c^{2}x^{2}\ln\left(\frac{bx+2a+2\sqrt{c\,x^{2}+bx+a}\;\sqrt{a}}{x}\right)+72\sqrt{c\,x^{2}+b}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+b*x^3+a*x^2)^(3/2),x)
```

```
[Out] -1/8*x*(c*x^2+b*x+a)*(48*a^(7/2)*x^2*c^2-104*a^(5/2)*x^3*b*c^2+16*a^(9/2)*c
-40*a^(7/2)*x*b*c-124*a^(5/2)*x^2*b^2*c+30*a^(3/2)*x^3*b^3*c-4*a^(7/2)*b^2+
10*a^(5/2)*x*b^3+30*a^(3/2)*x^2*b^4-48*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^
(1/2))/x)*(c*x^2+b*x+a)^(1/2)*x^2*a^3*c^2+72*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1
/2)*a^(1/2))/x)*(c*x^2+b*x+a)^(1/2)*x^2*a^2*b^2*c-15*ln((b*x+2*a+2*(c*x^2+b
*x+a)^(1/2)*a^(1/2))/x)*(c*x^2+b*x+a)^(1/2)*x^2*a*b^4)/(c*x^4+b*x^3+a*x^2)^
(3/2)/a^(9/2)/(4*a*c-b^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(cx^{4}+bx^{3}+ax^{2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^3 + a*x^2)^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(c\,x^4+b\,x^3+a\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^2 + b*x^3 + c*x^4)^(3/2),x)
```

```
[Out] int(1/(a*x^2 + b*x^3 + c*x^4)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ax^2+bx^3+cx^4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral((a*x**2 + b*x**3 + c*x**4)**(-3/2), x)
```

3.63 $\int \frac{1}{x\left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=271

$$\frac{5b\left(7b^2-12ac\right)\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{16a^{9/2}}+\frac{b\left(35b^2-116ac\right)\sqrt{ax^2+bx^3+cx^4}}{12a^3x^3\left(b^2-4ac\right)}-\frac{\left(7b^2-16ac\right)\sqrt{ax^2+bx^3+cx^4}}{3a^2x^4\left(b^2-4ac\right)}-$$

```
[Out] 5/16*b*(-12*a*c+7*b^2)*arctanh(1/2*x*(b*x+2*a)/a^(1/2)/(c*x^4+b*x^3+a*x^2)^
(1/2))/a^(9/2)+2*(b*c*x-2*a*c+b^2)/a/(-4*a*c+b^2)/x^2/(c*x^4+b*x^3+a*x^2)^(
1/2)-1/3*(-16*a*c+7*b^2)*(c*x^4+b*x^3+a*x^2)^(1/2)/a^2/(-4*a*c+b^2)/x^4+1/1
2*b*(-116*a*c+35*b^2)*(c*x^4+b*x^3+a*x^2)^(1/2)/a^3/(-4*a*c+b^2)/x^3-1/24*(
256*a^2*c^2-460*a*b^2*c+105*b^4)*(c*x^4+b*x^3+a*x^2)^(1/2)/a^4/(-4*a*c+b^2)
/x^2
```

Rubi [A] time = 0.45, antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1924, 1951, 12, 1904, 206}

$$-\frac{\left(256a^2c^2-460ab^2c+105b^4\right)\sqrt{ax^2+bx^3+cx^4}}{24a^4x^2\left(b^2-4ac\right)}+\frac{b\left(35b^2-116ac\right)\sqrt{ax^2+bx^3+cx^4}}{12a^3x^3\left(b^2-4ac\right)}-\frac{\left(7b^2-16ac\right)\sqrt{ax^2+bx^3}}{3a^2x^4\left(b^2-4ac\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a*x^2 + b*x^3 + c*x^4)^(3/2)),x]
```

```
[Out] (2*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*x^2*Sqrt[a*x^2 + b*x^3 + c*x^4])
 - ((7*b^2 - 16*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(3*a^2*(b^2 - 4*a*c)*x^4)
 + (b*(35*b^2 - 116*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(12*a^3*(b^2 - 4*a*c)
*x^3) - ((105*b^4 - 460*a*b^2*c + 256*a^2*c^2)*Sqrt[a*x^2 + b*x^3 + c*x^4])
/(24*a^4*(b^2 - 4*a*c)*x^2) + (5*b*(7*b^2 - 12*a*c)*ArcTanh[(x*(2*a + b*x))
/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])])/(16*a^(9/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1904

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, (x*(2*a + b*x^(n - 2)))/
Sqrt[a*x^2 + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n
 - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1924

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> -Simp[(x^(m - q + 1)*(b^2 - 2*a*c + b*c*x^(n - q))*(a*x^q +
 b*x^n + c*x^(2*n - q))^(p + 1))/(a*(n - q)*(p + 1)*(b^2 - 4*a*c)), x] + Di
st[1/(a*(n - q)*(p + 1)*(b^2 - 4*a*c)), Int[x^(m - q)*(b^2*(m + p*q + (n -
```

```
q)*(p + 1) + 1) - 2*a*c*(m + p*q + 2*(n - q)*(p + 1) + 1) + b*c*(m + p*q +
(n - q)*(2*p + 3) + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p + 1),
x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && !Intege
rQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p, -1] && RationalQ[m, q]
 && LtQ[m + p*q + 1, n - q]
```

Rule 1951

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[(A*x^(m - q + 1)*(a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1))/(a*(m + p*q + 1)), x] + Dist[1/(a*(m + p*q +
 1)), Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p
+ 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b*
x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q
] && EqQ[j, 2*n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
&& RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n - q
)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]
```

Rubi steps

$$\int \frac{1}{x\left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx = \frac{2\left(b^2-2ac+bcx\right)}{a\left(b^2-4ac\right)x^2\sqrt{ax^2+bx^3+cx^4}} - \frac{2\int \frac{-\frac{7b^2}{2}+8ac-3bcx}{x^3\sqrt{ax^2+bx^3+cx^4}}\,dx}{a\left(b^2-4ac\right)}$$

$$= \frac{2\left(b^2-2ac+bcx\right)}{a\left(b^2-4ac\right)x^2\sqrt{ax^2+bx^3+cx^4}} - \frac{\left(7b^2-16ac\right)\sqrt{ax^2+bx^3+cx^4}}{3a^2\left(b^2-4ac\right)x^4} + \frac{2\int \frac{-\frac{1}{4}b(3}{}}{}$$

$$= \frac{2\left(b^2-2ac+bcx\right)}{a\left(b^2-4ac\right)x^2\sqrt{ax^2+bx^3+cx^4}} - \frac{\left(7b^2-16ac\right)\sqrt{ax^2+bx^3+cx^4}}{3a^2\left(b^2-4ac\right)x^4} + \frac{b\left(35b^2\right.}{}$$

$$= \frac{2\left(b^2-2ac+bcx\right)}{a\left(b^2-4ac\right)x^2\sqrt{ax^2+bx^3+cx^4}} - \frac{\left(7b^2-16ac\right)\sqrt{ax^2+bx^3+cx^4}}{3a^2\left(b^2-4ac\right)x^4} + \frac{b\left(35b^2\right.}{}$$

$$= \frac{2\left(b^2-2ac+bcx\right)}{a\left(b^2-4ac\right)x^2\sqrt{ax^2+bx^3+cx^4}} - \frac{\left(7b^2-16ac\right)\sqrt{ax^2+bx^3+cx^4}}{3a^2\left(b^2-4ac\right)x^4} + \frac{b\left(35b^2\right.}{}$$

$$= \frac{2\left(b^2-2ac+bcx\right)}{a\left(b^2-4ac\right)x^2\sqrt{ax^2+bx^3+cx^4}} - \frac{\left(7b^2-16ac\right)\sqrt{ax^2+bx^3+cx^4}}{3a^2\left(b^2-4ac\right)x^4} + \frac{b\left(35b^2\right.}{}$$

$$= \frac{2\left(b^2-2ac+bcx\right)}{a\left(b^2-4ac\right)x^2\sqrt{ax^2+bx^3+cx^4}} - \frac{\left(7b^2-16ac\right)\sqrt{ax^2+bx^3+cx^4}}{3a^2\left(b^2-4ac\right)x^4} + \frac{b\left(35b^2\right.}{}$$

Mathematica [A] time = 0.20, size = 225, normalized size = 0.83

$$\frac{2\sqrt{a}\left(-32a^4c+8a^3\left(b^2+7bcx+16c^2x^2\right)+2a^2x\left(-7b^3-86b^2cx+244bc^2x^2+128c^3x^3\right)+5ab^2x^2\left(7b^2-106b\right.}{48a^{9/2}x^2\left(4ac-b^2\right)\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a*x^2 + b*x^3 + c*x^4)^(3/2)),x]
```

```
[Out] (2*Sqrt[a]*(-32*a^4*c + 105*b^4*x^3*(b + c*x) + 5*a*b^2*x^2*(7*b^2 - 106*b*
c*x - 92*c^2*x^2) + 8*a^3*(b^2 + 7*b*c*x + 16*c^2*x^2) + 2*a^2*x*(-7*b^3 -
86*b^2*c*x + 244*b*c^2*x^2 + 128*c^3*x^3)) - 15*b*(7*b^4 - 40*a*b^2*c + 48*
a^2*c^2)*x^3*Sqrt[a + x*(b + c*x)]*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a +
x*(b + c*x)])])/(48*a^(9/2)*(-b^2 + 4*a*c)*x^2*Sqrt[x^2*(a + x*(b + c*x))])
```

fricas [A] time = 1.02, size = 716, normalized size = 2.64

$$\left[-\frac{15\left(\left(7\,b^5c-40\,ab^3c^2+48\,a^2bc^3\right)x^6+\left(7\,b^6-40\,ab^4c+48\,a^2b^2c^2\right)x^5+\left(7\,ab^5-40\,a^2b^3c+48\,a^3bc^2\right)x^4\right)\sqrt{a}\,\log\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(15*((7*b^5*c - 40*a*b^3*c^2 + 48*a^2*b*c^3)*x^6 + (7*b^6 - 40*a*b^4
*c + 48*a^2*b^2*c^2)*x^5 + (7*a*b^5 - 40*a^2*b^3*c + 48*a^3*b*c^2)*x^4)*sqr
t(a)*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3 +
 a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) + 4*(8*a^4*b^2 - 32*a^5*c + (105*a*b^4*c
- 460*a^2*b^2*c^2 + 256*a^3*c^3)*x^4 + (105*a*b^5 - 530*a^2*b^3*c + 488*a^3
*b*c^2)*x^3 + (35*a^2*b^4 - 172*a^3*b^2*c + 128*a^4*c^2)*x^2 - 14*(a^3*b^3
- 4*a^4*b*c)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/((a^5*b^2*c - 4*a^6*c^2)*x^6 +
 (a^5*b^3 - 4*a^6*b*c)*x^5 + (a^6*b^2 - 4*a^7*c)*x^4), -1/48*(15*((7*b^5*c
- 40*a*b^3*c^2 + 48*a^2*b*c^3)*x^6 + (7*b^6 - 40*a*b^4*c + 48*a^2*b^2*c^2)*
x^5 + (7*a*b^5 - 40*a^2*b^3*c + 48*a^3*b*c^2)*x^4)*sqrt(-a)*arctan(1/2*sqrt
(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) +
 2*(8*a^4*b^2 - 32*a^5*c + (105*a*b^4*c - 460*a^2*b^2*c^2 + 256*a^3*c^3)*x^
4 + (105*a*b^5 - 530*a^2*b^3*c + 488*a^3*b*c^2)*x^3 + (35*a^2*b^4 - 172*a^3
*b^2*c + 128*a^4*c^2)*x^2 - 14*(a^3*b^3 - 4*a^4*b*c)*x)*sqrt(c*x^4 + b*x^3
+ a*x^2))/((a^5*b^2*c - 4*a^6*c^2)*x^6 + (a^5*b^3 - 4*a^6*b*c)*x^5 + (a^6*b
^2 - 4*a^7*c)*x^4)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^4+bx^3+ax^2\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^4 + b*x^3 + a*x^2)^(3/2)*x), x)
```

maple [A] time = 0.01, size = 340, normalized size = 1.25

$$-\frac{\left(c\,x^2+bx+a\right)\left(-512a^{\frac{7}{2}}c^3x^4+920a^{\frac{5}{2}}b^2c^2x^4-210a^{\frac{3}{2}}b^4c\,x^4+720\sqrt{c\,x^2+bx+a}\;a^3b\,c^2x^3\ln\left(\frac{bx+2a+2\sqrt{c\,x^2+bx+a}\,\sqrt{}}{x}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^4+b*x^3+a*x^2)^(3/2),x)
```

```
[Out] -1/48*(c*x^2+b*x+a)*(-512*a^(7/2)*x^4*c^3-256*a^(9/2)*x^2*c^2-976*a^(7/2)*x
^3*b*c^2+920*a^(5/2)*x^4*b^2*c^2+64*a^(11/2)*c-112*a^(9/2)*x*b*c+344*a^(7/2
)*x^2*b^2*c+1060*a^(5/2)*x^3*b^3*c-210*a^(3/2)*x^4*b^4*c-16*a^(9/2)*b^2+28*
a^(7/2)*x*b^3-70*a^(5/2)*x^2*b^4-210*a^(3/2)*x^3*b^5+720*ln((b*x+2*a+2*(c*x
^2+b*x+a)^(1/2)*a^(1/2))/x)*(c*x^2+b*x+a)^(1/2)*x^3*a^3*b*c^2-600*ln((b*x+2
```

```
*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)*(c*x^2+b*x+a)^(1/2)*x^3*a^2*b^3*c+105*
ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)*(c*x^2+b*x+a)^(1/2)*x^3*a*b^5
)/(c*x^4+b*x^3+a*x^2)^(3/2)/a^(11/2)/(4*a*c-b^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^4 + bx^3 + ax^2\right)^{\frac{3}{2}} x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((c*x^4 + b*x^3 + a*x^2)^(3/2)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x\left(c\,x^4 + b\,x^3 + a\,x^2\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a*x^2 + b*x^3 + c*x^4)^(3/2)),x)
[Out] int(1/(x*(a*x^2 + b*x^3 + c*x^4)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(x^2\left(a + bx + cx^2\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**4+b*x**3+a*x**2)**(3/2),x)
[Out] Integral(1/(x*(x**2*(a + b*x + c*x**2))**(3/2)), x)
```

3.64 $\int \frac{1}{x^2\left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=343

$$\frac{b\left(21b^2-68ac\right)\sqrt{ax^2+bx^3+cx^4}}{8a^3x^4\left(b^2-4ac\right)}-\frac{\left(9b^2-20ac\right)\sqrt{ax^2+bx^3+cx^4}}{4a^2x^5\left(b^2-4ac\right)}-\frac{15\left(16a^2c^2-56ab^2c+21b^4\right)\tanh^{-1}\left(\frac{x(2}{2\sqrt{a}\sqrt{a}}\right.}{128a^{11/2}}$$

```
[Out] -15/128*(16*a^2*c^2-56*a*b^2*c+21*b^4)*arctanh(1/2*x*(b*x+2*a)/a^(1/2)/(c*x
^4+b*x^3+a*x^2)^(1/2))/a^(11/2)+2*(b*c*x-2*a*c+b^2)/a/(-4*a*c+b^2)/x^3/(c*x
^4+b*x^3+a*x^2)^(1/2)-1/4*(-20*a*c+9*b^2)*(c*x^4+b*x^3+a*x^2)^(1/2)/a^2/(-4
*a*c+b^2)/x^5+1/8*b*(-68*a*c+21*b^2)*(c*x^4+b*x^3+a*x^2)^(1/2)/a^3/(-4*a*c+
b^2)/x^4-1/32*(240*a^2*c^2-448*a*b^2*c+105*b^4)*(c*x^4+b*x^3+a*x^2)^(1/2)/a
^4/(-4*a*c+b^2)/x^3+1/64*b*(1808*a^2*c^2-1680*a*b^2*c+315*b^4)*(c*x^4+b*x^3
+a*x^2)^(1/2)/a^5/(-4*a*c+b^2)/x^2
```

Rubi [A] time = 0.62, antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1924, 1951, 12, 1904, 206}

$$\frac{b\left(1808a^2c^2-1680ab^2c+315b^4\right)\sqrt{ax^2+bx^3+cx^4}}{64a^5x^2\left(b^2-4ac\right)}-\frac{\left(240a^2c^2-448ab^2c+105b^4\right)\sqrt{ax^2+bx^3+cx^4}}{32a^4x^3\left(b^2-4ac\right)}-\frac{15\left(16a^2c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a*x^2 + b*x^3 + c*x^4)^(3/2)),x]
```

```
[Out] (2*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*x^3*Sqrt[a*x^2 + b*x^3 + c*x^4])
 - ((9*b^2 - 20*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(4*a^2*(b^2 - 4*a*c)*x^5)
 + (b*(21*b^2 - 68*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(8*a^3*(b^2 - 4*a*c)*x
^4) - ((105*b^4 - 448*a*b^2*c + 240*a^2*c^2)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(
32*a^4*(b^2 - 4*a*c)*x^3) + (b*(315*b^4 - 1680*a*b^2*c + 1808*a^2*c^2)*Sqrt
[a*x^2 + b*x^3 + c*x^4])/(64*a^5*(b^2 - 4*a*c)*x^2) - (15*(21*b^4 - 56*a*b^
2*c + 16*a^2*c^2)*ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c
*x^4])])/(128*a^(11/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1904

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, (x*(2*a + b*x^(n - 2)))/
Sqrt[a*x^2 + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n
 - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1924

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> -Simp[(x^(m - q + 1)*(b^2 - 2*a*c + b*c*x^(n - q))*(a*x^q +
```

```
 b*x^n + c*x^(2*n - q))^(p + 1))/(a*(n - q)*(p + 1)*(b^2 - 4*a*c)), x] + Di
st[1/(a*(n - q)*(p + 1)*(b^2 - 4*a*c)), Int[x^(m - q)*(b^2*(m + p*q + (n -
q)*(p + 1) + 1) - 2*a*c*(m + p*q + 2*(n - q)*(p + 1) + 1) + b*c*(m + p*q +
(n - q)*(2*p + 3) + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p + 1),
x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] &&  !Intege
rQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p, -1] && RationalQ[m, q]
 && LtQ[m + p*q + 1, n - q]
```

Rule 1951

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[(A*x^(m - q + 1)*(a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1))/(a*(m + p*q + 1)), x] + Dist[1/(a*(m + p*q +
 1)), Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p
+ 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b*
x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q
] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
&& RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n - q
)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]
```

Rubi steps

$$\int \frac{1}{x^2\left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx = \frac{2\left(b^2-2ac+bcx\right)}{a\left(b^2-4ac\right)x^3\sqrt{ax^2+bx^3+cx^4}} - \frac{2\int \frac{-\frac{9b^2}{2}+10ac-4bcx}{x^4\sqrt{ax^2+bx^3+cx^4}}\,dx}{a\left(b^2-4ac\right)}$$

$$= \frac{2\left(b^2-2ac+bcx\right)}{a\left(b^2-4ac\right)x^3\sqrt{ax^2+bx^3+cx^4}} - \frac{\left(9b^2-20ac\right)\sqrt{ax^2+bx^3+cx^4}}{4a^2\left(b^2-4ac\right)x^5} + \int \frac{-\frac{3}{4}b(2$$

$$= \frac{2\left(b^2-2ac+bcx\right)}{a\left(b^2-4ac\right)x^3\sqrt{ax^2+bx^3+cx^4}} - \frac{\left(9b^2-20ac\right)\sqrt{ax^2+bx^3+cx^4}}{4a^2\left(b^2-4ac\right)x^5} + \frac{b\left(21b^2\right.}{}$$

$$= \frac{2\left(b^2-2ac+bcx\right)}{a\left(b^2-4ac\right)x^3\sqrt{ax^2+bx^3+cx^4}} - \frac{\left(9b^2-20ac\right)\sqrt{ax^2+bx^3+cx^4}}{4a^2\left(b^2-4ac\right)x^5} + \frac{b\left(21b^2\right.}{}$$

$$= \frac{2\left(b^2-2ac+bcx\right)}{a\left(b^2-4ac\right)x^3\sqrt{ax^2+bx^3+cx^4}} - \frac{\left(9b^2-20ac\right)\sqrt{ax^2+bx^3+cx^4}}{4a^2\left(b^2-4ac\right)x^5} + \frac{b\left(21b^2\right.}{}$$

$$= \frac{2\left(b^2-2ac+bcx\right)}{a\left(b^2-4ac\right)x^3\sqrt{ax^2+bx^3+cx^4}} - \frac{\left(9b^2-20ac\right)\sqrt{ax^2+bx^3+cx^4}}{4a^2\left(b^2-4ac\right)x^5} + \frac{b\left(21b^2\right.}{}$$

$$= \frac{2\left(b^2-2ac+bcx\right)}{a\left(b^2-4ac\right)x^3\sqrt{ax^2+bx^3+cx^4}} - \frac{\left(9b^2-20ac\right)\sqrt{ax^2+bx^3+cx^4}}{4a^2\left(b^2-4ac\right)x^5} + \frac{b\left(21b^2\right.}{}$$

$$= \frac{2\left(b^2-2ac+bcx\right)}{a\left(b^2-4ac\right)x^3\sqrt{ax^2+bx^3+cx^4}} - \frac{\left(9b^2-20ac\right)\sqrt{ax^2+bx^3+cx^4}}{4a^2\left(b^2-4ac\right)x^5} + \frac{b\left(21b^2\right.}{}$$

Mathematica [A] time = 0.25, size = 272, normalized size = 0.79

$$15x^4\left(-64a^3c^3+240a^2b^2c^2-140ab^4c+21b^6\right)\sqrt{a+x(b+cx)}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right)-2\sqrt{a}\left(64a^5c-16a^4\left(b\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a*x^2 + b*x^3 + c*x^4)^(3/2)),x]
```

```
[Out] (-2*Sqrt[a]*(64*a^5*c + 315*b^5*x^4*(b + c*x) + 105*a*b^3*x^3*(b^2 - 18*b*c
*x - 16*c^2*x^2) - 16*a^4*(b^2 + 6*b*c*x + 10*c^2*x^2) + 8*a^3*x*(3*b^3 + 2
6*b^2*c*x + 98*b*c^2*x^2 - 60*c^3*x^3) + 2*a^2*b*x^2*(-21*b^3 - 308*b^2*c*x
 + 1352*b*c^2*x^2 + 904*c^3*x^3)) + 15*(21*b^6 - 140*a*b^4*c + 240*a^2*b^2*
c^2 - 64*a^3*c^3)*x^4*Sqrt[a + x*(b + c*x)]*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*
Sqrt[a + x*(b + c*x)])])/(128*a^(11/2)*(-b^2 + 4*a*c)*x^3*Sqrt[x^2*(a + x*(
b + c*x))])
```

fricas [A] time = 1.31, size = 866, normalized size = 2.52

$$\left[\frac{15\left(\left(21\,b^{6}c-140\,ab^{4}c^{2}+240\,a^{2}b^{2}c^{3}-64\,a^{3}c^{4}\right)x^{7}+\left(21\,b^{7}-140\,ab^{5}c+240\,a^{2}b^{3}c^{2}-64\,a^{3}bc^{3}\right)x^{6}+\left(21\,ab^{6}-140\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/256*(15*((21*b^6*c - 140*a*b^4*c^2 + 240*a^2*b^2*c^3 - 64*a^3*c^4)*x^7 +
 (21*b^7 - 140*a*b^5*c + 240*a^2*b^3*c^2 - 64*a^3*b*c^3)*x^6 + (21*a*b^6 -
140*a^2*b^4*c + 240*a^3*b^2*c^2 - 64*a^4*c^3)*x^5)*sqrt(a)*log(-(8*a*b*x^2
+ (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*s
qrt(a))/x^3) - 4*(16*a^5*b^2 - 64*a^6*c - (315*a*b^5*c - 1680*a^2*b^3*c^2 +
 1808*a^3*b*c^3)*x^5 - (315*a*b^6 - 1890*a^2*b^4*c + 2704*a^3*b^2*c^2 - 480
*a^4*c^3)*x^4 - 7*(15*a^2*b^5 - 88*a^3*b^3*c + 112*a^4*b*c^2)*x^3 + 2*(21*a
^3*b^4 - 104*a^4*b^2*c + 80*a^5*c^2)*x^2 - 24*(a^4*b^3 - 4*a^5*b*c)*x)*sqrt
(c*x^4 + b*x^3 + a*x^2))/((a^6*b^2*c - 4*a^7*c^2)*x^7 + (a^6*b^3 - 4*a^7*b*
c)*x^6 + (a^7*b^2 - 4*a^8*c)*x^5), 1/128*(15*((21*b^6*c - 140*a*b^4*c^2 + 2
40*a^2*b^2*c^3 - 64*a^3*c^4)*x^7 + (21*b^7 - 140*a*b^5*c + 240*a^2*b^3*c^2
- 64*a^3*b*c^3)*x^6 + (21*a*b^6 - 140*a^2*b^4*c + 240*a^3*b^2*c^2 - 64*a^4*
c^3)*x^5)*sqrt(-a)*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(
-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) - 2*(16*a^5*b^2 - 64*a^6*c - (315*a*b^5*c
- 1680*a^2*b^3*c^2 + 1808*a^3*b*c^3)*x^5 - (315*a*b^6 - 1890*a^2*b^4*c + 27
04*a^3*b^2*c^2 - 480*a^4*c^3)*x^4 - 7*(15*a^2*b^5 - 88*a^3*b^3*c + 112*a^4*
b*c^2)*x^3 + 2*(21*a^3*b^4 - 104*a^4*b^2*c + 80*a^5*c^2)*x^2 - 24*(a^4*b^3
- 4*a^5*b*c)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/((a^6*b^2*c - 4*a^7*c^2)*x^7 +
 (a^6*b^3 - 4*a^7*b*c)*x^6 + (a^7*b^2 - 4*a^8*c)*x^5)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^{4}+bx^{3}+ax^{2}\right)^{\frac{3}{2}}x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^4 + b*x^3 + a*x^2)^(3/2)*x^2), x)
```

maple [A] time = 0.01, size = 446, normalized size = 1.30

$$-\frac{\left(c\,x^{2}+bx+a\right)\left(3616a^{\frac{7}{2}}b\,c^{3}x^{5}-3360a^{\frac{5}{2}}b^{3}c^{2}x^{5}+630a^{\frac{3}{2}}b^{5}c\,x^{5}+960\sqrt{c\,x^{2}+bx+a}\;a^{4}c^{3}x^{4}\ln\left(\frac{bx+2a+2\sqrt{c\,x^{2}+bx+a}}{x}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^4+b*x^3+a*x^2)^(3/2),x)
```

```
[Out] -1/128/x*(c*x^2+b*x+a)*(3616*a^(7/2)*x^5*b*c^3-3360*a^(5/2)*x^5*b^3*c^2+630
*a^(3/2)*x^5*b^5*c+960*(c*x^2+b*x+a)^(1/2)*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2
)*a^(1/2))/x)*x^4*a^4*c^3-3600*(c*x^2+b*x+a)^(1/2)*ln((b*x+2*a+2*(c*x^2+b*x
+a)^(1/2)*a^(1/2))/x)*x^4*a^3*b^2*c^2+2100*(c*x^2+b*x+a)^(1/2)*ln((b*x+2*a+
2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)*x^4*a^2*b^4*c-315*(c*x^2+b*x+a)^(1/2)*ln(
(b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)*x^4*a*b^6-960*a^(9/2)*x^4*c^3+54
08*a^(7/2)*x^4*b^2*c^2-3780*a^(5/2)*x^4*b^4*c+630*a^(3/2)*x^4*b^6+1568*a^(9
/2)*x^3*b*c^2-1232*a^(7/2)*x^3*b^3*c+210*a^(5/2)*x^3*b^5-320*a^(11/2)*x^2*c
^2+416*a^(9/2)*x^2*b^2*c-84*a^(7/2)*x^2*b^4-192*a^(11/2)*x*b*c+48*a^(9/2)*x
*b^3+128*a^(13/2)*c-32*a^(11/2)*b^2)/(c*x^4+b*x^3+a*x^2)^(3/2)/a^(13/2)/(4*
a*c-b^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^4 + bx^3 + ax^2\right)^{\frac{3}{2}} x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^4 + b*x^3 + a*x^2)^(3/2)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2 \left(c\, x^4 + b\, x^3 + a\, x^2\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a*x^2 + b*x^3 + c*x^4)^(3/2)),x)
```

```
[Out] int(1/(x^2*(a*x^2 + b*x^3 + c*x^4)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \left(x^2 \left(a + bx + cx^2\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**4+b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral(1/(x**2*(x**2*(a + b*x + c*x**2))**(3/2)), x)
```

3.65 $\int x^m \left(ax + bx^3 + cx^5\right) dx$

Optimal. Leaf size=37

$$\frac{ax^{m+2}}{m+2} + \frac{bx^{m+4}}{m+4} + \frac{cx^{m+6}}{m+6}$$

```
[Out] a*x^(2+m)/(2+m)+b*x^(4+m)/(4+m)+c*x^(6+m)/(6+m)
```

Rubi [A] time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {14}

$$\frac{ax^{m+2}}{m+2} + \frac{bx^{m+4}}{m+4} + \frac{cx^{m+6}}{m+6}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a*x + b*x^3 + c*x^5),x]
```

```
[Out] (a*x^(2 + m))/(2 + m) + (b*x^(4 + m))/(4 + m) + (c*x^(6 + m))/(6 + m)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \int x^m \left(ax + bx^3 + cx^5\right) dx &= \int \left(ax^{1+m} + bx^{3+m} + cx^{5+m}\right) dx \\ &= \frac{ax^{2+m}}{2+m} + \frac{bx^{4+m}}{4+m} + \frac{cx^{6+m}}{6+m} \end{aligned}$$

Mathematica [A] time = 0.03, size = 34, normalized size = 0.92

$$x^{m+2} \left(\frac{a}{m+2} + \frac{bx^2}{m+4} + \frac{cx^4}{m+6}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a*x + b*x^3 + c*x^5),x]
```

```
[Out] x^(2 + m)*(a/(2 + m) + (b*x^2)/(4 + m) + (c*x^4)/(6 + m))
```

fricas [A] time = 0.66, size = 71, normalized size = 1.92

$$\frac{\left(\left(cm^2 + 6\,cm + 8\,c\right)x^6 + \left(bm^2 + 8\,bm + 12\,b\right)x^4 + \left(am^2 + 10\,am + 24\,a\right)x^2\right)x^m}{m^3 + 12\,m^2 + 44\,m + 48}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c*x^5+b*x^3+a*x),x, algorithm="fricas")
```

```
[Out] ((c*m^2 + 6*c*m + 8*c)*x^6 + (b*m^2 + 8*b*m + 12*b)*x^4 + (a*m^2 + 10*a*m +
 24*a)*x^2)*x^m/(m^3 + 12*m^2 + 44*m + 48)
```

giac [B] time = 0.48, size = 107, normalized size = 2.89

$$\frac{cm^2x^6x^m + 6\,cmx^6x^m + bm^2x^4x^m + 8\,cx^6x^m + 8\,bmx^4x^m + am^2x^2x^m + 12\,bx^4x^m + 10\,amx^2x^m + 24\,ax^2x^m}{m^3 + 12\,m^2 + 44\,m + 48}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c*x^5+b*x^3+a*x),x, algorithm="giac")
```

```
[Out] (c*m^2*x^6*x^m + 6*c*m*x^6*x^m + b*m^2*x^4*x^m + 8*c*x^6*x^m + 8*b*m*x^4*x^
m + a*m^2*x^2*x^m + 12*b*x^4*x^m + 10*a*m*x^2*x^m + 24*a*x^2*x^m)/(m^3 + 12
*m^2 + 44*m + 48)
```

maple [B] time = 0.00, size = 77, normalized size = 2.08

$$\frac{\left(c\,m^2x^4+6cm\,x^4+b\,m^2x^2+8c\,x^4+8bm\,x^2+a\,m^2+12b\,x^2+10am+24a\right)x^{m+2}}{(m+6)\,(m+4)\,(m+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c*x^5+b*x^3+a*x),x)
```

```
[Out] x^(m+2)*(c*m^2*x^4+6*c*m*x^4+b*m^2*x^2+8*c*x^4+8*b*m*x^2+a*m^2+12*b*x^2+10*
a*m+24*a)/(m+6)/(m+4)/(m+2)
```

maxima [A] time = 0.45, size = 37, normalized size = 1.00

$$\frac{cx^{m+6}}{m+6}+\frac{bx^{m+4}}{m+4}+\frac{ax^{m+2}}{m+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c*x^5+b*x^3+a*x),x, algorithm="maxima")
```

```
[Out] c*x^(m + 6)/(m + 6) + b*x^(m + 4)/(m + 4) + a*x^(m + 2)/(m + 2)
```

mupad [B] time = 2.08, size = 89, normalized size = 2.41

$$x^m\left(\frac{a\,x^2\,\left(m^2+10\,m+24\right)}{m^3+12\,m^2+44\,m+48}+\frac{b\,x^4\,\left(m^2+8\,m+12\right)}{m^3+12\,m^2+44\,m+48}+\frac{c\,x^6\,\left(m^2+6\,m+8\right)}{m^3+12\,m^2+44\,m+48}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a*x + b*x^3 + c*x^5),x)
```

```
[Out] x^m*((a*x^2*(10*m + m^2 + 24))/(44*m + 12*m^2 + m^3 + 48) + (b*x^4*(8*m + m
^2 + 12))/(44*m + 12*m^2 + m^3 + 48) + (c*x^6*(6*m + m^2 + 8))/(44*m + 12*m
^2 + m^3 + 48))
```

sympy [A] time = 1.18, size = 280, normalized size = 7.57

$$\begin{cases} -\frac{a}{4x^4}-\frac{b}{2x^2}+c\log{(x)} \\ -\frac{a}{2x^2}+b\log{(x)}+\frac{cx^2}{2} \\ a\log{(x)}+\frac{bx^2}{2}+\frac{cx^4}{4} \\ \frac{am^2x^2x^m}{m^3+12m^2+44m+48}+\frac{10amx^2x^m}{m^3+12m^2+44m+48}+\frac{24ax^2x^m}{m^3+12m^2+44m+48}+\frac{bm^2x^4x^m}{m^3+12m^2+44m+48}+\frac{8bmx^4x^m}{m^3+12m^2+44m+48}+\frac{12bx^4x^m}{m^3+12m^2+44m+48}+ \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(c*x**5+b*x**3+a*x),x)
```

```
[Out] Piecewise((-a/(4*x**4) - b/(2*x**2) + c*log(x), Eq(m, -6)), (-a/(2*x**2) +
b*log(x) + c*x**2/2, Eq(m, -4)), (a*log(x) + b*x**2/2 + c*x**4/4, Eq(m, -2)
), (a*m**2*x**2*x**m/(m**3 + 12*m**2 + 44*m + 48) + 10*a*m*x**2*x**m/(m**3
```

```
+ 12*m**2 + 44*m + 48) + 24*a*x**2*x**m/(m**3 + 12*m**2 + 44*m + 48) + b*m*
*2*x**4*x**m/(m**3 + 12*m**2 + 44*m + 48) + 8*b*m*x**4*x**m/(m**3 + 12*m**2
 + 44*m + 48) + 12*b*x**4*x**m/(m**3 + 12*m**2 + 44*m + 48) + c*m**2*x**6*x
**m/(m**3 + 12*m**2 + 44*m + 48) + 6*c*m*x**6*x**m/(m**3 + 12*m**2 + 44*m +
 48) + 8*c*x**6*x**m/(m**3 + 12*m**2 + 44*m + 48), True))
```

3.66 $\int x^2 \left(ax + bx^3 + cx^5\right) dx$

Optimal. Leaf size=25

$$\frac{ax^4}{4} + \frac{bx^6}{6} + \frac{cx^8}{8}$$

```
[Out] 1/4*a*x^4+1/6*b*x^6+1/8*c*x^8
```

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {14}

$$\frac{ax^4}{4} + \frac{bx^6}{6} + \frac{cx^8}{8}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a*x + b*x^3 + c*x^5),x]
```

```
[Out] (a*x^4)/4 + (b*x^6)/6 + (c*x^8)/8
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \int x^2 \left(ax + bx^3 + cx^5\right) dx &= \int \left(ax^3 + bx^5 + cx^7\right) dx \\ &= \frac{ax^4}{4} + \frac{bx^6}{6} + \frac{cx^8}{8} \end{aligned}$$

Mathematica [A] time = 0.00, size = 25, normalized size = 1.00

$$\frac{ax^4}{4} + \frac{bx^6}{6} + \frac{cx^8}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a*x + b*x^3 + c*x^5),x]
```

```
[Out] (a*x^4)/4 + (b*x^6)/6 + (c*x^8)/8
```

fricas [A] time = 0.46, size = 19, normalized size = 0.76

$$\frac{1}{8}x^8c + \frac{1}{6}x^6b + \frac{1}{4}x^4a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^5+b*x^3+a*x),x, algorithm="fricas")
```

```
[Out] 1/8*x^8*c + 1/6*x^6*b + 1/4*x^4*a
```

giac [A] time = 0.38, size = 19, normalized size = 0.76

$$\frac{1}{8}\,cx^8 + \frac{1}{6}\,bx^6 + \frac{1}{4}\,ax^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^5+b*x^3+a*x),x, algorithm="giac")
```

```
[Out] 1/8*c*x^8 + 1/6*b*x^6 + 1/4*a*x^4
```

maple [A] time = 0.00, size = 20, normalized size = 0.80

$$\frac{1}{8}c\,x^{8}+\frac{1}{6}b\,x^{6}+\frac{1}{4}a\,x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^5+b*x^3+a*x),x)
```

```
[Out] 1/4*a*x^4+1/6*b*x^6+1/8*c*x^8
```

maxima [A] time = 0.43, size = 19, normalized size = 0.76

$$\frac{1}{8}\,cx^{8}+\frac{1}{6}\,bx^{6}+\frac{1}{4}\,ax^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^5+b*x^3+a*x),x, algorithm="maxima")
```

```
[Out] 1/8*c*x^8 + 1/6*b*x^6 + 1/4*a*x^4
```

mupad [B] time = 0.03, size = 19, normalized size = 0.76

$$\frac{c\,x^{8}}{8}+\frac{b\,x^{6}}{6}+\frac{a\,x^{4}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a*x + b*x^3 + c*x^5),x)
```

```
[Out] (a*x^4)/4 + (b*x^6)/6 + (c*x^8)/8
```

sympy [A] time = 0.06, size = 19, normalized size = 0.76

$$\frac{ax^{4}}{4}+\frac{bx^{6}}{6}+\frac{cx^{8}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**5+b*x**3+a*x),x)
```

```
[Out] a*x**4/4 + b*x**6/6 + c*x**8/8
```

3.67 $\int x\left(ax+bx^3+cx^5\right)\,dx$

Optimal. Leaf size=25

$$\frac{ax^3}{3}+\frac{bx^5}{5}+\frac{cx^7}{7}$$

```
[Out] 1/3*a*x^3+1/5*b*x^5+1/7*c*x^7
```

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {14}

$$\frac{ax^3}{3}+\frac{bx^5}{5}+\frac{cx^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a*x + b*x^3 + c*x^5),x]
```

```
[Out] (a*x^3)/3 + (b*x^5)/5 + (c*x^7)/7
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x\left(ax+bx^3+cx^5\right)\,dx &= \int \left(ax^2+bx^4+cx^6\right)\,dx\\ &= \frac{ax^3}{3}+\frac{bx^5}{5}+\frac{cx^7}{7}\end{aligned}$$

Mathematica [A] time = 0.00, size = 25, normalized size = 1.00

$$\frac{ax^3}{3}+\frac{bx^5}{5}+\frac{cx^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a*x + b*x^3 + c*x^5),x]
```

```
[Out] (a*x^3)/3 + (b*x^5)/5 + (c*x^7)/7
```

fricas [A] time = 0.52, size = 19, normalized size = 0.76

$$\frac{1}{7}x^7c+\frac{1}{5}x^5b+\frac{1}{3}x^3a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^5+b*x^3+a*x),x, algorithm="fricas")
```

```
[Out] 1/7*x^7*c + 1/5*x^5*b + 1/3*x^3*a
```

giac [A] time = 0.39, size = 19, normalized size = 0.76

$$\frac{1}{7}\,cx^7+\frac{1}{5}\,bx^5+\frac{1}{3}\,ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^5+b*x^3+a*x),x, algorithm="giac")
```

```
[Out] 1/7*c*x^7 + 1/5*b*x^5 + 1/3*a*x^3
```

maple [A] time = 0.00, size = 20, normalized size = 0.80

$$\frac{1}{7}c\,x^{7}+\frac{1}{5}b\,x^{5}+\frac{1}{3}a\,x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^5+b*x^3+a*x),x)
```

```
[Out] 1/3*a*x^3+1/5*b*x^5+1/7*c*x^7
```

maxima [A] time = 0.43, size = 19, normalized size = 0.76

$$\frac{1}{7}\,cx^{7}+\frac{1}{5}\,bx^{5}+\frac{1}{3}\,ax^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^5+b*x^3+a*x),x, algorithm="maxima")
```

```
[Out] 1/7*c*x^7 + 1/5*b*x^5 + 1/3*a*x^3
```

mupad [B] time = 0.03, size = 19, normalized size = 0.76

$$\frac{c\,x^{7}}{7}+\frac{b\,x^{5}}{5}+\frac{a\,x^{3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a*x + b*x^3 + c*x^5),x)
```

```
[Out] (a*x^3)/3 + (b*x^5)/5 + (c*x^7)/7
```

sympy [A] time = 0.06, size = 19, normalized size = 0.76

$$\frac{ax^{3}}{3}+\frac{bx^{5}}{5}+\frac{cx^{7}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**5+b*x**3+a*x),x)
```

```
[Out] a*x**3/3 + b*x**5/5 + c*x**7/7
```

3.68 $\int \left(ax + bx^3 + cx^5\right) \, dx$

Optimal. Leaf size=25

$$\frac{ax^2}{2} + \frac{bx^4}{4} + \frac{cx^6}{6}$$

```
[Out] 1/2*a*x^2+1/4*b*x^4+1/6*c*x^6
```

Rubi [A] time = 0.00, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\frac{ax^2}{2} + \frac{bx^4}{4} + \frac{cx^6}{6}$$

Antiderivative was successfully verified.

```
[In] Int[a*x + b*x^3 + c*x^5,x]
```

```
[Out] (a*x^2)/2 + (b*x^4)/4 + (c*x^6)/6
```

Rubi steps

$$\int \left(ax + bx^3 + cx^5\right) \, dx = \frac{ax^2}{2} + \frac{bx^4}{4} + \frac{cx^6}{6}$$

Mathematica [A] time = 0.00, size = 25, normalized size = 1.00

$$\frac{ax^2}{2} + \frac{bx^4}{4} + \frac{cx^6}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[a*x + b*x^3 + c*x^5,x]
```

```
[Out] (a*x^2)/2 + (b*x^4)/4 + (c*x^6)/6
```

fricas [A] time = 0.61, size = 19, normalized size = 0.76

$$\frac{1}{6} x^6 c + \frac{1}{4} x^4 b + \frac{1}{2} x^2 a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x^5+b*x^3+a*x,x, algorithm="fricas")
```

```
[Out] 1/6*x^6*c + 1/4*x^4*b + 1/2*x^2*a
```

giac [A] time = 0.38, size = 19, normalized size = 0.76

$$\frac{1}{6} cx^6 + \frac{1}{4} bx^4 + \frac{1}{2} ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x^5+b*x^3+a*x,x, algorithm="giac")
```

```
[Out] 1/6*c*x^6 + 1/4*b*x^4 + 1/2*a*x^2
```

maple [A] time = 0.00, size = 20, normalized size = 0.80

$$\frac{1}{6} c\, x^6 + \frac{1}{4} b\, x^4 + \frac{1}{2} a\, x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(c*x^5+b*x^3+a*x,x)
```

```
[Out] 1/2*a*x^2+1/4*b*x^4+1/6*c*x^6
```

maxima [A] time = 0.42, size = 19, normalized size = 0.76

$$\frac{1}{6}\,cx^6 + \frac{1}{4}\,bx^4 + \frac{1}{2}\,ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x^5+b*x^3+a*x,x, algorithm="maxima")
```

```
[Out] 1/6*c*x^6 + 1/4*b*x^4 + 1/2*a*x^2
```

mupad [B] time = 0.03, size = 19, normalized size = 0.76

$$\frac{c\,x^6}{6} + \frac{b\,x^4}{4} + \frac{a\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a*x + b*x^3 + c*x^5,x)
```

```
[Out] (a*x^2)/2 + (b*x^4)/4 + (c*x^6)/6
```

sympy [A] time = 0.06, size = 19, normalized size = 0.76

$$\frac{ax^2}{2} + \frac{bx^4}{4} + \frac{cx^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x**5+b*x**3+a*x,x)
```

```
[Out] a*x**2/2 + b*x**4/4 + c*x**6/6
```

3.69 $\int \frac{ax+bx^3+cx^5}{x} \, dx$

Optimal. Leaf size=20

$$ax + \frac{bx^3}{3} + \frac{cx^5}{5}$$

```
[Out] a*x+1/3*b*x^3+1/5*c*x^5
```

Rubi [A] time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {14}

$$ax + \frac{bx^3}{3} + \frac{cx^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^3 + c*x^5)/x,x]
```

```
[Out] a*x + (b*x^3)/3 + (c*x^5)/5
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \int \frac{ax + bx^3 + cx^5}{x} \, dx &= \int \left(a + bx^2 + cx^4\right) \, dx \\ &= ax + \frac{bx^3}{3} + \frac{cx^5}{5} \end{aligned}$$

Mathematica [A] time = 0.00, size = 20, normalized size = 1.00

$$ax + \frac{bx^3}{3} + \frac{cx^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^3 + c*x^5)/x,x]
```

```
[Out] a*x + (b*x^3)/3 + (c*x^5)/5
```

fricas [A] time = 0.56, size = 16, normalized size = 0.80

$$\frac{1}{5}\,cx^5 + \frac{1}{3}\,bx^3 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)/x,x, algorithm="fricas")
```

```
[Out] 1/5*c*x^5 + 1/3*b*x^3 + a*x
```

giac [A] time = 0.36, size = 16, normalized size = 0.80

$$\frac{1}{5}\,cx^5 + \frac{1}{3}\,bx^3 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)/x,x, algorithm="giac")
```

```
[Out] 1/5*c*x^5 + 1/3*b*x^3 + a*x
```

maple [A] time = 0.00, size = 17, normalized size = 0.85

$$\frac{1}{5}c\,x^{5}+\frac{1}{3}b\,x^{3}+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^5+b*x^3+a*x)/x,x)
```

```
[Out] a*x+1/3*b*x^3+1/5*c*x^5
```

maxima [A] time = 0.42, size = 16, normalized size = 0.80

$$\frac{1}{5}\,cx^{5}+\frac{1}{3}\,bx^{3}+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)/x,x, algorithm="maxima")
```

```
[Out] 1/5*c*x^5 + 1/3*b*x^3 + a*x
```

mupad [B] time = 0.03, size = 16, normalized size = 0.80

$$\frac{c\,x^5}{5}+\frac{b\,x^3}{3}+a\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^3 + c*x^5)/x,x)
```

```
[Out] a*x + (b*x^3)/3 + (c*x^5)/5
```

sympy [A] time = 0.07, size = 15, normalized size = 0.75

$$ax+\frac{bx^{3}}{3}+\frac{cx^{5}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**5+b*x**3+a*x)/x,x)
```

```
[Out] a*x + b*x**3/3 + c*x**5/5
```

3.70 $\int \frac{ax+bx^3+cx^5}{x^2}\,dx$

Optimal. Leaf size=21

$$a\log(x)+\frac{bx^2}{2}+\frac{cx^4}{4}$$

```
[Out] 1/2*b*x^2+1/4*c*x^4+a*ln(x)
```

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {14}

$$a\log(x)+\frac{bx^2}{2}+\frac{cx^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^3 + c*x^5)/x^2,x]
```

```
[Out] (b*x^2)/2 + (c*x^4)/4 + a*Log[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{ax+bx^3+cx^5}{x^2}\,dx &= \int \left(\frac{a}{x}+bx+cx^3\right)dx\\ &= \frac{bx^2}{2}+\frac{cx^4}{4}+a\log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$a\log(x)+\frac{bx^2}{2}+\frac{cx^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^3 + c*x^5)/x^2,x]
```

```
[Out] (b*x^2)/2 + (c*x^4)/4 + a*Log[x]
```

fricas [A] time = 0.60, size = 17, normalized size = 0.81

$$\frac{1}{4}cx^4+\frac{1}{2}bx^2+a\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)/x^2,x, algorithm="fricas")
```

```
[Out] 1/4*c*x^4 + 1/2*b*x^2 + a*log(x)
```

giac [A] time = 0.41, size = 20, normalized size = 0.95

$$\frac{1}{4}cx^4+\frac{1}{2}bx^2+\frac{1}{2}a\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)/x^2,x, algorithm="giac")
```

```
[Out] 1/4*c*x^4 + 1/2*b*x^2 + 1/2*a*log(x^2)
```

maple [A] time = 0.00, size = 18, normalized size = 0.86

$$\frac{c\,x^{4}}{4}+\frac{b\,x^{2}}{2}+a\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^5+b*x^3+a*x)/x^2,x)
```

```
[Out] 1/2*b*x^2+1/4*c*x^4+a*ln(x)
```

maxima [A] time = 0.43, size = 17, normalized size = 0.81

$$\frac{1}{4}\,cx^{4}+\frac{1}{2}\,bx^{2}+a\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)/x^2,x, algorithm="maxima")
```

```
[Out] 1/4*c*x^4 + 1/2*b*x^2 + a*log(x)
```

mupad [B] time = 0.03, size = 17, normalized size = 0.81

$$\frac{b\,x^2}{2}+\frac{c\,x^4}{4}+a\,\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^3 + c*x^5)/x^2,x)
```

```
[Out] (b*x^2)/2 + (c*x^4)/4 + a*log(x)
```

sympy [A] time = 0.10, size = 17, normalized size = 0.81

$$a\log{\left(x \right)}+\frac{bx^{2}}{2}+\frac{cx^{4}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**5+b*x**3+a*x)/x**2,x)
```

```
[Out] a*log(x) + b*x**2/2 + c*x**4/4
```

3.71 $\int \frac{ax+bx^3+cx^5}{x^3}\,dx$

Optimal. Leaf size=18

$$-\frac{a}{x}+bx+\frac{cx^3}{3}$$

```
[Out] -a/x+b*x+1/3*c*x^3
```

Rubi [A] time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {14}

$$-\frac{a}{x}+bx+\frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^3 + c*x^5)/x^3,x]
```

```
[Out] -(a/x) + b*x + (c*x^3)/3
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \int \frac{ax+bx^3+cx^5}{x^3}\,dx &= \int \left(b+\frac{a}{x^2}+cx^2\right)dx \\ &= -\frac{a}{x}+bx+\frac{cx^3}{3} \end{aligned}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$-\frac{a}{x}+bx+\frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^3 + c*x^5)/x^3,x]
```

```
[Out] -(a/x) + b*x + (c*x^3)/3
```

fricas [A] time = 0.61, size = 20, normalized size = 1.11

$$\frac{cx^4+3\,bx^2-3\,a}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)/x^3,x, algorithm="fricas")
```

```
[Out] 1/3*(c*x^4 + 3*b*x^2 - 3*a)/x
```

giac [A] time = 0.59, size = 16, normalized size = 0.89

$$\frac{1}{3}\,cx^3+bx-\frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)/x^3,x, algorithm="giac")
```

```
[Out] 1/3*c*x^3 + b*x - a/x
```

maple [A] time = 0.00, size = 17, normalized size = 0.94

$$\frac{c\,x^3}{3}+bx-\frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^5+b*x^3+a*x)/x^3,x)
```

```
[Out] -a/x+b*x+1/3*c*x^3
```

maxima [A] time = 0.43, size = 16, normalized size = 0.89

$$\frac{1}{3}\,cx^3+bx-\frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)/x^3,x, algorithm="maxima")
```

```
[Out] 1/3*c*x^3 + b*x - a/x
```

mupad [B] time = 0.03, size = 16, normalized size = 0.89

$$b\,x-\frac{a}{x}+\frac{c\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^3 + c*x^5)/x^3,x)
```

```
[Out] b*x - a/x + (c*x^3)/3
```

sympy [A] time = 0.10, size = 12, normalized size = 0.67

$$-\frac{a}{x}+bx+\frac{cx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**5+b*x**3+a*x)/x**3,x)
```

```
[Out] -a/x + b*x + c*x**3/3
```

3.72 $\int x^m \left(ax+bx^3+cx^5\right)^2 \, dx$

Optimal. Leaf size=76

$$\frac{a^2x^{m+3}}{m+3}+\frac{x^{m+7}\left(2ac+b^2\right)}{m+7}+\frac{2abx^{m+5}}{m+5}+\frac{2bcx^{m+9}}{m+9}+\frac{c^2x^{m+11}}{m+11}$$

```
[Out] a^2*x^(3+m)/(3+m)+2*a*b*x^(5+m)/(5+m)+(2*a*c+b^2)*x^(7+m)/(7+m)+2*b*c*x^(9+
m)/(9+m)+c^2*x^(11+m)/(11+m)
```

Rubi [A] time = 0.05, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1585, 1108}

$$\frac{a^2x^{m+3}}{m+3}+\frac{x^{m+7}\left(2ac+b^2\right)}{m+7}+\frac{2abx^{m+5}}{m+5}+\frac{2bcx^{m+9}}{m+9}+\frac{c^2x^{m+11}}{m+11}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] (a^2*x^(3 + m))/(3 + m) + (2*a*b*x^(5 + m))/(5 + m) + ((b^2 + 2*a*c)*x^(7 +
 m))/(7 + m) + (2*b*c*x^(9 + m))/(9 + m) + (c^2*x^(11 + m))/(11 + m)
```

Rule 1108

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\begin{aligned}\int x^m \left(ax+bx^3+cx^5\right)^2 \, dx &= \int x^{2+m} \left(a+bx^2+cx^4\right)^2 \, dx\\ &= \int \left(a^2x^{2+m}+2abx^{4+m}+\left(b^2+2ac\right)x^{6+m}+2bcx^{8+m}+c^2x^{10+m}\right) dx\\ &= \frac{a^2x^{3+m}}{3+m}+\frac{2abx^{5+m}}{5+m}+\frac{\left(b^2+2ac\right)x^{7+m}}{7+m}+\frac{2bcx^{9+m}}{9+m}+\frac{c^2x^{11+m}}{11+m}\end{aligned}$$

Mathematica [A] time = 0.07, size = 69, normalized size = 0.91

$$x^{m+3}\left(\frac{a^2}{m+3}+\frac{x^4\left(2ac+b^2\right)}{m+7}+\frac{2abx^2}{m+5}+\frac{2bcx^6}{m+9}+\frac{c^2x^8}{m+11}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] x^(3 + m)*(a^2/(3 + m) + (2*a*b*x^2)/(5 + m) + ((b^2 + 2*a*c)*x^4)/(7 + m)
+ (2*b*c*x^6)/(9 + m) + (c^2*x^8)/(11 + m))
```

fricas [B] time = 0.67, size = 241, normalized size = 3.17

$$\frac{\left(\left(c^2m^4 + 24\,c^2m^3 + 206\,c^2m^2 + 744\,c^2m + 945\,c^2\right)x^{11} + 2\left(bcm^4 + 26\,bcm^3 + 236\,bcm^2 + 886\,bcm + 1155\,bc\right)x^9 + \cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] ((c^2*m^4 + 24*c^2*m^3 + 206*c^2*m^2 + 744*c^2*m + 945*c^2)*x^11 + 2*(b*c*m
^4 + 26*b*c*m^3 + 236*b*c*m^2 + 886*b*c*m + 1155*b*c)*x^9 + ((b^2 + 2*a*c)*
m^4 + 28*(b^2 + 2*a*c)*m^3 + 274*(b^2 + 2*a*c)*m^2 + 1485*b^2 + 2970*a*c +
1092*(b^2 + 2*a*c)*m)*x^7 + 2*(a*b*m^4 + 30*a*b*m^3 + 320*a*b*m^2 + 1410*a*
b*m + 2079*a*b)*x^5 + (a^2*m^4 + 32*a^2*m^3 + 374*a^2*m^2 + 1888*a^2*m + 34
65*a^2)*x^3)*x^m/(m^5 + 35*m^4 + 470*m^3 + 3010*m^2 + 9129*m + 10395)
```

giac [B] time = 0.52, size = 399, normalized size = 5.25

$$\frac{c^2m^4x^{11}x^m + 24\,c^2m^3x^{11}x^m + 2\,bcm^4x^9x^m + 206\,c^2m^2x^{11}x^m + 52\,bcm^3x^9x^m + 744\,c^2mx^{11}x^m + b^2m^4x^7x^m + 2\,ac\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] (c^2*m^4*x^11*x^m + 24*c^2*m^3*x^11*x^m + 2*b*c*m^4*x^9*x^m + 206*c^2*m^2*x
^11*x^m + 52*b*c*m^3*x^9*x^m + 744*c^2*m*x^11*x^m + b^2*m^4*x^7*x^m + 2*a*c
*m^4*x^7*x^m + 472*b*c*m^2*x^9*x^m + 945*c^2*x^11*x^m + 28*b^2*m^3*x^7*x^m
+ 56*a*c*m^3*x^7*x^m + 1772*b*c*m*x^9*x^m + 2*a*b*m^4*x^5*x^m + 274*b^2*m^2
*x^7*x^m + 548*a*c*m^2*x^7*x^m + 2310*b*c*x^9*x^m + 60*a*b*m^3*x^5*x^m + 10
92*b^2*m*x^7*x^m + 2184*a*c*m*x^7*x^m + a^2*m^4*x^3*x^m + 640*a*b*m^2*x^5*x
^m + 1485*b^2*x^7*x^m + 2970*a*c*x^7*x^m + 32*a^2*m^3*x^3*x^m + 2820*a*b*m*
x^5*x^m + 374*a^2*m^2*x^3*x^m + 4158*a*b*x^5*x^m + 1888*a^2*m*x^3*x^m + 346
5*a^2*x^3*x^m)/(m^5 + 35*m^4 + 470*m^3 + 3010*m^2 + 9129*m + 10395)
```

maple [B] time = 0.00, size = 300, normalized size = 3.95

$$\frac{\left(c^2m^4x^8 + 24c^2m^3x^8 + 2bc\,m^4x^6 + 206c^2m^2x^8 + 52bc\,m^3x^6 + 744c^2m\,x^8 + 2ac\,m^4x^4 + b^2m^4x^4 + 472bc\,m^2x^6 + \cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c*x^5+b*x^3+a*x)^2,x)
```

```
[Out] x^(m+3)*(c^2*m^4*x^8+24*c^2*m^3*x^8+2*b*c*m^4*x^6+206*c^2*m^2*x^8+52*b*c*m^
3*x^6+744*c^2*m*x^8+2*a*c*m^4*x^4+b^2*m^4*x^4+472*b*c*m^2*x^6+945*c^2*x^8+5
6*a*c*m^3*x^4+28*b^2*m^3*x^4+1772*b*c*m*x^6+2*a*b*m^4*x^2+548*a*c*m^2*x^4+2
74*b^2*m^2*x^4+2310*b*c*x^6+60*a*b*m^3*x^2+2184*a*c*m*x^4+1092*b^2*m*x^4+a^
2*m^4+640*a*b*m^2*x^2+2970*a*c*x^4+1485*b^2*x^4+32*a^2*m^3+2820*a*b*m*x^2+3
74*a^2*m^2+4158*a*b*x^2+1888*a^2*m+3465*a^2)/(m+11)/(m+9)/(m+7)/(m+5)/(m+3)
```

maxima [A] time = 0.44, size = 85, normalized size = 1.12

$$\frac{c^2x^{m+11}}{m+11} + \frac{2\,bcx^{m+9}}{m+9} + \frac{b^2x^{m+7}}{m+7} + \frac{2\,acx^{m+7}}{m+7} + \frac{2\,abx^{m+5}}{m+5} + \frac{a^2x^{m+3}}{m+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] c^2*x^(m + 11)/(m + 11) + 2*b*c*x^(m + 9)/(m + 9) + b^2*x^(m + 7)/(m + 7) +
 2*a*c*x^(m + 7)/(m + 7) + 2*a*b*x^(m + 5)/(m + 5) + a^2*x^(m + 3)/(m + 3)
```

mupad [B] time = 2.19, size = 271, normalized size = 3.57

$$\frac{a^2 x^m x^3 \left(m^4+32 m^3+374 m^2+1888 m+3465\right)}{m^5+35 m^4+470 m^3+3010 m^2+9129 m+10395}+\frac{c^2 x^m x^{11} \left(m^4+24 m^3+206 m^2+744 m+945\right)}{m^5+35 m^4+470 m^3+3010 m^2+9129 m+10395}+\frac{x^m}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a*x + b*x^3 + c*x^5)^2,x)
```

```
[Out] (a^2*x^m*x^3*(1888*m + 374*m^2 + 32*m^3 + m^4 + 3465))/(9129*m + 3010*m^2 +
 470*m^3 + 35*m^4 + m^5 + 10395) + (c^2*x^m*x^11*(744*m + 206*m^2 + 24*m^3
+ m^4 + 945))/(9129*m + 3010*m^2 + 470*m^3 + 35*m^4 + m^5 + 10395) + (x^m*x
^7*(2*a*c + b^2)*(1092*m + 274*m^2 + 28*m^3 + m^4 + 1485))/(9129*m + 3010*m
^2 + 470*m^3 + 35*m^4 + m^5 + 10395) + (2*a*b*x^m*x^5*(1410*m + 320*m^2 + 3
0*m^3 + m^4 + 2079))/(9129*m + 3010*m^2 + 470*m^3 + 35*m^4 + m^5 + 10395) +
 (2*b*c*x^m*x^9*(886*m + 236*m^2 + 26*m^3 + m^4 + 1155))/(9129*m + 3010*m^2
 + 470*m^3 + 35*m^4 + m^5 + 10395)
```

sympy [A] time = 4.29, size = 1377, normalized size = 18.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] Piecewise((-a**2/(8*x**8) - a*b/(3*x**6) - a*c/(2*x**4) - b**2/(4*x**4) - b
*c/x**2 + c**2*log(x), Eq(m, -11)), (-a**2/(6*x**6) - a*b/(2*x**4) - a*c/x*
*2 - b**2/(2*x**2) + 2*b*c*log(x) + c**2*x**2/2, Eq(m, -9)), (-a**2/(4*x**4
) - a*b/x**2 + 2*a*c*log(x) + b**2*log(x) + b*c*x**2 + c**2*x**4/4, Eq(m, -
7)), (-a**2/(2*x**2) + 2*a*b*log(x) + a*c*x**2 + b**2*x**2/2 + b*c*x**4/2 +
 c**2*x**6/6, Eq(m, -5)), (a**2*log(x) + a*b*x**2 + a*c*x**4/2 + b**2*x**4/
4 + b*c*x**6/3 + c**2*x**8/8, Eq(m, -3)), (a**2*m**4*x**3*x**m/(m**5 + 35*m
**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 32*a**2*m**3*x**3*x**m/(m**5
 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 374*a**2*m**2*x**3*x*
*m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 1888*a**2*m*x
**3*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 3465*a*
*2*x**3*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 2*a
*b*m**4*x**5*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395)
+ 60*a*b*m**3*x**5*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 1
0395) + 640*a*b*m**2*x**5*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 912
9*m + 10395) + 2820*a*b*m*x**5*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2
+ 9129*m + 10395) + 4158*a*b*x**5*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m*
*2 + 9129*m + 10395) + 2*a*c*m**4*x**7*x**m/(m**5 + 35*m**4 + 470*m**3 + 30
10*m**2 + 9129*m + 10395) + 56*a*c*m**3*x**7*x**m/(m**5 + 35*m**4 + 470*m**
3 + 3010*m**2 + 9129*m + 10395) + 548*a*c*m**2*x**7*x**m/(m**5 + 35*m**4 +
470*m**3 + 3010*m**2 + 9129*m + 10395) + 2184*a*c*m*x**7*x**m/(m**5 + 35*m*
*4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 2970*a*c*x**7*x**m/(m**5 + 35
*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + b**2*m**4*x**7*x**m/(m**5
+ 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 28*b**2*m**3*x**7*x**m
/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 274*b**2*m**2*x
**7*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 1092*b*
*2*m*x**7*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 1
485*b**2*x**7*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395)
 + 2*b*c*m**4*x**9*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 1
0395) + 52*b*c*m**3*x**9*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129
*m + 10395) + 472*b*c*m**2*x**9*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2
 + 9129*m + 10395) + 1772*b*c*m*x**9*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010
*m**2 + 9129*m + 10395) + 2310*b*c*x**9*x**m/(m**5 + 35*m**4 + 470*m**3 + 3
010*m**2 + 9129*m + 10395) + c**2*m**4*x**11*x**m/(m**5 + 35*m**4 + 470*m**
3 + 3010*m**2 + 9129*m + 10395) + 24*c**2*m**3*x**11*x**m/(m**5 + 35*m**4 +
```

```
 470*m**3 + 3010*m**2 + 9129*m + 10395) + 206*c**2*m**2*x**11*x**m/(m**5 +
35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 744*c**2*m*x**11*x**m/(m
**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 945*c**2*x**11*x**
m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395), True))
```

3.73 $\int x^2 \left(ax+bx^3+cx^5\right)^2 \, dx$

Optimal. Leaf size=54

$$\frac{a^2x^5}{5}+\frac{1}{9}x^9\left(2ac+b^2\right)+\frac{2}{7}abx^7+\frac{2}{11}bcx^{11}+\frac{c^2x^{13}}{13}$$

```
[Out] 1/5*a^2*x^5+2/7*a*b*x^7+1/9*(2*a*c+b^2)*x^9+2/11*b*c*x^11+1/13*c^2*x^13
```

Rubi [A] time = 0.04, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1585, 1108}

$$\frac{a^2x^5}{5}+\frac{1}{9}x^9\left(2ac+b^2\right)+\frac{2}{7}abx^7+\frac{2}{11}bcx^{11}+\frac{c^2x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] (a^2*x^5)/5 + (2*a*b*x^7)/7 + ((b^2 + 2*a*c)*x^9)/9 + (2*b*c*x^11)/11 + (c^
2*x^13)/13
```

Rule 1108

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\begin{aligned}\int x^2\left(ax+bx^3+cx^5\right)^2\,dx &= \int x^4\left(a+bx^2+cx^4\right)^2\,dx\\ &= \int \left(a^2x^4+2abx^6+\left(b^2+2ac\right)x^8+2bcx^{10}+c^2x^{12}\right)dx\\ &= \frac{a^2x^5}{5}+\frac{2}{7}abx^7+\frac{1}{9}\left(b^2+2ac\right)x^9+\frac{2}{11}bcx^{11}+\frac{c^2x^{13}}{13}\end{aligned}$$

Mathematica [A] time = 0.01, size = 54, normalized size = 1.00

$$\frac{a^2x^5}{5}+\frac{1}{9}x^9\left(2ac+b^2\right)+\frac{2}{7}abx^7+\frac{2}{11}bcx^{11}+\frac{c^2x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] (a^2*x^5)/5 + (2*a*b*x^7)/7 + ((b^2 + 2*a*c)*x^9)/9 + (2*b*c*x^11)/11 + (c^
2*x^13)/13
```

fricas [A] time = 0.41, size = 46, normalized size = 0.85

$$\frac{1}{13}x^{13}c^2+\frac{2}{11}x^{11}cb+\frac{1}{9}x^9b^2+\frac{2}{9}x^9ca+\frac{2}{7}x^7ba+\frac{1}{5}x^5a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/13*x^13*c^2 + 2/11*x^11*c*b + 1/9*x^9*b^2 + 2/9*x^9*c*a + 2/7*x^7*b*a + 1
/5*x^5*a^2
```

giac [A] time = 0.41, size = 46, normalized size = 0.85

$$\frac{1}{13}\,c^2x^{13} + \frac{2}{11}\,bcx^{11} + \frac{1}{9}\,b^2x^9 + \frac{2}{9}\,acx^9 + \frac{2}{7}\,abx^7 + \frac{1}{5}\,a^2x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] 1/13*c^2*x^13 + 2/11*b*c*x^11 + 1/9*b^2*x^9 + 2/9*a*c*x^9 + 2/7*a*b*x^7 + 1
/5*a^2*x^5
```

maple [A] time = 0.00, size = 45, normalized size = 0.83

$$\frac{c^2x^{13}}{13} + \frac{2bc\,x^{11}}{11} + \frac{2ab\,x^7}{7} + \frac{\left(2ac + b^2\right)x^9}{9} + \frac{a^2x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^5+b*x^3+a*x)^2,x)
```

```
[Out] 1/5*a^2*x^5+2/7*a*b*x^7+1/9*(2*a*c+b^2)*x^9+2/11*b*c*x^11+1/13*c^2*x^13
```

maxima [A] time = 0.42, size = 44, normalized size = 0.81

$$\frac{1}{13}\,c^2x^{13} + \frac{2}{11}\,bcx^{11} + \frac{1}{9}\left(b^2 + 2\,ac\right)x^9 + \frac{2}{7}\,abx^7 + \frac{1}{5}\,a^2x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/13*c^2*x^13 + 2/11*b*c*x^11 + 1/9*(b^2 + 2*a*c)*x^9 + 2/7*a*b*x^7 + 1/5*a
^2*x^5
```

mupad [B] time = 0.03, size = 45, normalized size = 0.83

$$x^9\left(\frac{b^2}{9} + \frac{2\,a\,c}{9}\right) + \frac{a^2\,x^5}{5} + \frac{c^2\,x^{13}}{13} + \frac{2\,a\,b\,x^7}{7} + \frac{2\,b\,c\,x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a*x + b*x^3 + c*x^5)^2,x)
```

```
[Out] x^9*((2*a*c)/9 + b^2/9) + (a^2*x^5)/5 + (c^2*x^13)/13 + (2*a*b*x^7)/7 + (2*
b*c*x^11)/11
```

sympy [A] time = 0.08, size = 51, normalized size = 0.94

$$\frac{a^2x^5}{5} + \frac{2abx^7}{7} + \frac{2bcx^{11}}{11} + \frac{c^2x^{13}}{13} + x^9\left(\frac{2ac}{9} + \frac{b^2}{9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] a**2*x**5/5 + 2*a*b*x**7/7 + 2*b*c*x**11/11 + c**2*x**13/13 + x**9*(2*a*c/9
 + b**2/9)
```

3.74 $\int x \left(ax+bx^3+cx^5\right)^2 \, dx$

Optimal. Leaf size=54

$$\frac{a^2x^4}{4}+\frac{1}{8}x^8\left(2ac+b^2\right)+\frac{1}{3}abx^6+\frac{1}{5}bcx^{10}+\frac{c^2x^{12}}{12}$$

```
[Out] 1/4*a^2*x^4+1/3*a*b*x^6+1/8*(2*a*c+b^2)*x^8+1/5*b*c*x^10+1/12*c^2*x^12
```

Rubi [A] time = 0.05, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1585, 1114, 631}

$$\frac{a^2x^4}{4}+\frac{1}{8}x^8\left(2ac+b^2\right)+\frac{1}{3}abx^6+\frac{1}{5}bcx^{10}+\frac{c^2x^{12}}{12}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] (a^2*x^4)/4 + (a*b*x^6)/3 + ((b^2 + 2*a*c)*x^8)/8 + (b*c*x^10)/5 + (c^2*x^1
2)/12
```

Rule 631

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbo
l] :> Int[ExpandIntegrand[(d + e*x)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || Eq
Q[a, 0])
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\begin{aligned}
\int x\left(ax+bx^3+cx^5\right)^2\,dx &= \int x^3\left(a+bx^2+cx^4\right)^2\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int x\left(a+bx+cx^2\right)^2\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(a^2x+2abx^2+\left(b^2+2ac\right)x^3+2bcx^4+c^2x^5\right)\,dx,x,x^2\right)\\
&= \frac{a^2x^4}{4}+\frac{1}{3}abx^6+\frac{1}{8}\left(b^2+2ac\right)x^8+\frac{1}{5}bcx^{10}+\frac{c^2x^{12}}{12}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 48, normalized size = 0.89

$$\frac{1}{120}x^4\left(30a^2+15x^4\left(2ac+b^2\right)+40abx^2+24bcx^6+10c^2x^8\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] (x^4*(30*a^2 + 40*a*b*x^2 + 15*(b^2 + 2*a*c)*x^4 + 24*b*c*x^6 + 10*c^2*x^8)
)/120
```

fricas [A] time = 0.50, size = 46, normalized size = 0.85

$$\frac{1}{12}x^{12}c^2 + \frac{1}{5}x^{10}cb + \frac{1}{8}x^8b^2 + \frac{1}{4}x^8ca + \frac{1}{3}x^6ba + \frac{1}{4}x^4a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/12*x^12*c^2 + 1/5*x^10*c*b + 1/8*x^8*b^2 + 1/4*x^8*c*a + 1/3*x^6*b*a + 1/
4*x^4*a^2
```

giac [A] time = 0.38, size = 46, normalized size = 0.85

$$\frac{1}{12}\,c^2x^{12} + \frac{1}{5}\,bcx^{10} + \frac{1}{8}\,b^2x^8 + \frac{1}{4}\,acx^8 + \frac{1}{3}\,abx^6 + \frac{1}{4}\,a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] 1/12*c^2*x^12 + 1/5*b*c*x^10 + 1/8*b^2*x^8 + 1/4*a*c*x^8 + 1/3*a*b*x^6 + 1/
4*a^2*x^4
```

maple [A] time = 0.00, size = 45, normalized size = 0.83

$$\frac{c^2x^{12}}{12} + \frac{bc\,x^{10}}{5} + \frac{ab\,x^6}{3} + \frac{\left(2ac + b^2\right)x^8}{8} + \frac{a^2x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^5+b*x^3+a*x)^2,x)
```

```
[Out] 1/4*a^2*x^4+1/3*a*b*x^6+1/8*(2*a*c+b^2)*x^8+1/5*b*c*x^10+1/12*c^2*x^12
```

maxima [A] time = 0.45, size = 44, normalized size = 0.81

$$\frac{1}{12}\,c^2x^{12} + \frac{1}{5}\,bcx^{10} + \frac{1}{8}\left(b^2 + 2\,ac\right)x^8 + \frac{1}{3}\,abx^6 + \frac{1}{4}\,a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/12*c^2*x^12 + 1/5*b*c*x^10 + 1/8*(b^2 + 2*a*c)*x^8 + 1/3*a*b*x^6 + 1/4*a^
2*x^4
```

mupad [B] time = 0.02, size = 45, normalized size = 0.83

$$x^8\left(\frac{b^2}{8} + \frac{a\,c}{4}\right) + \frac{a^2\,x^4}{4} + \frac{c^2\,x^{12}}{12} + \frac{a\,b\,x^6}{3} + \frac{b\,c\,x^{10}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a*x + b*x^3 + c*x^5)^2,x)
```

```
[Out] x^8*((a*c)/4 + b^2/8) + (a^2*x^4)/4 + (c^2*x^12)/12 + (a*b*x^6)/3 + (b*c*x^
10)/5
```

sympy [A] time = 0.08, size = 46, normalized size = 0.85

$$\frac{a^2x^4}{4} + \frac{abx^6}{3} + \frac{bcx^{10}}{5} + \frac{c^2x^{12}}{12} + x^8\left(\frac{ac}{4} + \frac{b^2}{8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] a**2*x**4/4 + a*b*x**6/3 + b*c*x**10/5 + c**2*x**12/12 + x**8*(a*c/4 + b**2
/8)
```

3.75 $\int \left(ax+bx^3+cx^5\right)^2 dx$

Optimal. Leaf size=54

$$\frac{a^2x^3}{3}+\frac{1}{7}x^7\left(2ac+b^2\right)+\frac{2}{5}abx^5+\frac{2}{9}bcx^9+\frac{c^2x^{11}}{11}$$

```
[Out] 1/3*a^2*x^3+2/5*a*b*x^5+1/7*(2*a*c+b^2)*x^7+2/9*b*c*x^9+1/11*c^2*x^11
```

Rubi [A] time = 0.03, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1594, 1108}

$$\frac{a^2x^3}{3}+\frac{1}{7}x^7\left(2ac+b^2\right)+\frac{2}{5}abx^5+\frac{2}{9}bcx^9+\frac{c^2x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] (a^2*x^3)/3 + (2*a*b*x^5)/5 + ((b^2 + 2*a*c)*x^7)/7 + (2*b*c*x^9)/9 + (c^2*
x^11)/11
```

Rule 1108

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rubi steps

$$\begin{aligned}\int \left(ax+bx^3+cx^5\right)^2 dx &= \int x^2\left(a+bx^2+cx^4\right)^2 dx\\ &= \int \left(a^2x^2+2abx^4+\left(b^2+2ac\right)x^6+2bcx^8+c^2x^{10}\right) dx\\ &= \frac{a^2x^3}{3}+\frac{2}{5}abx^5+\frac{1}{7}\left(b^2+2ac\right)x^7+\frac{2}{9}bcx^9+\frac{c^2x^{11}}{11}\end{aligned}$$

Mathematica [A] time = 0.01, size = 54, normalized size = 1.00

$$\frac{a^2x^3}{3}+\frac{1}{7}x^7\left(2ac+b^2\right)+\frac{2}{5}abx^5+\frac{2}{9}bcx^9+\frac{c^2x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] (a^2*x^3)/3 + (2*a*b*x^5)/5 + ((b^2 + 2*a*c)*x^7)/7 + (2*b*c*x^9)/9 + (c^2*
x^11)/11
```

fricas [A] time = 0.55, size = 46, normalized size = 0.85

$$\frac{1}{11}x^{11}c^2+\frac{2}{9}x^9cb+\frac{1}{7}x^7b^2+\frac{2}{7}x^7ca+\frac{2}{5}x^5ba+\frac{1}{3}x^3a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/11*x^11*c^2 + 2/9*x^9*c*b + 1/7*x^7*b^2 + 2/7*x^7*c*a + 2/5*x^5*b*a + 1/3
*x^3*a^2
```

giac [A] time = 0.36, size = 46, normalized size = 0.85

$$\frac{1}{11}c^2x^{11} + \frac{2}{9}bcx^9 + \frac{1}{7}b^2x^7 + \frac{2}{7}acx^7 + \frac{2}{5}abx^5 + \frac{1}{3}a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] 1/11*c^2*x^11 + 2/9*b*c*x^9 + 1/7*b^2*x^7 + 2/7*a*c*x^7 + 2/5*a*b*x^5 + 1/3
*a^2*x^3
```

maple [A] time = 0.00, size = 45, normalized size = 0.83

$$\frac{c^2x^{11}}{11} + \frac{2bc\,x^9}{9} + \frac{2ab\,x^5}{5} + \frac{\left(2ac + b^2\right)x^7}{7} + \frac{a^2x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^5+b*x^3+a*x)^2,x)
```

```
[Out] 1/3*a^2*x^3+2/5*a*b*x^5+1/7*(2*a*c+b^2)*x^7+2/9*b*c*x^9+1/11*c^2*x^11
```

maxima [A] time = 0.43, size = 48, normalized size = 0.89

$$\frac{1}{11}c^2x^{11} + \frac{2}{9}bcx^9 + \frac{1}{7}b^2x^7 + \frac{1}{3}a^2x^3 + \frac{2}{35}\left(5\,cx^7 + 7\,bx^5\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/11*c^2*x^11 + 2/9*b*c*x^9 + 1/7*b^2*x^7 + 1/3*a^2*x^3 + 2/35*(5*c*x^7 + 7
*b*x^5)*a
```

mupad [B] time = 0.02, size = 45, normalized size = 0.83

$$x^7\left(\frac{b^2}{7} + \frac{2\,a\,c}{7}\right) + \frac{a^2\,x^3}{3} + \frac{c^2\,x^{11}}{11} + \frac{2\,a\,b\,x^5}{5} + \frac{2\,b\,c\,x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^3 + c*x^5)^2,x)
```

```
[Out] x^7*((2*a*c)/7 + b^2/7) + (a^2*x^3)/3 + (c^2*x^11)/11 + (2*a*b*x^5)/5 + (2*
b*c*x^9)/9
```

sympy [A] time = 0.07, size = 51, normalized size = 0.94

$$\frac{a^2x^3}{3} + \frac{2abx^5}{5} + \frac{2bcx^9}{9} + \frac{c^2x^{11}}{11} + x^7\left(\frac{2ac}{7} + \frac{b^2}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] a**2*x**3/3 + 2*a*b*x**5/5 + 2*b*c*x**9/9 + c**2*x**11/11 + x**7*(2*a*c/7 +
 b**2/7)
```

3.76 $\int \frac{\left(ax+bx^3+cx^5\right)^2}{x} dx$

Optimal. Leaf size=54

$$\frac{a^2x^2}{2}+\frac{1}{6}x^6\left(2ac+b^2\right)+\frac{1}{2}abx^4+\frac{1}{4}bcx^8+\frac{c^2x^{10}}{10}$$

```
[Out] 1/2*a^2*x^2+1/2*a*b*x^4+1/6*(2*a*c+b^2)*x^6+1/4*b*c*x^8+1/10*c^2*x^10
```

Rubi [A] time = 0.05, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1585, 1107, 611}

$$\frac{a^2x^2}{2}+\frac{1}{6}x^6\left(2ac+b^2\right)+\frac{1}{2}abx^4+\frac{1}{4}bcx^8+\frac{c^2x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^3 + c*x^5)^2/x,x]
```

```
[Out] (a^2*x^2)/2 + (a*b*x^4)/2 + ((b^2 + 2*a*c)*x^6)/6 + (b*c*x^8)/4 + (c^2*x^10
)/10
```

Rule 611

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegr
and[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && IGtQ[p, 0] && (EqQ[a, 0] ||  !PerfectSquareQ[b^2 - 4*a*c])
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(ax+bx^3+cx^5\right)^2}{x} dx &= \int x\left(a+bx^2+cx^4\right)^2 dx\\
&= \frac{1}{2}\text{Subst}\left(\int \left(a+bx+cx^2\right)^2 dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(a^2+2abx+b^2\left(1+\frac{2ac}{b^2}\right)x^2+2bcx^3+c^2x^4\right) dx, x, x^2\right)\\
&= \frac{a^2x^2}{2}+\frac{1}{2}abx^4+\frac{1}{6}\left(b^2+2ac\right)x^6+\frac{1}{4}bcx^8+\frac{c^2x^{10}}{10}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 48, normalized size = 0.89

$$\frac{1}{60}x^2\left(30a^2+10x^4\left(2ac+b^2\right)+30abx^2+15bcx^6+6c^2x^8\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^3 + c*x^5)^2/x,x]
```

```
[Out] (x^2*(30*a^2 + 30*a*b*x^2 + 10*(b^2 + 2*a*c)*x^4 + 15*b*c*x^6 + 6*c^2*x^8))
/60
```

fricas [A] time = 0.61, size = 44, normalized size = 0.81

$$\frac{1}{10}\,c^2x^{10} + \frac{1}{4}\,bcx^8 + \frac{1}{6}\left(b^2 + 2\,ac\right)x^6 + \frac{1}{2}\,abx^4 + \frac{1}{2}\,a^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)^2/x,x, algorithm="fricas")
```

```
[Out] 1/10*c^2*x^10 + 1/4*b*c*x^8 + 1/6*(b^2 + 2*a*c)*x^6 + 1/2*a*b*x^4 + 1/2*a^2
*x^2
```

giac [A] time = 0.52, size = 46, normalized size = 0.85

$$\frac{1}{10}\,c^2x^{10} + \frac{1}{4}\,bcx^8 + \frac{1}{6}\,b^2x^6 + \frac{1}{3}\,acx^6 + \frac{1}{2}\,abx^4 + \frac{1}{2}\,a^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)^2/x,x, algorithm="giac")
```

```
[Out] 1/10*c^2*x^10 + 1/4*b*c*x^8 + 1/6*b^2*x^6 + 1/3*a*c*x^6 + 1/2*a*b*x^4 + 1/2
*a^2*x^2
```

maple [A] time = 0.00, size = 45, normalized size = 0.83

$$\frac{c^2x^{10}}{10} + \frac{bc\,x^8}{4} + \frac{ab\,x^4}{2} + \frac{\left(2ac + b^2\right)x^6}{6} + \frac{a^2x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^5+b*x^3+a*x)^2/x,x)
```

```
[Out] 1/2*a^2*x^2+1/2*a*b*x^4+1/6*(2*a*c+b^2)*x^6+1/4*b*c*x^8+1/10*c^2*x^10
```

maxima [A] time = 0.43, size = 44, normalized size = 0.81

$$\frac{1}{10}\,c^2x^{10} + \frac{1}{4}\,bcx^8 + \frac{1}{6}\left(b^2 + 2\,ac\right)x^6 + \frac{1}{2}\,abx^4 + \frac{1}{2}\,a^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)^2/x,x, algorithm="maxima")
```

```
[Out] 1/10*c^2*x^10 + 1/4*b*c*x^8 + 1/6*(b^2 + 2*a*c)*x^6 + 1/2*a*b*x^4 + 1/2*a^2
*x^2
```

mupad [B] time = 0.02, size = 45, normalized size = 0.83

$$x^6\left(\frac{b^2}{6} + \frac{a\,c}{3}\right) + \frac{a^2\,x^2}{2} + \frac{c^2\,x^{10}}{10} + \frac{a\,b\,x^4}{2} + \frac{b\,c\,x^8}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^3 + c*x^5)^2/x,x)
```

```
[Out] x^6*((a*c)/3 + b^2/6) + (a^2*x^2)/2 + (c^2*x^10)/10 + (a*b*x^4)/2 + (b*c*x^
8)/4
```

<u>**sympy**</u> **[A]** time = 0.08, size = 46, normalized size = 0.85

$$\frac{a^2x^2}{2}+\frac{abx^4}{2}+\frac{bcx^8}{4}+\frac{c^2x^{10}}{10}+x^6\left(\frac{ac}{3}+\frac{b^2}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**5+b*x**3+a*x)**2/x,x)
```

```
[Out] a**2*x**2/2 + a*b*x**4/2 + b*c*x**8/4 + c**2*x**10/10 + x**6*(a*c/3 + b**2/
6)
```

3.77 $\int \frac{(ax+bx^3+cx^5)^2}{x^2}\,dx$

Optimal. Leaf size=49

$$a^2x+\frac{1}{5}x^5\left(2ac+b^2\right)+\frac{2}{3}abx^3+\frac{2}{7}bcx^7+\frac{c^2x^9}{9}$$

```
[Out] a^2*x+2/3*a*b*x^3+1/5*(2*a*c+b^2)*x^5+2/7*b*c*x^7+1/9*c^2*x^9
```

Rubi [A] time = 0.03, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1585, 1090}

$$a^2x+\frac{1}{5}x^5\left(2ac+b^2\right)+\frac{2}{3}abx^3+\frac{2}{7}bcx^7+\frac{c^2x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^3 + c*x^5)^2/x^2,x]
```

```
[Out] a^2*x + (2*a*b*x^3)/3 + ((b^2 + 2*a*c)*x^5)/5 + (2*b*c*x^7)/7 + (c^2*x^9)/9
```

Rule 1090

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandInteg
rand[(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*
c, 0] && IGtQ[p, 0]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(ax+bx^3+cx^5\right)^2}{x^2}\,dx&=\int \left(a+bx^2+cx^4\right)^2\,dx\\&=\int \left(a^2+2abx^2+b^2\left(1+\frac{2ac}{b^2}\right)x^4+2bcx^6+c^2x^8\right)dx\\&=a^2x+\frac{2}{3}abx^3+\frac{1}{5}\left(b^2+2ac\right)x^5+\frac{2}{7}bcx^7+\frac{c^2x^9}{9}\end{aligned}$$

Mathematica [A] time = 0.01, size = 49, normalized size = 1.00

$$a^2x+\frac{1}{5}x^5\left(2ac+b^2\right)+\frac{2}{3}abx^3+\frac{2}{7}bcx^7+\frac{c^2x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^3 + c*x^5)^2/x^2,x]
```

```
[Out] a^2*x + (2*a*b*x^3)/3 + ((b^2 + 2*a*c)*x^5)/5 + (2*b*c*x^7)/7 + (c^2*x^9)/9
```

fricas [A] time = 0.60, size = 41, normalized size = 0.84

$$\frac{1}{9}c^2x^9+\frac{2}{7}bcx^7+\frac{1}{5}\left(b^2+2\,ac\right)x^5+\frac{2}{3}\,abx^3+a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)^2/x^2,x, algorithm="fricas")
[Out] 1/9*c^2*x^9 + 2/7*b*c*x^7 + 1/5*(b^2 + 2*a*c)*x^5 + 2/3*a*b*x^3 + a^2*x
```

giac [A] time = 0.51, size = 43, normalized size = 0.88

$$\frac{1}{9}c^2x^9 + \frac{2}{7}bcx^7 + \frac{1}{5}b^2x^5 + \frac{2}{5}acx^5 + \frac{2}{3}abx^3 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)^2/x^2,x, algorithm="giac")
[Out] 1/9*c^2*x^9 + 2/7*b*c*x^7 + 1/5*b^2*x^5 + 2/5*a*c*x^5 + 2/3*a*b*x^3 + a^2*x
```

maple [A] time = 0.00, size = 42, normalized size = 0.86

$$\frac{c^2x^9}{9} + \frac{2bc\,x^7}{7} + \frac{2ab\,x^3}{3} + \frac{\left(2ac+b^2\right)x^5}{5} + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^5+b*x^3+a*x)^2/x^2,x)
[Out] a^2*x+2/3*a*b*x^3+1/5*(2*a*c+b^2)*x^5+2/7*b*c*x^7+1/9*c^2*x^9
```

maxima [A] time = 0.43, size = 41, normalized size = 0.84

$$\frac{1}{9}c^2x^9 + \frac{2}{7}bcx^7 + \frac{1}{5}\left(b^2+2\,ac\right)x^5 + \frac{2}{3}abx^3 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)^2/x^2,x, algorithm="maxima")
[Out] 1/9*c^2*x^9 + 2/7*b*c*x^7 + 1/5*(b^2 + 2*a*c)*x^5 + 2/3*a*b*x^3 + a^2*x
```

mupad [B] time = 0.02, size = 42, normalized size = 0.86

$$a^2\,x + x^5\left(\frac{b^2}{5} + \frac{2\,a\,c}{5}\right) + \frac{c^2\,x^9}{9} + \frac{2\,a\,b\,x^3}{3} + \frac{2\,b\,c\,x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^3 + c*x^5)^2/x^2,x)
[Out] a^2*x + x^5*((2*a*c)/5 + b^2/5) + (c^2*x^9)/9 + (2*a*b*x^3)/3 + (2*b*c*x^7)
/7
```

sympy [A] time = 0.08, size = 48, normalized size = 0.98

$$a^2x + \frac{2abx^3}{3} + \frac{2bcx^7}{7} + \frac{c^2x^9}{9} + x^5\left(\frac{2ac}{5} + \frac{b^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**5+b*x**3+a*x)**2/x**2,x)
[Out] a**2*x + 2*a*b*x**3/3 + 2*b*c*x**7/7 + c**2*x**9/9 + x**5*(2*a*c/5 + b**2/5
)
```

3.78 $\int \frac{x^8}{ax+bx^3+cx^5}\,dx$

Optimal. Leaf size=100

$$\frac{b\left(b^2-3ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\sqrt{b^2-4ac}}+\frac{\left(b^2-ac\right)\log\left(a+bx^2+cx^4\right)}{4c^3}-\frac{bx^2}{2c^2}+\frac{x^4}{4c}$$

```
[Out] -1/2*b*x^2/c^2+1/4*x^4/c+1/4*(-a*c+b^2)*ln(c*x^4+b*x^2+a)/c^3+1/2*b*(-3*a*c
+b^2)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/c^3/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1585, 1114, 701, 634, 618, 206, 628}

$$\frac{\left(b^2-ac\right)\log\left(a+bx^2+cx^4\right)}{4c^3}+\frac{b\left(b^2-3ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\sqrt{b^2-4ac}}-\frac{bx^2}{2c^2}+\frac{x^4}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a*x + b*x^3 + c*x^5),x]
```

```
[Out] -(b*x^2)/(2*c^2) + x^4/(4*c) + (b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[
b^2 - 4*a*c]])/(2*c^3*Sqrt[b^2 - 4*a*c]) + ((b^2 - a*c)*Log[a + b*x^2 + c*x
^4])/(4*c^3)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 701

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Int[PolynomialDivide[(d + e*x)^m, a + b*x + c*x^2, x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 NeQ[2*c*d - b*e, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^8}{ax+bx^3+cx^5}\,dx &= \int \frac{x^7}{a+bx^2+cx^4}\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{x^3}{a+bx+cx^2}\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{b}{c^2}+\frac{x}{c}+\frac{ab+\left(b^2-ac\right)x}{c^2\left(a+bx+cx^2\right)}\right)dx, x, x^2\right)\\
&= -\frac{bx^2}{2c^2}+\frac{x^4}{4c}+\frac{\text{Subst}\left(\int \frac{ab+\left(b^2-ac\right)x}{a+bx+cx^2}\,dx, x, x^2\right)}{2c^2}\\
&= -\frac{bx^2}{2c^2}+\frac{x^4}{4c}-\frac{\left(b\left(b^2-3ac\right)\right)\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^2\right)}{4c^3}+\frac{\left(b^2-ac\right)\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx\right.}{4c^3}\\
&= -\frac{bx^2}{2c^2}+\frac{x^4}{4c}+\frac{\left(b^2-ac\right)\log\left(a+bx^2+cx^4\right)}{4c^3}+\frac{\left(b\left(b^2-3ac\right)\right)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+\right.}{2c^3}\\
&= -\frac{bx^2}{2c^2}+\frac{x^4}{4c}+\frac{b\left(b^2-3ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\sqrt{b^2-4ac}}+\frac{\left(b^2-ac\right)\log\left(a+bx^2+cx^4\right)}{4c^3}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 93, normalized size = 0.93

$$\frac{-\frac{2b\left(b^2-3ac\right)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+\left(b^2-ac\right)\log\left(a+bx^2+cx^4\right)+cx^2\left(cx^2-2b\right)}{4c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a*x + b*x^3 + c*x^5),x]
```

```
[Out] (c*x^2*(-2*b + c*x^2) - (2*b*(b^2 - 3*a*c)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 +
 4*a*c]])/Sqrt[-b^2 + 4*a*c] + (b^2 - a*c)*Log[a + b*x^2 + c*x^4])/(4*c^3)
```

fricas [A] time = 0.59, size = 313, normalized size = 3.13

$$\left[\frac{\left(b^2c^2-4\,ac^3\right)x^4-2\left(b^3c-4\,abc^2\right)x^2-\left(b^3-3\,abc\right)\sqrt{b^2-4\,ac}\,\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac-\left(2\,cx^2+b\right)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)+\left(b^4-5\right.}{4\left(b^2c^3-4\,ac^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^5+b*x^3+a*x),x, algorithm="fricas")
```

```
[Out] [1/4*((b^2*c^2 - 4*a*c^3)*x^4 - 2*(b^3*c - 4*a*b*c^2)*x^2 - (b^3 - 3*a*b*c)
*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)
*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*lo
g(c*x^4 + b*x^2 + a))/(b^2*c^3 - 4*a*c^4), 1/4*((b^2*c^2 - 4*a*c^3)*x^4 - 2
*(b^3*c - 4*a*b*c^2)*x^2 + 2*(b^3 - 3*a*b*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2*
c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (b^4 - 5*a*b^2*c + 4*a^2*c^2
)*log(c*x^4 + b*x^2 + a))/(b^2*c^3 - 4*a*c^4)]
```

giac [A] time = 0.47, size = 92, normalized size = 0.92

$$\frac{cx^4 - 2\,bx^2}{4\,c^2} + \frac{\left(b^2 - ac\right)\log\left(cx^4 + bx^2 + a\right)}{4\,c^3} - \frac{\left(b^3 - 3\,abc\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2 + 4\,ac}\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^5+b*x^3+a*x),x, algorithm="giac")
```

```
[Out] 1/4*(c*x^4 - 2*b*x^2)/c^2 + 1/4*(b^2 - a*c)*log(c*x^4 + b*x^2 + a)/c^3 - 1/
2*(b^3 - 3*a*b*c)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a
*c)*c^3)
```

maple [A] time = 0.00, size = 142, normalized size = 1.42

$$\frac{x^4}{4c} + \frac{3ab\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac - b^2}\;c^2} - \frac{b^3\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac - b^2}\;c^3} - \frac{b\,x^2}{2c^2} - \frac{a\ln\left(c\,x^4 + b\,x^2 + a\right)}{4c^2} + \frac{b^2\ln\left(c\,x^4 + b\,x^2 + a\right)}{4c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(c*x^5+b*x^3+a*x),x)
```

```
[Out] 1/4/c*x^4-1/2*b/c^2*x^2-1/4/c^2*ln(c*x^4+b*x^2+a)*a+1/4/c^3*ln(c*x^4+b*x^2+
a)*b^2+3/2/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*b-
1/2/c^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{cx^4 - 2\,bx^2}{4\,c^2} - \frac{-\int\frac{\left(b^2-ac\right)x^3+abx}{cx^4+bx^2+a}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^5+b*x^3+a*x),x, algorithm="maxima")
```

```
[Out] 1/4*(c*x^4 - 2*b*x^2)/c^2 - integrate(-((b^2 - a*c)*x^3 + a*b*x)/(c*x^4 + b
*x^2 + a), x)/c^2
```

mupad [B] time = 2.20, size = 842, normalized size = 8.42

$$\frac{x^4}{4\,c} - \frac{\ln\left(c\,x^4 + b\,x^2 + a\right)\,\left(8\,a^2\,c^2 - 10\,a\,b^2\,c + 2\,b^4\right)}{2\,\left(16\,a\,c^4 - 4\,b^2\,c^3\right)} - \frac{b\,x^2}{2\,c^2} + \frac{b\,\mathrm{atan}\left(\frac{2\,c^4\,\left(4\,a\,c-b^2\right)\left(\frac{b\,\left(3\,a\,c-b^2\right)\left(\frac{8\,a^2\,c^4-8\,a\,b^2\,c^3}{c^4}-\frac{8\,a\,c^2\,\left(8\,a^2\,c^2-10\,a \cdots\right.}{16\,a\,c^4-4\,b^2 \cdots}\right.}{8\,c^3\,\sqrt{4\,a\,c-b^2}} \cdots\right.}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(a*x + b*x^3 + c*x^5),x)
```

```
[Out] x^4/(4*c) - (log(a + b*x^2 + c*x^4)*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(2*(1
6*a*c^4 - 4*b^2*c^3)) - (b*x^2)/(2*c^2) + (b*atan((2*c^4*(4*a*c - b^2)*(((b
*(3*a*c - b^2)*((8*a^2*c^4 - 8*a*b^2*c^3)/c^4 - (8*a*c^2*(2*b^4 + 8*a^2*c^2
 - 10*a*b^2*c))/(16*a*c^4 - 4*b^2*c^3)))/(8*c^3*(4*a*c - b^2)^(1/2)) - (a*b
*(3*a*c - b^2)*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(c*(4*a*c - b^2)^(1/2)*(16
*a*c^4 - 4*b^2*c^3)))/a - x^2*(((b*((6*b^3*c^3 - 10*a*b*c^4)/c^4 + (4*b*c^2
*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(16*a*c^4 - 4*b^2*c^3))*(3*a*c - b^2))/(
8*c^3*(4*a*c - b^2)^(1/2)) + (b^2*(3*a*c - b^2)*(2*b^4 + 8*a^2*c^2 - 10*a*b
^2*c))/(2*c*(4*a*c - b^2)^(1/2)*(16*a*c^4 - 4*b^2*c^3)))/a + (b*((b^5 + 2*a
^2*b*c^2 - 3*a*b^3*c)/c^4 + (((6*b^3*c^3 - 10*a*b*c^4)/c^4 + (4*b*c^2*(2*b^
4 + 8*a^2*c^2 - 10*a*b^2*c))/(16*a*c^4 - 4*b^2*c^3))*(2*b^4 + 8*a^2*c^2 - 1
0*a*b^2*c))/(2*(16*a*c^4 - 4*b^2*c^3)) - (b^3*(3*a*c - b^2)^2)/(2*c^4*(4*a*
c - b^2))))/(2*a*(4*a*c - b^2)^(1/2))) + (b*((((8*a^2*c^4 - 8*a*b^2*c^3)/c^
4 - (8*a*c^2*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(16*a*c^4 - 4*b^2*c^3))*(2*b
^4 + 8*a^2*c^2 - 10*a*b^2*c))/(2*(16*a*c^4 - 4*b^2*c^3)) - (a*b^4 + a^3*c^2
 - 2*a^2*b^2*c)/c^4 + (a*b^2*(3*a*c - b^2)^2)/(c^4*(4*a*c - b^2))))/(2*a*(4
*a*c - b^2)^(1/2))))/(b^6 + 9*a^2*b^2*c^2 - 6*a*b^4*c))*(3*a*c - b^2))/(2*c
^3*(4*a*c - b^2)^(1/2))
```

sympy [B] time = 2.95, size = 391, normalized size = 3.91

$$-\frac{bx^2}{2c^2}+\left(-\frac{b\sqrt{-4ac+b^2}\left(3ac-b^2\right)}{4c^3\left(4ac-b^2\right)}-\frac{ac-b^2}{4c^3}\right)\log\left(x^2+\frac{2a^2c-ab^2+8ac^3\left(-\frac{b\sqrt{-4ac+b^2}\left(3ac-b^2\right)}{4c^3\left(4ac-b^2\right)}-\frac{ac-b^2}{4c^3}\right)-2b^2c^2\left(-\right.}{3abc-b^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(c*x**5+b*x**3+a*x),x)
```

```
[Out] -b*x**2/(2*c**2) + (-b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(4*c**3*(4*a*c -
b**2)) - (a*c - b**2)/(4*c**3))*log(x**2 + (2*a**2*c - a*b**2 + 8*a*c**3*(-
b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(4*c**3*(4*a*c - b**2)) - (a*c - b**2)
/(4*c**3)) - 2*b**2*c**2*(-b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(4*c**3*(4*
a*c - b**2)) - (a*c - b**2)/(4*c**3)))/(3*a*b*c - b**3)) + (b*sqrt(-4*a*c +
 b**2)*(3*a*c - b**2)/(4*c**3*(4*a*c - b**2)) - (a*c - b**2)/(4*c**3))*log(
x**2 + (2*a**2*c - a*b**2 + 8*a*c**3*(b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/
(4*c**3*(4*a*c - b**2)) - (a*c - b**2)/(4*c**3)) - 2*b**2*c**2*(b*sqrt(-4*a
*c + b**2)*(3*a*c - b**2)/(4*c**3*(4*a*c - b**2)) - (a*c - b**2)/(4*c**3)))
/(3*a*b*c - b**3)) + x**4/(4*c)
```

3.79 $\int \frac{x^7}{ax+bx^3+cx^5}\,dx$

Optimal. Leaf size=203

$$\frac{\left(-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,c^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{bx}{c^2}+\frac{x^3}{3c}$$

```
[Out] -b*x/c^2+1/3*x^3/c+1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2
))*(b^2-a*c-b*(-3*a*c+b^2)/(-4*a*c+b^2)^(1/2))/c^(5/2)*2^(1/2)/(b-(-4*a*c+b
^2)^(1/2))^(1/2)+1/2*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))
*(b^2-a*c+b*(-3*a*c+b^2)/(-4*a*c+b^2)^(1/2))/c^(5/2)*2^(1/2)/(b+(-4*a*c+b^2
)^(1/2))^(1/2)
```

Rubi [A] time = 0.60, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1585, 1122, 1279, 1166, 205}

$$\frac{\left(-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,c^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{bx}{c^2}+\frac{x^3}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a*x + b*x^3 + c*x^5),x]
```

```
[Out] -((b*x)/c^2) + x^3/(3*c) + ((b^2 - a*c - (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4*a*c
])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2
)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^2 - a*c + (b*(b^2 - 3*a*c))/Sqrt[b^2 -
 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*
c^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
```

```
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\int \frac{x^7}{ax+bx^3+cx^5}\,dx = \int \frac{x^6}{a+bx^2+cx^4}\,dx$$

$$= \frac{x^3}{3c} - \frac{\int \frac{x^2(3a+3bx^2)}{a+bx^2+cx^4}\,dx}{3c}$$

$$= -\frac{bx}{c^2} + \frac{x^3}{3c} + \frac{\int \frac{3ab+3(b^2-ac)x^2}{a+bx^2+cx^4}\,dx}{3c^2}$$

$$= -\frac{bx}{c^2} + \frac{x^3}{3c} + \frac{\left(b^2-ac-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2c^2} + \frac{\left(b^2-ac+\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4a}}$$

$$= -\frac{bx}{c^2} + \frac{x^3}{3c} + \frac{\left(b^2-ac-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(b^2-ac+\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}}{\sqrt{b+\sqrt{}}}\right.}{\sqrt{2}\,c^{5/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

Mathematica [A] time = 0.16, size = 250, normalized size = 1.23

$$\frac{\left(b^2\sqrt{b^2-4ac}-ac\sqrt{b^2-4ac}+3abc-b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{5/2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(b^2\sqrt{b^2-4ac}-ac\sqrt{b^2-4ac}-3abc+b^3\right)\tan^{-1}}{\sqrt{2}\,c^{5/2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a*x + b*x^3 + c*x^5),x]
```

```
[Out] -((b*x)/c^2) + x^3/(3*c) + ((-b^3 + 3*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - a*c*S
qrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/
(Sqrt[2]*c^(5/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^3 - 3
*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqr
t[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b^2 - 4*a*c]*Sq
rt[b + Sqrt[b^2 - 4*a*c]])
```

fricas [B] time = 0.65, size = 1564, normalized size = 7.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^5+b*x^3+a*x),x, algorithm="fricas")
```

```
[Out] 1/6*(2*c*x^3 - 3*sqrt(1/2)*c^2*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^2*
c^5 - 4*a*c^6)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4
*c^4)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*log(2*(a^2*b^4 - 3*a^3*b
^2*c + a^4*c^2)*x + sqrt(1/2)*(b^7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*c
^3 - (b^4*c^5 - 6*a*b^2*c^6 + 8*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4
*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*c^11)))*sqrt(-(b^5 - 5*a*b^
3*c + 5*a^2*b*c^2 + (b^2*c^5 - 4*a*c^6)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*
c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))
) + 3*sqrt(1/2)*c^2*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^2*c^5 - 4*a*c
^6)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^2*
c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*log(2*(a^2*b^4 - 3*a^3*b^2*c + a^4*
c^2)*x - sqrt(1/2)*(b^7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*c^3 - (b^4*c
^5 - 6*a*b^2*c^6 + 8*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^
3*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*c^11)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2
*b*c^2 + (b^2*c^5 - 4*a*c^6)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3
*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))) - 3*sqrt(
1/2)*c^2*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^2*c^5 - 4*a*c^6)*sqrt((b
^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*
c^11)))/(b^2*c^5 - 4*a*c^6))*log(2*(a^2*b^4 - 3*a^3*b^2*c + a^4*c^2)*x + sq
rt(1/2)*(b^7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*c^3 + (b^4*c^5 - 6*a*b^
2*c^6 + 8*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 +
 a^4*c^4)/(b^2*c^10 - 4*a*c^11)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b
^2*c^5 - 4*a*c^6)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 +
a^4*c^4)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))) + 3*sqrt(1/2)*c^2*sq
rt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^2*c^5 - 4*a*c^6)*sqrt((b^8 - 6*a*b^
6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*c^11)))/(b^
2*c^5 - 4*a*c^6))*log(2*(a^2*b^4 - 3*a^3*b^2*c + a^4*c^2)*x - sqrt(1/2)*(b^
7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*c^3 + (b^4*c^5 - 6*a*b^2*c^6 + 8*a
^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(
b^2*c^10 - 4*a*c^11)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^2*c^5 - 4*
a*c^6)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b
^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))) - 6*b*x)/c^2
```

giac [B] time = 2.04, size = 2457, normalized size = 12.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^5+b*x^3+a*x),x, algorithm="giac")
```

```
[Out] -1/8*(2*b^6*c^4 - 14*a*b^4*c^5 + 24*a^2*b^2*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c^2 + 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*b^5*c^3 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a*b^3*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*b^4*c^4 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*a*b^2*c^5 - 2*(b^2 - 4*a*c)*b^4*c^4 + 6*(b^2 - 4*a*c)*a*b^2*c^5 -
 (2*b^6*c^2 - 18*a*b^4*c^3 + 48*a^2*b^2*c^4 - 32*a^3*c^5 - sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6 + 9*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*b^4*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a^2*b*c^3 + 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a*b^2*c^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^2 + 10*(b^2 - 4*a*c)*a*b^2*c^3 - 8*(b^2 -
 4*a*c)*a^2*c^4)*c^2 - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2
```

```
 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 - 2*sqrt(2)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 + 2*a*b^5*c^3 + 16*sqrt(2)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^3*b*c^4 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
^2*b^2*c^4 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 - 16*a^2*b^3
*c^4 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 + 32*a^3*b*c^5 -
 2*(b^2 - 4*a*c)*a*b^3*c^3 + 8*(b^2 - 4*a*c)*a^2*b*c^4)*abs(c))*arctan(2*sq
rt(1/2)*x/sqrt((b*c^3 + sqrt(b^2*c^6 - 4*a*c^7))/c^4))/((a*b^4*c^4 - 8*a^2*
b^2*c^5 - 2*a*b^3*c^5 + 16*a^3*c^6 + 8*a^2*b*c^6 + a*b^2*c^6 - 4*a^2*c^7)*c
^2) + 1/8*(2*b^6*c^4 - 14*a*b^4*c^5 + 24*a^2*b^2*c^6 - sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c^2 + 7*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^3 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*b^4*c^4 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a*b^2*c^5 - 2*(b^2 - 4*a*c)*b^4*c^4 + 6*(b^2 - 4*a*c)*a*b^2*
c^5 - (2*b^6*c^2 - 18*a*b^4*c^3 + 48*a^2*b^2*c^4 - 32*a^3*c^5 - sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6 + 9*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^2*b*c^3 + 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a*b^2*c^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^2 + 10*(b^2 - 4*a*c)*a*b^2*c^3 - 8*(
b^2 - 4*a*c)*a^2*c^4)*c^2 + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^
5*c^2 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 - 2*sqrt(2)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 - 2*a*b^5*c^3 + 16*sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^2*b^2*c^4 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 + 16*a^
2*b^3*c^4 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 - 32*a^3*b*
c^5 + 2*(b^2 - 4*a*c)*a*b^3*c^3 - 8*(b^2 - 4*a*c)*a^2*b*c^4)*abs(c))*arctan
(2*sqrt(1/2)*x/sqrt((b*c^3 - sqrt(b^2*c^6 - 4*a*c^7))/c^4))/((a*b^4*c^4 - 8
*a^2*b^2*c^5 - 2*a*b^3*c^5 + 16*a^3*c^6 + 8*a^2*b*c^6 + a*b^2*c^6 - 4*a^2*c
^7)*c^2) + 1/3*(c^2*x^3 - 3*b*c*x)/c^3
```

maple [B] time = 0.02, size = 467, normalized size = 2.30

$$-\frac{3\sqrt{2}\, ab \operatorname{arctanh}\left(\frac{\sqrt{2}\, cx}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}\; c}-\frac{3\sqrt{2}\, ab \arctan\left(\frac{\sqrt{2}\, cx}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}\; c}+\frac{\sqrt{2}\, b^3 \operatorname{arctanh}\left(\frac{\sqrt{2}\, c\cdots}{\sqrt{\left(-b+\sqrt{-4\cdots}\right.}}\right)}{2\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac\cdots}\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(c*x^5+b*x^3+a*x),x)
```

```
[Out] 1/3/c*x^3-b/c^2*x+1/2/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2
^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a-1/2/c^2*2^(1/2)/((-b+(-4*a*
c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*
x)*b^2-3/2/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*a
rctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*b+1/2/c^2/(-4*a*c+b
^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^3-1/2/c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a+1/2/c^2*2^(1
/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))
*c)^(1/2)*c*x)*b^2-3/2/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))
```

```
*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*b+1/2/c^2/
(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/
((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{cx^3 - 3\,bx}{3\,c^2} - \frac{-\int \frac{(b^2-ac)x^2+ab}{cx^4+bx^2+a}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^5+b*x^3+a*x),x, algorithm="maxima")
```

```
[Out] 1/3*(c*x^3 - 3*b*x)/c^2 - integrate(-((b^2 - a*c)*x^2 + a*b)/(c*x^4 + b*x^2
 + a), x)/c^2
```

mupad [B] time = 2.72, size = 4127, normalized size = 20.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a*x + b*x^3 + c*x^5),x)
```

```
[Out] x^3/(3*c) - atan(((((4*a*b^3*c^3 - 16*a^2*b*c^4)/c^3 - (2*x*(4*b^3*c^5 - 16
*a*b*c^6)*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3
*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c -
b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2))/c^3)*(-(b^7
 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(
-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(
8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) - (2*x*(b^6 - 2*a^3*c^3 + 9*
a^2*b^2*c^2 - 6*a*b^4*c))/c^3)*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a
^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c -
3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)
))^(1/2)*1i - (((4*a*b^3*c^3 - 16*a^2*b*c^4)/c^3 + (2*x*(4*b^3*c^5 - 16*a*b
*c^6)*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2
 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)
^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2))/c^3)*(-(b^7 + b
^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*
a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(1
6*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (2*x*(b^6 - 2*a^3*c^3 + 9*a^2*
b^2*c^2 - 6*a*b^4*c))/c^3)*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b
*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*
b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(
1/2)*1i)/((((4*a*b^3*c^3 - 16*a^2*b*c^4)/c^3 - (2*x*(4*b^3*c^5 - 16*a*b*c^6
)*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a
^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^
(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2))/c^3)*(-(b^7 + b^4*(
-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c
- b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^
2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) - (2*x*(b^6 - 2*a^3*c^3 + 9*a^2*b^2*
c^2 - 6*a*b^4*c))/c^3)*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3
 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*
c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)
 + (((4*a*b^3*c^3 - 16*a^2*b*c^4)/c^3 + (2*x*(4*b^3*c^5 - 16*a*b*c^6)*(-(b^
7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*
(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/
(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2))/c^3)*(-(b^7 + b^4*(-(4*a*c
 - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^
3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 +
 b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (2*x*(b^6 - 2*a^3*c^3 + 9*a^2*b^2*c^2 - 6
```

```
*a*b^4*c))/c^3)*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a
^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*
a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (2*(
a^4*c - a^3*b^2))/c^3))*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^
3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2
*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2
)*2i - atan(((((4*a*b^3*c^3 - 16*a^2*b*c^4)/c^3 - (2*x*(4*b^3*c^5 - 16*a*b*
c^6)*((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 +
 a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3
)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2))/c^3)*((b^4*(-(4*a
*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c
 - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a
^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) - (2*x*(b^6 - 2*a^3*c^3 + 9*a^2*b^2
*c^2 - 6*a*b^4*c))/c^3)*((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3
 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*
c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)
*1i - (((4*a*b^3*c^3 - 16*a^2*b*c^4)/c^3 + (2*x*(4*b^3*c^5 - 16*a*b*c^6)*((
b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^
2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2)
)/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2))/c^3)*((b^4*(-(4*a*c - b^
2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)
^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7
+ b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (2*x*(b^6 - 2*a^3*c^3 + 9*a^2*b^2*c^2 -
6*a*b^4*c))/c^3)*((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a
^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*
a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*1i)/((
((4*a*b^3*c^3 - 16*a^2*b*c^4)/c^3 - (2*x*(4*b^3*c^5 - 16*a*b*c^6)*((b^4*(-(
4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*
a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(1
6*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2))/c^3)*((b^4*(-(4*a*c - b^2)^3)^(
1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/
2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c
^5 - 8*a*b^2*c^6)))^(1/2) - (2*x*(b^6 - 2*a^3*c^3 + 9*a^2*b^2*c^2 - 6*a*b^4
*c))/c^3)*((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*
c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b
^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (((4*a*b^3*
c^3 - 16*a^2*b*c^4)/c^3 + (2*x*(4*b^3*c^5 - 16*a*b*c^6)*((b^4*(-(4*a*c - b^
2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)
^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7
+ b^4*c^5 - 8*a*b^2*c^6)))^(1/2))/c^3)*((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7
 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b
^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b
^2*c^6)))^(1/2) + (2*x*(b^6 - 2*a^3*c^3 + 9*a^2*b^2*c^2 - 6*a*b^4*c))/c^3)*
((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*
c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/
2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (2*(a^4*c - a^3*b^2))
/c^3))*((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2
 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)
^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*2i - (b*x)/c^2
```

sympy [**A**] time = 4.25, size = 194, normalized size = 0.96

$$-\frac{bx}{c^2}+\mathrm{RootSum}\left(t^4\left(256a^2c^7-128ab^2c^6+16b^4c^5\right)+t^2\left(-80a^3bc^3+100a^2b^3c^2-36ab^5c+4b^7\right)+a^5,\left(t\mapsto t\log\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(c*x**5+b*x**3+a*x),x)
```

```
[Out] -b*x/c**2 + RootSum(_t**4*(256*a**2*c**7 - 128*a*b**2*c**6 + 16*b**4*c**5)
+ _t**2*(-80*a**3*b*c**3 + 100*a**2*b**3*c**2 - 36*a*b**5*c + 4*b**7) + a**
```

```
5, Lambda(_t, _t*log(x + (-64*_t**3*a**2*c**7 + 48*_t**3*a*b**2*c**6 - 8*_t
**3*b**4*c**5 + 14*_t*a**3*b*c**3 - 28*_t*a**2*b**3*c**2 + 14*_t*a*b**5*c -
 2*_t*b**7)/(a**4*c**2 - 3*a**3*b**2*c + a**2*b**4)))) + x**3/(3*c)
```

3.80 $\int \frac{x^6}{ax+bx^3+cx^5}\,dx$

Optimal. Leaf size=81

$$-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx^2+cx^4\right)}{4c^2}+\frac{x^2}{2c}$$

```
[Out] 1/2*x^2/c-1/4*b*ln(c*x^4+b*x^2+a)/c^2-1/2*(-2*a*c+b^2)*arctanh((2*c*x^2+b)/
(-4*a*c+b^2)^(1/2))/c^2/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1585, 1114, 703, 634, 618, 206, 628}

$$-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx^2+cx^4\right)}{4c^2}+\frac{x^2}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a*x + b*x^3 + c*x^5),x]
```

```
[Out] x^2/(2*c) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^2
*Sqrt[b^2 - 4*a*c]) - (b*Log[a + b*x^2 + c*x^4])/(4*c^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 703

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^
(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^6}{ax+bx^3+cx^5}\,dx &= \int \frac{x^5}{a+bx^2+cx^4}\,dx\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2}{a+bx+cx^2}\,dx, x, x^2\right)\\
&= \frac{x^2}{2c}+\frac{\operatorname{Subst}\left(\int \frac{-a-bx}{a+bx+cx^2}\,dx, x, x^2\right)}{2c}\\
&= \frac{x^2}{2c}-\frac{b\operatorname{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)}{4c^2}+\frac{\left(b^2-2ac\right)\operatorname{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^2\right)}{4c^2}\\
&= \frac{x^2}{2c}-\frac{b\log\left(a+bx^2+cx^4\right)}{4c^2}-\frac{\left(b^2-2ac\right)\operatorname{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^2\right)}{2c^2}\\
&= \frac{x^2}{2c}-\frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx^2+cx^4\right)}{4c^2}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 78, normalized size = 0.96

$$\frac{\frac{2\left(b^2-2ac\right)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}-b\log\left(a+bx^2+cx^4\right)+2cx^2}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a*x + b*x^3 + c*x^5),x]
```

```
[Out] (2*c*x^2 + (2*(b^2 - 2*a*c)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[
-b^2 + 4*a*c] - b*Log[a + b*x^2 + c*x^4])/(4*c^2)
```

fricas [A] time = 0.68, size = 254, normalized size = 3.14

$$\left[\frac{2\left(b^2c-4\,ac^2\right)x^2-\left(b^2-2\,ac\right)\sqrt{b^2-4\,ac}\,\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac+\left(2\,cx^2+b\right)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)-\left(b^3-4\,abc\right)\log\left(cx^4+bx\right.}{4\left(b^2c^2-4\,ac^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^5+b*x^3+a*x),x, algorithm="fricas")
```

```
[Out] [1/4*(2*(b^2*c - 4*a*c^2)*x^2 - (b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*
x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b
*x^2 + a)) - (b^3 - 4*a*b*c)*log(c*x^4 + b*x^2 + a))/(b^2*c^2 - 4*a*c^3), 1
```

```
/4*(2*(b^2*c - 4*a*c^2)*x^2 - 2*(b^2 - 2*a*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2
*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (b^3 - 4*a*b*c)*log(c*x^4 +
 b*x^2 + a))/(b^2*c^2 - 4*a*c^3)]
```

giac [A] time = 0.63, size = 75, normalized size = 0.93

$$\frac{x^2}{2\,c} - \frac{b\log\left(cx^4 + bx^2 + a\right)}{4\,c^2} + \frac{\left(b^2 - 2\,ac\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2 + 4\,ac}\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^5+b*x^3+a*x),x, algorithm="giac")
```

```
[Out] 1/2*x^2/c - 1/4*b*log(c*x^4 + b*x^2 + a)/c^2 + 1/2*(b^2 - 2*a*c)*arctan((2*
c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^2)
```

maple [A] time = 0.00, size = 111, normalized size = 1.37

$$-\frac{a\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\;c} + \frac{b^2\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}\;c^2} + \frac{x^2}{2c} - \frac{b\ln\left(c\,x^4 + b\,x^2 + a\right)}{4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(c*x^5+b*x^3+a*x),x)
```

```
[Out] 1/2/c*x^2-1/4*b*ln(c*x^4+b*x^2+a)/c^2-1/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2
+b)/(4*a*c-b^2)^(1/2))*a+1/2/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*
c-b^2)^(1/2))*b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{x^2}{2\,c} - \frac{\frac{b\log\left(cx^4+bx^2+a\right)}{4\,c} - \frac{\left(b^2-2\,ac\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}\,c}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^5+b*x^3+a*x),x, algorithm="maxima")
```

```
[Out] 1/2*x^2/c - integrate((b*x^3 + a*x)/(c*x^4 + b*x^2 + a), x)/c
```

mupad [B] time = 2.44, size = 655, normalized size = 8.09

$$\frac{x^2}{2\,c}+\frac{\ln\left(c\,x^4+b\,x^2+a\right)\,\left(2\,b^3-8\,a\,b\,c\right)}{2\,\left(16\,a\,c^3-4\,b^2\,c^2\right)}+\frac{\mathrm{atan}\left(\frac{2\,c^2\,\left(4\,a\,c-b^2\right)\left(\frac{\frac{\left(8\,a\,b+\frac{8\,a\,c^2\,\left(2\,b^3-8\,a\,b\,c\right)}{16\,a\,c^3-4\,b^2\,c^2}\right)\left(2\,a\,c-b^2\right)}{8\,c^2\,\sqrt{4\,a\,c-b^2}}+\frac{a\,\left(2\,b^3-8\,a\,b\,c\right)\left(2\,a\,c-b^2\right)}{\sqrt{4\,a\,c-b^2}\,\left(16\,a\,c^3-4\,b^2\,c^2\right)}}{a}-x^2\left(\frac{\left(2\,a\,c-b^2\right)\cdots}{\cdots}\right.\right.}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a*x + b*x^3 + c*x^5),x)
```

```
[Out] x^2/(2*c) + (log(a + b*x^2 + c*x^4)*(2*b^3 - 8*a*b*c))/(2*(16*a*c^3 - 4*b^2
*c^2)) + (atan((2*c^2*(4*a*c - b^2)*((((8*a*b + (8*a*c^2*(2*b^3 - 8*a*b*c))
/(16*a*c^3 - 4*b^2*c^2))*(2*a*c - b^2))/(8*c^2*(4*a*c - b^2)^(1/2)) + (a*(2
*b^3 - 8*a*b*c)*(2*a*c - b^2))/((4*a*c - b^2)^(1/2)*(16*a*c^3 - 4*b^2*c^2))
)/a - x^2*((((2*a*c - b^2)*((4*a*c^3 - 6*b^2*c^2)/c^2 - (4*b*c^2*(2*b^3 - 8
*a*b*c))/(16*a*c^3 - 4*b^2*c^2)))/(8*c^2*(4*a*c - b^2)^(1/2)) - (b*(2*b^3 -
 8*a*b*c)*(2*a*c - b^2))/(2*(4*a*c - b^2)^(1/2)*(16*a*c^3 - 4*b^2*c^2)))/a
+ (b*(((2*b^3 - 8*a*b*c)*((4*a*c^3 - 6*b^2*c^2)/c^2 - (4*b*c^2*(2*b^3 - 8*a
*b*c))/(16*a*c^3 - 4*b^2*c^2)))/(2*(16*a*c^3 - 4*b^2*c^2)) - (b^3 - a*b*c)/
c^2 + (b*(2*a*c - b^2)^2)/(2*c^2*(4*a*c - b^2))))/(2*a*(4*a*c - b^2)^(1/2))
) + (b*((a*b^2)/c^2 + ((2*b^3 - 8*a*b*c)*(8*a*b + (8*a*c^2*(2*b^3 - 8*a*b*c
))/(16*a*c^3 - 4*b^2*c^2)))/(2*(16*a*c^3 - 4*b^2*c^2)) - (a*(2*a*c - b^2)^2
)/(c^2*(4*a*c - b^2))))/(2*a*(4*a*c - b^2)^(1/2))))/(b^4 + 4*a^2*c^2 - 4*a*
b^2*c))*(2*a*c - b^2))/(2*c^2*(4*a*c - b^2)^(1/2))
```

sympy [B] time = 1.95, size = 316, normalized size = 3.90

$$\left(-\frac{b}{4c^2} - \frac{\sqrt{-4ac+b^2}\left(2ac-b^2\right)}{4c^2\left(4ac-b^2\right)}\right)\log\left(x^2 + \frac{-ab - 8ac^2\left(-\frac{b}{4c^2} - \frac{\sqrt{-4ac+b^2}\left(2ac-b^2\right)}{4c^2\left(4ac-b^2\right)}\right) + 2b^2c\left(-\frac{b}{4c^2} - \frac{\sqrt{-4ac+b^2}\left(2ac-b\right.}{4c^2\left(4ac-b^2\right)}\right.}{2ac-b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(c*x**5+b*x**3+a*x),x)
```

```
[Out] (-b/(4*c**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*c**2*(4*a*c - b**2)))*
log(x**2 + (-a*b - 8*a*c**2*(-b/(4*c**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**
2)/(4*c**2*(4*a*c - b**2))) + 2*b**2*c*(-b/(4*c**2) - sqrt(-4*a*c + b**2)*(
2*a*c - b**2)/(4*c**2*(4*a*c - b**2))))/(2*a*c - b**2)) + (-b/(4*c**2) + sq
rt(-4*a*c + b**2)*(2*a*c - b**2)/(4*c**2*(4*a*c - b**2)))*log(x**2 + (-a*b
- 8*a*c**2*(-b/(4*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*c**2*(4*a*c
 - b**2))) + 2*b**2*c*(-b/(4*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*
c**2*(4*a*c - b**2))))/(2*a*c - b**2)) + x**2/(2*c)
```

3.81 $\int \frac{x^5}{ax+bx^3+cx^5} \, dx$

Optimal. Leaf size=179

$$-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x}{c}$$

```
[Out] x/c-1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b+(2*a*c-b^
2)/(-4*a*c+b^2)^(1/2))/c^(3/2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/2*arc
tan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b+(-2*a*c+b^2)/(-4*a*c
+b^2)^(1/2))/c^(3/2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1585, 1122, 1166, 205}

$$-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a*x + b*x^3 + c*x^5),x]
```

```
[Out] x/c - ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqr
t[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) -
((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b +
Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
```

```
Q[r - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5}{ax+bx^3+cx^5}\,dx &= \int \frac{x^4}{a+bx^2+cx^4}\,dx \\
&= \frac{x}{c}-\frac{\int \frac{a+bx^2}{a+bx^2+cx^4}\,dx}{c} \\
&= \frac{x}{c}-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2c}-\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2c} \\
&= \frac{x}{c}-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}
\end{aligned}
$$

Mathematica [A] time = 0.11, size = 202, normalized size = 1.13

$$
-\frac{\left(b\sqrt{b^2-4ac}+2ac-b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(b\sqrt{b^2-4ac}-2ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x}{c}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a*x + b*x^3 + c*x^5),x]
```

```
[Out] x/c - ((-b^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt
[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[
b^2 - 4*a*c]]) - ((b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[
c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt
[b + Sqrt[b^2 - 4*a*c]])
```

fricas [B] time = 0.65, size = 1059, normalized size = 5.92

$$
-\frac{\sqrt{\frac{1}{2}}\,c\sqrt{-\frac{b^3-3\,abc+\left(b^2c^3-4\,ac^4\right)\sqrt{\frac{b^4-2\,ab^2c+a^2c^2}{b^2c^6-4\,ac^7}}}{b^2c^3-4\,ac^4}}\,\log\left(-2\left(ab^2-a^2c\right)x+\sqrt{\frac{1}{2}}\left(b^4-5\,ab^2c+4\,a^2c^2-\left(b^3c^3-4\,abc^4\right)\sqrt{\cdots}\right.\right.}{\cdots}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^5+b*x^3+a*x),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(1/2)*c*sqrt(-(b^3 - 3*a*b*c + (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*
a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-2*(a*b^2
 - a^2*c)*x + sqrt(1/2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2 - (b^3*c^3 - 4*a*b*c^4
)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(b^3 - 3*a*b
*c + (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^
7)))/(b^2*c^3 - 4*a*c^4))) - sqrt(1/2)*c*sqrt(-(b^3 - 3*a*b*c + (b^2*c^3 -
4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 -
4*a*c^4))*log(-2*(a*b^2 - a^2*c)*x - sqrt(1/2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2
 - (b^3*c^3 - 4*a*b*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^
7)))*sqrt(-(b^3 - 3*a*b*c + (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2
*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) + sqrt(1/2)*c*sqrt(-(b^3
```

```
- 3*a*b*c - (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 -
 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-2*(a*b^2 - a^2*c)*x + sqrt(1/2)*(b^4
- 5*a*b^2*c + 4*a^2*c^2 + (b^3*c^3 - 4*a*b*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2
*c^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(b^3 - 3*a*b*c - (b^2*c^3 - 4*a*c^4)*sqrt
((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) -
sqrt(1/2)*c*sqrt(-(b^3 - 3*a*b*c - (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*
c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-2*(a*b^2 - a^2
*c)*x - sqrt(1/2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2 + (b^3*c^3 - 4*a*b*c^4)*sqrt
((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(b^3 - 3*a*b*c - (
b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(
b^2*c^3 - 4*a*c^4))) - 2*x)/c
```

giac [B] time = 1.50, size = 2109, normalized size = 11.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^5+b*x^3+a*x),x, algorithm="giac")
```

```
[Out] x/c + 1/8*(2*b^5*c^4 - 12*a*b^3*c^5 + 16*a^2*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*b^3*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*a*b*c^5 - 2*(b^2 - 4*a*c)*b^3*c^4 + 4*(b^2 - 4*a*c)*a*b*c^5 - (2*
b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*b^4*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a^2*b*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c
^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 -
2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*c^2 - 2*(sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a^2*b^2*c^3 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*a
*b^4*c^3 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 8*sqrt(2)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a*b^2*c^4 - 16*a^2*b^2*c^4 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a^2*c^5 + 32*a^3*c^5 - 2*(b^2 - 4*a*c)*a*b^2*c^3 + 8*(b^2 - 4*a*c)*a^2*
c^4)*abs(c))*arctan(2*sqrt(1/2)*x/sqrt((b*c + sqrt(b^2*c^2 - 4*a*c^3))/c^2)
)/((a*b^4*c^3 - 8*a^2*b^2*c^4 - 2*a*b^3*c^4 + 16*a^3*c^5 + 8*a^2*b*c^5 + a*
b^2*c^5 - 4*a^2*c^6)*c^2) - 1/8*(2*b^5*c^4 - 12*a*b^3*c^5 + 16*a^2*b*c^6 -
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 6*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 8*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 4*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^5 - 2*(b^2 - 4*a*c)*b^3*c^4 + 4*(b^2
- 4*a*c)*a*b*c^5 - (2*b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4 - sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*c^
2 + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - 8*sqrt(2)*sqrt(b
```

```
*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a*b^3*c^3 - 2*a*b^4*c^3 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^3*c^4 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 + 16*a^2*b^2*c^4 - 4*sqrt(2)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^5 - 32*a^3*c^5 + 2*(b^2 - 4*a*c)*a*b^2*c^3
- 8*(b^2 - 4*a*c)*a^2*c^4)*abs(c))*arctan(2*sqrt(1/2)*x/sqrt((b*c - sqrt(b^
2*c^2 - 4*a*c^3))/c^2))/((a*b^4*c^3 - 8*a^2*b^2*c^4 - 2*a*b^3*c^4 + 16*a^3*
c^5 + 8*a^2*b*c^5 + a*b^2*c^5 - 4*a^2*c^6)*c^2)
```

maple [B] time = 0.01, size = 343, normalized size = 1.92

$$\frac{\sqrt{2}\,a\,\mathrm{arctanh}\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\frac{\sqrt{2}\,a\arctan\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\sqrt{2}\,b^2\,\mathrm{arctanh}\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right.}}\right.}{2\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b\right.}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(c*x^5+b*x^3+a*x),x)
```

```
[Out] 1/c*x+1/2/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+
(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b+1/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a
*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c
*x)*a-1/2/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*ar
ctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2-1/2/c*2^(1/2)/((b+
(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2
)*c*x)*b+1/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arct
an(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a-1/2/c/(-4*a*c+b^2)^(1/2)
*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(
1/2))*c)^(1/2)*c*x)*b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{x}{c}-\frac{\left(\sqrt{2}\,\sqrt{bc+\sqrt{b^2-4\,ac}\,c}\,b^4-8\,\sqrt{2}\,\sqrt{bc+\sqrt{b^2-4\,ac}\,c}\,ab^2c-2\,\sqrt{2}\,\sqrt{bc+\sqrt{b^2-4\,ac}\,c}\,b^3c-2\,b^4c+16\,\sqrt{2}\,\sqrt{bc+\sqrt{b^2-4\,ac}\,c}\,a^2c^2+8\,\sqrt{2}\,\sqrt{bc+\sqrt{b^2-4\,ac}\,c}\,ab\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^5+b*x^3+a*x),x, algorithm="maxima")
```

```
[Out] x/c - integrate((b*x^2 + a)/(c*x^4 + b*x^2 + a), x)/c
```

mupad [B] time = 2.58, size = 3026, normalized size = 16.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a*x + b*x^3 + c*x^5),x)
```

```
[Out] x/c - atan(((((16*a^2*c^3 - 4*a*b^2*c^2)/c - (2*x*(4*b^3*c^3 - 16*a*b*c^4)*
(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4
*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*(
-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*
a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (2*x
*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) +
12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b
^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i - (((16*a^2*c^3 - 4*a*b^2*c^2)/c + (2*x*(4
```

```
*b^3*c^3 - 16*a*b*c^4)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2
 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a
*b^2*c^4)))^(1/2))/c)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2
- 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*
b^2*c^4)))^(1/2) + (2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c)*(-(b^5 + b^2*(-(4
*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1
/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i)/((((16*a^2*c^3 - 4
*a*b^2*c^2)/c - (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(b^5 + b^2*(-(4*a*c - b^2)^
3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*
a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(b^5 + b^2*(-(4*a*c - b^2)^3
)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a
^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c
))/c)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*
c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)
 + (((16*a^2*c^3 - 4*a*b^2*c^2)/c + (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(b^5 +
b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^
2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(b^5 + b
^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2
)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (2*x*(b^4 + 2
*a^2*c^2 - 4*a*b^2*c))/c)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*
c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 -
8*a*b^2*c^4)))^(1/2) + (2*a^2*b)/c))*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2)
+ 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 +
 b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*2i - atan(((((16*a^2*c^3 - 4*a*b^2*c^2)/c -
 (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a
^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c
^3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^
2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^
3 - 8*a*b^2*c^4)))^(1/2) - (2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c)*(-(b^5 -
b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^
2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i - (((16*a^2
*c^3 - 4*a*b^2*c^2)/c + (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(b^5 - b^2*(-(4*a*c
 - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))
/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(b^5 - b^2*(-(4*a*c
- b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/
(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (2*x*(b^4 + 2*a^2*c^2 - 4
*a*b^2*c))/c)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^
3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)
))^(1/2)*1i)/((((16*a^2*c^3 - 4*a*b^2*c^2)/c - (2*x*(4*b^3*c^3 - 16*a*b*c^4
)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-
(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)
*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(
4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (2
*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2)
+ 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 +
 b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (((16*a^2*c^3 - 4*a*b^2*c^2)/c + (2*x*(4*
b^3*c^3 - 16*a*b*c^4)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2
- 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*
b^2*c^4)))^(1/2))/c)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 -
 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b
^2*c^4)))^(1/2) + (2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c)*(-(b^5 - b^2*(-(4*
a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/
2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (2*a^2*b)/c))*(-(b^5
- b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c -
b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*2i
```

sympy [A] time = 2.18, size = 129, normalized size = 0.72

$$\mathrm{RootSum}\left(t^4\left(256a^2c^5-128ab^2c^4+16b^4c^3\right)+t^2\left(48a^2bc^2-28ab^3c+4b^5\right)+a^3,\left(t\mapsto t\log\left(x+\frac{32t^3abc^4-8t^3b\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(c*x**5+b*x**3+a*x),x)
```

```
[Out] RootSum(_t**4*(256*a**2*c**5 - 128*a*b**2*c**4 + 16*b**4*c**3) + _t**2*(48*
a**2*b*c**2 - 28*a*b**3*c + 4*b**5) + a**3, Lambda(_t, _t*log(x + (32*_t**3
*a*b*c**4 - 8*_t**3*b**3*c**3 - 4*_t*a**2*c**2 + 8*_t*a*b**2*c - 2*_t*b**4)
/(a**2*c - a*b**2)))) + x/c
```

3.82 $\int \frac{x^4}{ax+bx^3+cx^5}\,dx$

Optimal. Leaf size=63

$$\frac{b\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}}+\frac{\log\left(a+bx^2+cx^4\right)}{4c}$$

```
[Out] 1/4*ln(c*x^4+b*x^2+a)/c+1/2*b*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/c/(-4
*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1585, 1114, 634, 618, 206, 628}

$$\frac{b\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}}+\frac{\log\left(a+bx^2+cx^4\right)}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a*x + b*x^3 + c*x^5),x]
```

```
[Out] (b*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c*Sqrt[b^2 - 4*a*c]) + Log[
a + b*x^2 + c*x^4]/(4*c)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
```

```
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{ax+bx^3+cx^5}\,dx &= \int \frac{x^3}{a+bx^2+cx^4}\,dx\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{a+bx+cx^2}\,dx,x,x^2\right)\\ &= \frac{\operatorname{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^2\right)}{4c}-\frac{b\operatorname{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,x^2\right)}{4c}\\ &= \frac{\log\left(a+bx^2+cx^4\right)}{4c}+\frac{b\operatorname{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx^2\right)}{2c}\\ &= \frac{b\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}}+\frac{\log\left(a+bx^2+cx^4\right)}{4c}\end{aligned}$$

Mathematica [A] time = 0.02, size = 62, normalized size = 0.98

$$\frac{\log\left(a+bx^2+cx^4\right)-\frac{2b\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{4c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a*x + b*x^3 + c*x^5),x]
```

```
[Out] ((-2*b*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + Log[a
 + b*x^2 + c*x^4])/(4*c)
```

fricas [A] time = 0.79, size = 197, normalized size = 3.13

$$\left[\frac{\sqrt{b^2-4\,ac}\,b\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac+\left(2\,cx^2+b\right)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)+\left(b^2-4\,ac\right)\log\left(cx^4+bx^2+a\right)}{4\left(b^2c-4\,ac^2\right)},\ \frac{2\sqrt{-b^2+4\,ac}\,b\arctan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^5+b*x^3+a*x),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x
^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + (b^2 - 4*a*c)*log(c*x^4 +
 b*x^2 + a))/(b^2*c - 4*a*c^2), 1/4*(2*sqrt(-b^2 + 4*a*c)*b*arctan(-(2*c*x^
2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (b^2 - 4*a*c)*log(c*x^4 + b*x^2
+ a))/(b^2*c - 4*a*c^2)]
```

giac [A] time = 0.42, size = 59, normalized size = 0.94

$$-\frac{b\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\sqrt{-b^2+4\,ac}\,c}+\frac{\log\left(cx^4+bx^2+a\right)}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^5+b*x^3+a*x),x, algorithm="giac")
```

```
[Out] -1/2*b*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c) + 1/
4*log(c*x^4 + b*x^2 + a)/c
```

maple [A] time = 0.00, size = 60, normalized size = 0.95

$$-\frac{b \arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}\;c}+\frac{\ln\left(c\,x^4+b\,x^2+a\right)}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^5+b*x^3+a*x),x)
```

```
[Out] 1/4*ln(c*x^4+b*x^2+a)/c-1/2*b/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c
-b^2)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{cx^5+bx^3+ax}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^5+b*x^3+a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(c*x^5 + b*x^3 + a*x), x)
```

mupad [B] time = 0.17, size = 118, normalized size = 1.87

$$\frac{4\,a\,c\,\ln\left(c\,x^4+b\,x^2+a\right)}{16\,a\,c^2-4\,b^2\,c}-\frac{b^2\,\ln\left(c\,x^4+b\,x^2+a\right)}{16\,a\,c^2-4\,b^2\,c}-\frac{b\,\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,x^2}{\sqrt{4\,a\,c-b^2}}\right)}{2\,c\,\sqrt{4\,a\,c-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a*x + b*x^3 + c*x^5),x)
```

```
[Out] (4*a*c*log(a + b*x^2 + c*x^4))/(16*a*c^2 - 4*b^2*c) - (b^2*log(a + b*x^2 +
c*x^4))/(16*a*c^2 - 4*b^2*c) - (b*atan(b/(4*a*c - b^2)^(1/2) + (2*c*x^2)/(4
*a*c - b^2)^(1/2)))/(2*c*(4*a*c - b^2)^(1/2))
```

sympy [B] time = 0.92, size = 223, normalized size = 3.54

$$\left(-\frac{b\sqrt{-4ac+b^2}}{4c\left(4ac-b^2\right)}+\frac{1}{4c}\right)\log\left(x^2+\frac{-8ac\left(-\frac{b\sqrt{-4ac+b^2}}{4c\left(4ac-b^2\right)}+\frac{1}{4c}\right)+2a+2b^2\left(-\frac{b\sqrt{-4ac+b^2}}{4c\left(4ac-b^2\right)}+\frac{1}{4c}\right)}{b}\right)+\left(\frac{b\sqrt{-4ac+b^2}}{4c\left(4ac-b^2\right)}+\frac{1}{4c}\right)\mathrm{l}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**5+b*x**3+a*x),x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)/(4*c*(4*a*c - b**2)) + 1/(4*c))*log(x**2 + (-8*a*c*
(-b*sqrt(-4*a*c + b**2)/(4*c*(4*a*c - b**2)) + 1/(4*c)) + 2*a + 2*b**2*(-b*
sqrt(-4*a*c + b**2)/(4*c*(4*a*c - b**2)) + 1/(4*c)))/b) + (b*sqrt(-4*a*c +
b**2)/(4*c*(4*a*c - b**2)) + 1/(4*c))*log(x**2 + (-8*a*c*(b*sqrt(-4*a*c + b
**2)/(4*c*(4*a*c - b**2)) + 1/(4*c)) + 2*a + 2*b**2*(b*sqrt(-4*a*c + b**2)/
(4*c*(4*a*c - b**2)) + 1/(4*c)))/b)
```

3.83 $\int \frac{x^3}{ax+bx^3+cx^5} dx$

Optimal. Leaf size=150

$$\frac{\sqrt{\sqrt{b^2-4ac}+b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}-\frac{\sqrt{b-\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

```
[Out] -1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b-(-4*a*c+b^2)
^(1/2))^(1/2)*2^(1/2)/c^(1/2)/(-4*a*c+b^2)^(1/2)+1/2*arctan(x*2^(1/2)*c^(1/
2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b+(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2)/c^(1/
2)/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1585, 1130, 205}

$$\frac{\sqrt{\sqrt{b^2-4ac}+b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}-\frac{\sqrt{b-\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a*x + b*x^3 + c*x^5),x]
```

```
[Out] -((Sqrt[b - Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2
 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])) + (Sqrt[b + Sqrt[b^2 - 4*
a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqr
t[c]*Sqrt[b^2 - 4*a*c])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1130

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\int \frac{x^3}{ax+bx^3+cx^5}\,dx = \int \frac{x^2}{a+bx^2+cx^4}\,dx$$
$$= -\left(\frac{1}{2}\left(-1+\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx\right)+\frac{1}{2}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2}}$$
$$= -\frac{\sqrt{b-\sqrt{b^2-4ac}}\,\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}+\frac{\sqrt{b+\sqrt{b^2-4ac}}\,\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Mathematica [A] time = 0.08, size = 165, normalized size = 1.10

$$\frac{\left(\sqrt{b^2-4ac}-b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{\sqrt{b^2-4ac}+b}\,\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a*x + b*x^3 + c*x^5),x]
```

```
[Out] ((-b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*
a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (
Sqrt[b + Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 -
4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])
```

fricas [B] time = 0.76, size = 559, normalized size = 3.73

$$\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b+\frac{b^2c-4ac^2}{\sqrt{b^2c^2-4ac^3}}}{b^2c-4ac^2}}\log\left(\frac{\sqrt{\frac{1}{2}}\left(b^2c-4ac^2\right)\sqrt{-\frac{b+\frac{b^2c-4ac^2}{\sqrt{b^2c^2-4ac^3}}}{b^2c-4ac^2}}}{\sqrt{b^2c^2-4ac^3}}+x\right)-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b+\frac{b^2c-4ac^2}{\sqrt{b^2c^2-4ac^3}}}{b^2c-4ac^2}}\log\left(-\frac{\sqrt{\frac{1}{2}}\left(b^2c\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^5+b*x^3+a*x),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(1/2)*sqrt(-(b + (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c
- 4*a*c^2))*log(sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt(-(b + (b^2*c - 4*a*c^2)/sq
rt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a*c^3) + x) - 1/
2*sqrt(1/2)*sqrt(-(b + (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c -
4*a*c^2))*log(-sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt(-(b + (b^2*c - 4*a*c^2)/sqr
t(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a*c^3) + x) - 1/2
*sqrt(1/2)*sqrt(-(b - (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4
*a*c^2))*log(sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt(-(b - (b^2*c - 4*a*c^2)/sqrt(
b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a*c^3) + x) + 1/2*s
qrt(1/2)*sqrt(-(b - (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a
*c^2))*log(-sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt(-(b - (b^2*c - 4*a*c^2)/sqrt(b
^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a*c^3) + x)
```

giac [B] time = 1.81, size = 503, normalized size = 3.35

$$-\frac{\left(2b^2c^2-8ac^3-\sqrt{2}\sqrt{b^2-4ac}\sqrt{bc+\sqrt{b^2-4ac}\,c}\,b^2+4\sqrt{2}\sqrt{b^2-4ac}\sqrt{bc+\sqrt{b^2-4ac}\,c}\,ac+2\sqrt{2}\sqrt{b^2-4a}\right.}{2\left(b^4-8ab^2c-2b^3c+16a^2c^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^5+b*x^3+a*x),x, algorithm="giac")
```

```
[Out] -1/2*(2*b^2*c^2 - 8*a*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c - s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*c^2 - 2*(b^2 - 4*a
*c)*c^2)*arctan(2*sqrt(1/2)*x/sqrt((b + sqrt(b^2 - 4*a*c))/c))/((b^4 - 8*a*
b^2*c - 2*b^3*c + 16*a^2*c^2 + 8*a*b*c^2 + b^2*c^2 - 4*a*c^3)*abs(c)) + 1/2
*(2*b^2*c^2 - 8*a*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*
c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c - sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*c^2 - 2*(b^2 - 4*a*c)*
c^2)*arctan(2*sqrt(1/2)*x/sqrt((b - sqrt(b^2 - 4*a*c))/c))/((b^4 - 8*a*b^2*
c - 2*b^3*c + 16*a^2*c^2 + 8*a*b*c^2 + b^2*c^2 - 4*a*c^3)*abs(c))
```

maple [A] time = 0.01, size = 208, normalized size = 1.39

$$\frac{\sqrt{2}\, b \operatorname{arctanh}\left(\frac{\sqrt{2}\, cx}{\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{2\sqrt{-4ac+b^{2}}\,\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}}+\frac{\sqrt{2}\, b \arctan\left(\frac{\sqrt{2}\, cx}{\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{2\sqrt{-4ac+b^{2}}\,\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}}-\frac{\sqrt{2}\, \operatorname{arctanh}\left(\frac{\sqrt{2}\, cx}{\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{2\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^5+b*x^3+a*x),x)
```

```
[Out] -1/2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*c*x)+1/2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)
^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b+1
/2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)
^(1/2))*c)^(1/2)*c*x)+1/2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3}}{cx^{5}+bx^{3}+ax}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^5+b*x^3+a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(c*x^5 + b*x^3 + a*x), x)
```

mupad [B] time = 2.21, size = 416, normalized size = 2.77

$$-2\,\mathrm{atanh}\left(\frac{\left(x\,\left(4\,a\,c^2-2\,b^2\,c\right)+\frac{x\,\left(8\,b^3\,c^2-32\,a\,b\,c^3\right)\,\left(b^3+\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-4\,a\,b\,c\right)}{8\,\left(16\,a^2\,c^3-8\,a\,b^2\,c^2+b^4\,c\right)}\right)\,\sqrt{-\frac{b^3+\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-4\,a\,b\,c}{8\,\left(16\,a^2\,c^3-8\,a\,b^2\,c^2+b^4\,c\right)}}}{a\,c}\right)\,\sqrt{-\frac{b^3+\sqrt{-}}{8\,\left(16\,a^2\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*x + b*x^3 + c*x^5),x)
```

```
[Out] - 2*atanh(((x*(4*a*c^2 - 2*b^2*c) + (x*(8*b^3*c^2 - 32*a*b*c^3)*(b^3 + (-(4
*a*c - b^2)^3)^(1/2) - 4*a*b*c))/(8*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-
(b^3 + (-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c)/(8*(b^4*c + 16*a^2*c^3 - 8*a*b^2
*c^2)))^(1/2))/(a*c))*(-(b^3 + (-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c)/(8*(b^4*
c + 16*a^2*c^3 - 8*a*b^2*c^2)))^(1/2) - 2*atanh(((x*(4*a*c^2 - 2*b^2*c) - (
x*(8*b^3*c^2 - 32*a*b*c^3)*((-(4*a*c - b^2)^3)^(1/2) - b^3 + 4*a*b*c))/(8*(
b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(((-(4*a*c - b^2)^3)^(1/2) - b^3 + 4*a*
b*c)/(8*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))^(1/2))/(a*c))*(((-(4*a*c - b^2
)^3)^(1/2) - b^3 + 4*a*b*c)/(8*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))^(1/2)
```

sympy [**A**] time = 0.82, size = 75, normalized size = 0.50

$$\operatorname{RootSum}\left(t^4\left(256a^2c^3-128ab^2c^2+16b^4c\right)+t^2\left(-16abc+4b^3\right)+a,\left(t\mapsto t\log\left(64t^3ac^2-16t^3b^2c-2tb+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**5+b*x**3+a*x),x)
```

```
[Out] RootSum(_t**4*(256*a**2*c**3 - 128*a*b**2*c**2 + 16*b**4*c) + _t**2*(-16*a*
b*c + 4*b**3) + a, Lambda(_t, _t*log(64*_t**3*a*c**2 - 16*_t**3*b**2*c - 2*
_t*b + x)))
```

3.84 $\int \frac{x^2}{ax+bx^3+cx^5}\,dx$

Optimal. Leaf size=36

$$-\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

```
[Out] -arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1585, 1107, 618, 206}

$$-\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a*x + b*x^3 + c*x^5),x]
```

```
[Out] -(ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]]/Sqrt[b^2 - 4*a*c])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{ax+bx^3+cx^5}\,dx &= \int \frac{x}{a+bx^2+cx^4}\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^2\right)\\
&= -\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^2\right)\\
&= -\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 1.08

$$\frac{\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a*x + b*x^3 + c*x^5),x]
[Out] ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]]/Sqrt[-b^2 + 4*a*c]
```

fricas [A] time = 0.62, size = 129, normalized size = 3.58

$$\left[\frac{\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac-\left(2\,cx^2+b\right)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)}{2\,\sqrt{b^2-4\,ac}}, -\frac{\sqrt{-b^2+4\,ac}\,\arctan\left(-\frac{\left(2\,cx^2+b\right)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)}{b^2-4\,ac}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^5+b*x^3+a*x),x, algorithm="fricas")
[Out] [1/2*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*
a*c))/(c*x^4 + b*x^2 + a))/sqrt(b^2 - 4*a*c), -sqrt(-b^2 + 4*a*c)*arctan(-(
2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c))/(b^2 - 4*a*c)]
```

giac [A] time = 0.49, size = 35, normalized size = 0.97

$$\frac{\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^5+b*x^3+a*x),x, algorithm="giac")
[Out] arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/sqrt(-b^2 + 4*a*c)
```

maple [A] time = 0.00, size = 36, normalized size = 1.00

$$\frac{\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^5+b*x^3+a*x),x)
[Out] 1/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{cx^5+bx^3+ax}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^5+b*x^3+a*x),x, algorithm="maxima")
[Out] integrate(x^2/(c*x^5 + b*x^3 + a*x), x)
```

mupad [B] time = 2.04, size = 41, normalized size = 1.14

$$\frac{\operatorname{atan}\left(\frac{2\,a\,c\,x^2+a\,b}{a\,\sqrt{4\,a\,c-b^2}}\right)}{\sqrt{4\,a\,c-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x + b*x^3 + c*x^5),x)
```

```
[Out] atan((a*b + 2*a*c*x^2)/(a*(4*a*c - b^2)^(1/2)))/(4*a*c - b^2)^(1/2)
```

sympy [B] time = 0.50, size = 131, normalized size = 3.64

$$-\frac{\sqrt{-\frac{1}{4ac-b^2}}\log\left(x^2+\frac{-4ac\sqrt{-\frac{1}{4ac-b^2}}+b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2c}\right)}{2}+\frac{\sqrt{-\frac{1}{4ac-b^2}}\log\left(x^2+\frac{4ac\sqrt{-\frac{1}{4ac-b^2}}-b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2c}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**5+b*x**3+a*x),x)
```

```
[Out] -sqrt(-1/(4*a*c - b**2))*log(x**2 + (-4*a*c*sqrt(-1/(4*a*c - b**2)) + b**2*
sqrt(-1/(4*a*c - b**2)) + b)/(2*c))/2 + sqrt(-1/(4*a*c - b**2))*log(x**2 +
(4*a*c*sqrt(-1/(4*a*c - b**2)) - b**2*sqrt(-1/(4*a*c - b**2)) + b)/(2*c))/2
```

3.85 $\int \frac{x}{ax+bx^3+cx^5}\,dx$

Optimal. Leaf size=150

$$\frac{\sqrt{2}\,\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\,\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\,\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\,\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[Out] arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)*c^(1/2)/(-4*
a*c+b^2)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-arctan(x*2^(1/2)*c^(1/2)/(b+(-4
*a*c+b^2)^(1/2))^(1/2))*2^(1/2)*c^(1/2)/(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^
(1/2))^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1585, 1093, 205}

$$\frac{\sqrt{2}\,\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\,\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\,\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\,\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a*x + b*x^3 + c*x^5),x]
```

```
[Out] (Sqrt[2]*Sqrt[c]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(
Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*Sqrt[c]*ArcTan[(S
qrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b +
 Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\int \frac{x}{ax+bx^3+cx^5}\,dx = \int \frac{1}{a+bx^2+cx^4}\,dx$$
$$= \frac{c\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{\sqrt{b^2-4ac}} - \frac{c\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{\sqrt{b^2-4ac}}$$
$$= \frac{\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}$$

Mathematica [A] time = 0.08, size = 129, normalized size = 0.86

$$\frac{\sqrt{2}\sqrt{c}\left(\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a*x + b*x^3 + c*x^5),x]
```

```
[Out] (Sqrt[2]*Sqrt[c]*(ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]/S
qrt[b - Sqrt[b^2 - 4*a*c]] - ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 -
 4*a*c]]]/Sqrt[b + Sqrt[b^2 - 4*a*c]]))/Sqrt[b^2 - 4*a*c]
```

fricas [B] time = 0.77, size = 613, normalized size = 4.09

$$-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b+\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\log\left(2cx+\sqrt{\frac{1}{2}}\left(b^2-4ac-\frac{ab^3-4a^2bc}{\sqrt{a^2b^2-4a^3c}}\right)\sqrt{-\frac{b+\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\right)+\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b+}{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^5+b*x^3+a*x),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(1/2)*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2
 - 4*a^2*c))*log(2*c*x + sqrt(1/2)*(b^2 - 4*a*c - (a*b^3 - 4*a^2*b*c)/sqrt(
a^2*b^2 - 4*a^3*c))*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(
a*b^2 - 4*a^2*c))) + 1/2*sqrt(1/2)*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^
2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*log(2*c*x - sqrt(1/2)*(b^2 - 4*a*c - (a*b^
3 - 4*a^2*b*c)/sqrt(a^2*b^2 - 4*a^3*c))*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a
^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))) - 1/2*sqrt(1/2)*sqrt(-(b - (a*b^2 -
4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*log(2*c*x + sqrt(1/2)*
(b^2 - 4*a*c + (a*b^3 - 4*a^2*b*c)/sqrt(a^2*b^2 - 4*a^3*c))*sqrt(-(b - (a*b
^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))) + 1/2*sqrt(1/2)*
sqrt(-(b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*lo
g(2*c*x - sqrt(1/2)*(b^2 - 4*a*c + (a*b^3 - 4*a^2*b*c)/sqrt(a^2*b^2 - 4*a^3
*c))*sqrt(-(b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c
)))
```

giac [B] time = 1.86, size = 1026, normalized size = 6.84

$$\left(\sqrt{2}\sqrt{bc+\sqrt{b^2-4ac}\,c}\,b^4 - 8\sqrt{2}\sqrt{bc+\sqrt{b^2-4ac}\,c}\,ab^2c - 2\sqrt{2}\sqrt{bc+\sqrt{b^2-4ac}\,c}\,b^3c - 2b^4c + 16\sqrt{2}\sqrt{bc}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^5+b*x^3+a*x),x, algorithm="giac")
```

```
[Out] 1/4*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c
 - 2*b^4*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*b^2*c^2 + 16*a*b^2*c^2 + 2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*c^3 - 32*a^2*c^3 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*b*c^2 + 2*(b^2 - 4*a*c)*b^2*c - 8*(b^2 - 4*a*c)*a*c^2 - 2*(b^2 - 4*a*c)*
b*c^2)*arctan(2*sqrt(1/2)*x/sqrt((b + sqrt(b^2 - 4*a*c))/c))/((a*b^4 - 8*a^
2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*abs
(c)) + 1/4*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*b^3*c + 2*b^4*c + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*
sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*b^2*c^2 - 16*a*b^2*c^2 - 2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a*c^3 + 32*a^2*c^3 + 8*a*b*c^3 + sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a*b*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*b^2*c + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b^2*c + 8*(b^2 - 4*a*c)*a*c^2 + 2*(b^2 -
4*a*c)*b*c^2)*arctan(2*sqrt(1/2)*x/sqrt((b - sqrt(b^2 - 4*a*c))/c))/((a*b^4
 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c
^3)*abs(c))
```

maple [A] time = 0.01, size = 116, normalized size = 0.77

$$-\frac{\sqrt{2}\, c \operatorname{arctanh}\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}} - \frac{\sqrt{2}\, c \arctan\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^5+b*x^3+a*x),x)
```

```
[Out] -c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(
1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)-c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((
b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*c*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{cx^5 + bx^3 + ax}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^5+b*x^3+a*x),x, algorithm="maxima")
```

```
[Out] integrate(x/(c*x^5 + b*x^3 + a*x), x)
```

mupad [B] time = 2.48, size = 763, normalized size = 5.09

$$-\operatorname{atan}\Bigg(\frac{b^4\, x\, 1\mathrm{i} + b\, x\, \sqrt{-64\, a^3\, c^3 + 48\, a^2\, b^2\, c^2 - 12\, a\, b^4\, c + b^6}\; 1\mathrm{i} + a^2\, c^2\, x\, 16\mathrm{i} - }{4\, a\, b^4 \sqrt{-\frac{b^3 + \sqrt{-64\, a^3\, c^3 + 48\, a^2\, b^2\, c^2 - 12\, a\, b^4\, c + b^6} - 4\, a\, b\, c}{128\, a^3\, c^2 - 64\, a^2\, b^2\, c + 8\, a\, b^4}} + 64\, a^3\, c^2 \sqrt{-\frac{b^3 + \sqrt{-64\, a^3\, c^3 + 48\, a^2\, b^2\, c^2 - 12\, a\, b^4\, c + b^6} - 4\, a\, b\, c}{128\, a^3\, c^2 - 64\, a^2\, b^2\, c + 8\, a\, b^4}} - 32\, a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x + b*x^3 + c*x^5),x)
```

```
[Out] - atan((b^4*x*1i + b*x*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/
2)*1i + a^2*c^2*x*16i - a*b^2*c*x*8i)/(4*a*b^4*(-(b^3 + (b^6 - 64*a^3*c^3 +
 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2 - 64*
a^2*b^2*c))^(1/2) + 64*a^3*c^2*(-(b^3 + (b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2
- 12*a*b^4*c)^(1/2) - 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2
) - 32*a^2*b^2*c*(-(b^3 + (b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^
(1/2) - 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2)))*(-(b^3 + (
b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c)/(8*a*b^4 +
 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2)*2i - atan((b^4*x*1i - b*x*(b^6 - 64*a^3
*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2)*1i + a^2*c^2*x*16i - a*b^2*c*x*8i
)/(4*a*b^4*(((b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - b^3 +
 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2) + 64*a^3*c^2*(((b^6
 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - b^3 + 4*a*b*c)/(8*a*b^
4 + 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2) - 32*a^2*b^2*c*(((b^6 - 64*a^3*c^3 +
 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - b^3 + 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2
 - 64*a^2*b^2*c))^(1/2)))*(((b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c
)^(1/2) - b^3 + 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2)*2i
```

sympy [A] time = 1.19, size = 87, normalized size = 0.58

$$\operatorname{RootSum}\left(t^4\left(256a^3c^2-128a^2b^2c+16ab^4\right)+t^2\left(-16abc+4b^3\right)+c,\left(t\mapsto t\log\left(x+\frac{32t^3a^2bc-8t^3ab^3+4tac}{c}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**5+b*x**3+a*x),x)
```

```
[Out] RootSum(_t**4*(256*a**3*c**2 - 128*a**2*b**2*c + 16*a*b**4) + _t**2*(-16*a*
b*c + 4*b**3) + c, Lambda(_t, _t*log(x + (32*_t**3*a**2*b*c - 8*_t**3*a*b**
3 + 4*_t*a*c - 2*_t*b**2)/c)))
```

3.86 $\int \frac{1}{ax+bx^3+cx^5}\,dx$

Optimal. Leaf size=69

$$\frac{b\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}}-\frac{\log\left(a+bx^2+cx^4\right)}{4a}+\frac{\log(x)}{a}$$

```
[Out] ln(x)/a-1/4*ln(c*x^4+b*x^2+a)/a+1/2*b*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2
))/a/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1594, 1114, 705, 29, 634, 618, 206, 628}

$$\frac{b\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}}-\frac{\log\left(a+bx^2+cx^4\right)}{4a}+\frac{\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^3 + c*x^5)^(-1),x]
```

```
[Out] (b*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a*Sqrt[b^2 - 4*a*c]) + Log[
x]/a - Log[a + b*x^2 + c*x^4]/(4*a)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
```

```
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{ax+bx^3+cx^5}\,dx &= \int \frac{1}{x\left(a+bx^2+cx^4\right)}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\left(a+bx+cx^2\right)}\,dx,x,x^2\right) \\
&= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx,x,x^2\right)}{2a}+\frac{\text{Subst}\left(\int \frac{-b-cx}{a+bx+cx^2}\,dx,x,x^2\right)}{2a} \\
&= \frac{\log(x)}{a}-\frac{\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^2\right)}{4a}-\frac{b\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,x^2\right)}{4a} \\
&= \frac{\log(x)}{a}-\frac{\log\left(a+bx^2+cx^4\right)}{4a}+\frac{b\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx^2\right)}{2a} \\
&= \frac{b\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}}+\frac{\log(x)}{a}-\frac{\log\left(a+bx^2+cx^4\right)}{4a}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 113, normalized size = 1.64

$$\frac{-\left(\sqrt{b^2-4ac}+b\right)\log\left(-\sqrt{b^2-4ac}+b+2cx^2\right)+\left(b-\sqrt{b^2-4ac}\right)\log\left(\sqrt{b^2-4ac}+b+2cx^2\right)+4\log(x)\sqrt{b}}{4a\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^3 + c*x^5)^(-1),x]
```

```
[Out] (4*Sqrt[b^2 - 4*a*c]*Log[x] - (b + Sqrt[b^2 - 4*a*c])*Log[b - Sqrt[b^2 - 4*
a*c] + 2*c*x^2] + (b - Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x
^2])/(4*a*Sqrt[b^2 - 4*a*c])
```

fricas [A] time = 0.58, size = 223, normalized size = 3.23

$$\left[\frac{\sqrt{b^2-4\,ac}\,b\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac+\left(2\,cx^2+b\right)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)-\left(b^2-4\,ac\right)\log\left(cx^4+bx^2+a\right)+4\left(b^2-4\,ac\right)\log(x)}{4\left(ab^2-4\,a^2c\right)},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^5+b*x^3+a*x),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x
^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - (b^2 - 4*a*c)*log(c*x^4 +
 b*x^2 + a) + 4*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2*c), 1/4*(2*sqrt(-b^2 +
 4*a*c)*b*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (b^2 -
4*a*c)*log(c*x^4 + b*x^2 + a) + 4*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2*c)]
```

giac [A] time = 0.43, size = 68, normalized size = 0.99

$$-\frac{b\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}\,a}-\frac{\log\left(cx^4+bx^2+a\right)}{4\,a}+\frac{\log\left(x^2\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^5+b*x^3+a*x),x, algorithm="giac")
```

```
[Out] -1/2*b*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a) - 1/
4*log(c*x^4 + b*x^2 + a)/a + 1/2*log(x^2)/a
```

maple [A] time = 0.01, size = 66, normalized size = 0.96

$$-\frac{b\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}\;a}+\frac{\ln(x)}{a}-\frac{\ln\left(c\,x^4+b\,x^2+a\right)}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^5+b*x^3+a*x),x)
```

```
[Out] 1/a*ln(x)-1/4*ln(c*x^4+b*x^2+a)/a-1/2/a*b/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2
+b)/(4*a*c-b^2)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\frac{b\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}}+\frac{1}{4}\,\log\left(cx^4+bx^2+a\right)}{a}+\frac{\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^5+b*x^3+a*x),x, algorithm="maxima")
```

```
[Out] -integrate((c*x^3 + b*x)/(c*x^4 + b*x^2 + a), x)/a + log(x)/a
```

mupad [B] time = 2.70, size = 1014, normalized size = 14.70

$$\frac{\ln(x)}{a}+\frac{\ln\left(c\,x^4+b\,x^2+a\right)\,\left(8\,a\,c-2\,b^2\right)}{2\,\left(4\,a\,b^2-16\,a^2\,c\right)}+\frac{b\,\mathrm{atan}\left(\frac{16\,a^3\,x^2\left(\frac{\left(3\,b^3-8\,a\,b\,c\right)\left(\frac{\left(8\,a\,c-2\,b^2\right)^2\left(10\,b\,c^3-\frac{\left(12\,b^3\,c^2-40\,a\,b\,c^3\right)\left(8\,a\,c-2\,b^2\right)}{2\,\left(4\,a\,b^2-16\,a^2\,c\right)}\right)}{4\,\left(4\,a\,b^2-16\,a^2\,c\right)^2}-\frac{b^2\left(10\,b\,c^3-\frac{(12\,b \cdots}{\cdots}\right.}{16\cdots}\right.}{8\,a^3\,c^2\,\left(25\,a\,c-6\,b^2\right)}\cdots\right.}{\cdots}\right)}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x + b*x^3 + c*x^5),x)
```

```
[Out] log(x)/a + (log(a + b*x^2 + c*x^4)*(8*a*c - 2*b^2))/(2*(4*a*b^2 - 16*a^2*c)
) + (b*atan((16*a^3*x^2*(((3*b^3 - 8*a*b*c)*(((8*a*c - 2*b^2)^2*(10*b*c^3 -
 ((12*b^3*c^2 - 40*a*b*c^3)*(8*a*c - 2*b^2))/(2*(4*a*b^2 - 16*a^2*c))))/(4*
(4*a*b^2 - 16*a^2*c)^2) - (b^2*(10*b*c^3 - ((12*b^3*c^2 - 40*a*b*c^3)*(8*a*
c - 2*b^2))/(2*(4*a*b^2 - 16*a^2*c))))/(16*a^2*(4*a*c - b^2)) + (b^2*(12*b^
3*c^2 - 40*a*b*c^3)*(8*a*c - 2*b^2))/(16*a^2*(4*a*b^2 - 16*a^2*c)*(4*a*c -
b^2))))/(8*a^3*c^2*(25*a*c - 6*b^2)) - ((3*b^4 + 10*a^2*c^2 - 14*a*b^2*c)*(
(b^3*(12*b^3*c^2 - 40*a*b*c^3))/(64*a^3*(4*a*c - b^2)^(3/2)) - (b*(12*b^3*c
^2 - 40*a*b*c^3)*(8*a*c - 2*b^2)^2)/(16*a*(4*a*b^2 - 16*a^2*c)^2*(4*a*c - b
^2)^(1/2)) + (b*(8*a*c - 2*b^2)*(10*b*c^3 - ((12*b^3*c^2 - 40*a*b*c^3)*(8*a
*c - 2*b^2))/(2*(4*a*b^2 - 16*a^2*c))))/(4*a*(4*a*b^2 - 16*a^2*c)*(4*a*c -
b^2)^(1/2))))/(8*a^3*c^2*(4*a*c - b^2)^(1/2)*(25*a*c - 6*b^2)))*(4*a*c - b^
2)^(3/2))/(b^2*c^2) + (2*(3*b^3 - 8*a*b*c)*(4*a*c - b^2)^(3/2)*(((8*a*c - 2
*b^2)^2*(4*b^2*c^2 - (2*a*b^2*c^2*(8*a*c - 2*b^2))/(4*a*b^2 - 16*a^2*c)))/(
4*(4*a*b^2 - 16*a^2*c)^2) - (b^2*(4*b^2*c^2 - (2*a*b^2*c^2*(8*a*c - 2*b^2))
/(4*a*b^2 - 16*a^2*c)))/(16*a^2*(4*a*c - b^2)) + (b^4*c^2*(8*a*c - 2*b^2))/
(4*a*(4*a*b^2 - 16*a^2*c)*(4*a*c - b^2))))/(b^2*c^4*(25*a*c - 6*b^2)) - (2*
(4*a*c - b^2)*(3*b^4 + 10*a^2*c^2 - 14*a*b^2*c)*((b^5*c^2)/(16*a^2*(4*a*c -
 b^2)^(3/2)) - (b^3*c^2*(8*a*c - 2*b^2)^2)/(4*(4*a*b^2 - 16*a^2*c)^2*(4*a*c
 - b^2)^(1/2)) + (b*(8*a*c - 2*b^2)*(4*b^2*c^2 - (2*a*b^2*c^2*(8*a*c - 2*b^
2))/(4*a*b^2 - 16*a^2*c)))/(4*a*(4*a*b^2 - 16*a^2*c)*(4*a*c - b^2)^(1/2))))
/(b^2*c^4*(25*a*c - 6*b^2))))/(2*a*(4*a*c - b^2)^(1/2))
```

sympy [B] time = 4.28, size = 253, normalized size = 3.67

$$\left(-\frac{b\sqrt{-4ac+b^2}}{4a\left(4ac-b^2\right)}-\frac{1}{4a}\right)\log\left(x^2+\frac{-8a^2c\left(-\frac{b\sqrt{-4ac+b^2}}{4a\left(4ac-b^2\right)}-\frac{1}{4a}\right)+2ab^2\left(-\frac{b\sqrt{-4ac+b^2}}{4a\left(4ac-b^2\right)}-\frac{1}{4a}\right)-2ac+b^2}{bc}\right)+\left(\frac{b\sqrt{-4ac+}}{4a\left(4ac-\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**5+b*x**3+a*x),x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4*a))*log(x**2 + (-8*a**2
*c*(-b*sqrt(-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4*a)) + 2*a*b**2*(-b*s
qrt(-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4*a)) - 2*a*c + b**2)/(b*c)) +
 (b*sqrt(-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4*a))*log(x**2 + (-8*a**2
*c*(b*sqrt(-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4*a)) + 2*a*b**2*(b*sqr
t(-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4*a)) - 2*a*c + b**2)/(b*c)) + l
og(x)/a
```

3.87 $\int \frac{1}{x(ax+bx^3+cx^5)}\,dx$

Optimal. Leaf size=174

$$-\frac{\sqrt{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,a\sqrt{\sqrt{b^2-4ac}+b}}-\frac{1}{ax}$$

```
[Out] -1/a/x-1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(
1+b/(-4*a*c+b^2)^(1/2))/a*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/2*arctan(x
*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(1-b/(-4*a*c+b^2)^(1
/2))/a*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1585, 1123, 1166, 205}

$$-\frac{\sqrt{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,a\sqrt{\sqrt{b^2-4ac}+b}}-\frac{1}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a*x + b*x^3 + c*x^5)),x]
```

```
[Out] -(1/(a*x)) - (Sqrt[c]*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/
Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sq
rt[c]*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^
2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1123

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(a*d*(m + 1)), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
```

```
Q[r - p]
```

Rubi steps

$$\int \frac{1}{x\left(ax+bx^3+cx^5\right)}\,dx = \int \frac{1}{x^2\left(a+bx^2+cx^4\right)}\,dx$$
$$= -\frac{1}{ax} + \frac{\int \frac{-b-cx^2}{a+bx^2+cx^4}\,dx}{a}$$
$$= -\frac{1}{ax} - \frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2a} - \frac{\left(c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}}{2a}$$
$$= -\frac{1}{ax} - \frac{\sqrt{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4a}}}\right.}{\sqrt{2}\,a\sqrt{b+\sqrt{b^2-4ac}}}$$

Mathematica [A] time = 0.40, size = 191, normalized size = 1.10

$$-\frac{\frac{\sqrt{2}\sqrt{c}\left(\sqrt{b^2-4ac}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(\sqrt{b^2-4ac}-b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{2}{x}}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a*x + b*x^3 + c*x^5)),x]
```

```
[Out] -1/2*(2/x + (Sqrt[2]*Sqrt[c]*(b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c
]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4
*a*c]]) + (Sqrt[2]*Sqrt[c]*(-b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]
*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*
a*c]]))/a
```

fricas [B] time = 1.19, size = 1116, normalized size = 6.41

$$-\frac{\sqrt{\frac{1}{2}}\,ax\sqrt{-\frac{b^3-3\,abc+\left(a^3b^2-4\,a^4c\right)\sqrt{\frac{b^4-2\,ab^2c+a^2c^2}{a^6b^2-4\,a^7c}}}{a^3b^2-4\,a^4c}}\,\log\left(-2\left(b^2c^2-ac^3\right)x+\sqrt{\frac{1}{2}}\left(b^5-5\,ab^3c+4\,a^2bc^2-\left(a^3b^4-6\,a^4b^2\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^5+b*x^3+a*x),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(1/2)*a*x*sqrt(-(b^3 - 3*a*b*c + (a^3*b^2 - 4*a^4*c)*sqrt((b^4 -
2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(-2*(b^2
*c^2 - a*c^3)*x + sqrt(1/2)*(b^5 - 5*a*b^3*c + 4*a^2*b*c^2 - (a^3*b^4 - 6*a
^4*b^2*c + 8*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c))
)*sqrt(-(b^3 - 3*a*b*c + (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^
2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))) - sqrt(1/2)*a*x*sqrt(-(b^3 -
 3*a*b*c + (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 -
4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(-2*(b^2*c^2 - a*c^3)*x - sqrt(1/2)*(b^5
 - 5*a*b^3*c + 4*a^2*b*c^2 - (a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*sqrt((b^4
- 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))*sqrt(-(b^3 - 3*a*b*c + (a^3*b^
2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^
```

```
2 - 4*a^4*c))) + sqrt(1/2)*a*x*sqrt(-(b^3 - 3*a*b*c - (a^3*b^2 - 4*a^4*c)*s
qrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*
log(-2*(b^2*c^2 - a*c^3)*x + sqrt(1/2)*(b^5 - 5*a*b^3*c + 4*a^2*b*c^2 + (a^
3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2
- 4*a^7*c)))*sqrt(-(b^3 - 3*a*b*c - (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2
*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))) - sqrt(1/2)*a*x*s
qrt(-(b^3 - 3*a*b*c - (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/
(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(-2*(b^2*c^2 - a*c^3)*x - sqr
t(1/2)*(b^5 - 5*a*b^3*c + 4*a^2*b*c^2 + (a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)
*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))*sqrt(-(b^3 - 3*a*b*
c - (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c
)))/(a^3*b^2 - 4*a^4*c))) + 2)/(a*x)
```

giac [B] time = 1.88, size = 1839, normalized size = 10.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^5+b*x^3+a*x),x, algorithm="giac")
```

```
[Out] -1/8*(2*a^2*b^4*c^2 - 8*a^3*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^2*b^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^3*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^2*b^3*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a^2*b^2*c^2 - 2*(b^2 - 4*a*c)*a^2*b^2*c^2 + (2*b^4*c^2 - 16*a*b^2*c
^3 + 32*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*b^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c
+ 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c - 16*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 8*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 4*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2 + 8*
(b^2 - 4*a*c)*a*c^3)*a^2 + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5
 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c - 2*sqrt(2)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 2*a*b^5*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^3*b*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2
*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 16*a^2*b^3*c^2 -
 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 32*a^3*b*c^3 + 2*(b^
2 - 4*a*c)*a*b^3*c - 8*(b^2 - 4*a*c)*a^2*b*c^2)*abs(a))*arctan(2*sqrt(1/2)*
x/sqrt((a*b + sqrt(a^2*b^2 - 4*a^3*c))/(a*c)))/((a^3*b^4 - 8*a^4*b^2*c - 2*
a^3*b^3*c + 16*a^5*c^2 + 8*a^4*b*c^2 + a^3*b^2*c^2 - 4*a^4*c^3)*abs(a)*abs(
c)) - 1/8*(2*a^2*b^4*c^2 - 8*a^3*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a^2*b^3*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a^2*b^2*c^2 - 2*(b^2 - 4*a*c)*a^2*b^2*c^2 + (2*b^4*c^2 - 16*a*
b^2*c^3 + 32*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*b^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b
^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c -
16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 8*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 4*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2
 + 8*(b^2 - 4*a*c)*a*c^3)*a^2 + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a*b^5 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c - 2*sqrt(2)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*a*b^5*c + 16*sqrt(2)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^3*b*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
2*b^2*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 16*a^2*b^3*
c^2 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 32*a^3*b*c^3 -
2*(b^2 - 4*a*c)*a*b^3*c + 8*(b^2 - 4*a*c)*a^2*b*c^2)*abs(a))*arctan(2*sqrt(
```

```
1/2)*x/sqrt((a*b - sqrt(a^2*b^2 - 4*a^3*c))/(a*c)))/((a^3*b^4 - 8*a^4*b^2*c
 - 2*a^3*b^3*c + 16*a^5*c^2 + 8*a^4*b*c^2 + a^3*b^2*c^2 - 4*a^4*c^3)*abs(a)
*abs(c)) - 1/(a*x)
```

maple [A] time = 0.02, size = 232, normalized size = 1.33

$$\frac{\sqrt{2}\, bc \operatorname{arctanh}\left(\frac{\sqrt{2}\, cx}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}\; a}+\frac{\sqrt{2}\, bc \arctan\left(\frac{\sqrt{2}\, cx}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}\; a}+\frac{\sqrt{2}\, c \operatorname{arctanh}\left(\frac{\sqrt{2}\, cx}{\sqrt{\left(-b+\sqrt{-4ac+\ldots}\right.}}\right.}{2\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^5+b*x^3+a*x),x)
```

```
[Out] -1/a/x+1/2/a*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((
-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)+1/2/a*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-
b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^
(1/2)*c*x)*b-1/2/a*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2
)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)+1/2/a*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/
((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^
(1/2)*c*x)*b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^5+b*x^3+a*x),x, algorithm="maxima")
```

```
[Out] -integrate((c*x^2 + b)/(c*x^4 + b*x^2 + a), x)/a - 1/(a*x)
```

mupad [B] time = 2.86, size = 2997, normalized size = 17.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a*x + b*x^3 + c*x^5)),x)
```

```
[Out] - atan(((x*(4*a^4*c^4 - 2*a^3*b^2*c^3) + (-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1
/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4
+ 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(4*a^4*b^3*c^2 - 16*a^5*b*c^3 + x*(32*a
^6*b*c^3 - 8*a^5*b^3*c^2)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*
c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 -
8*a^4*b^2*c)))^(1/2)))*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2
 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a
^4*b^2*c)))^(1/2)*1i + (x*(4*a^4*c^4 - 2*a^3*b^2*c^3) + (-(b^5 + b^2*(-(4*a
*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2
))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(16*a^5*b*c^3 - 4*a^4*b^
3*c^2 + x*(32*a^6*b*c^3 - 8*a^5*b^3*c^2)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1
/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4
+ 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)))*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2)
 + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 1
6*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*1i)/((x*(4*a^4*c^4 - 2*a^3*b^2*c^3) + (-(b
^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c
 - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(16*a^5*b
*c^3 - 4*a^4*b^3*c^2 + x*(32*a^6*b*c^3 - 8*a^5*b^3*c^2)*(-(b^5 + b^2*(-(4*a
*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2
```

```
))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)))*(-(b^5 + b^2*(-(4*a*c
- b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/
(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) - (x*(4*a^4*c^4 - 2*a^3*b^2
*c^3) + (-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c -
a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/
2)*(4*a^4*b^3*c^2 - 16*a^5*b*c^3 + x*(32*a^6*b*c^3 - 8*a^5*b^3*c^2)*(-(b^5
+ b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c -
b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)))*(-(b^5 + b
^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2
)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) + 2*a^3*c^4))*(
-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*
a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*2i - a
tan(((x*(4*a^4*c^4 - 2*a^3*b^2*c^3) + (-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2)
 + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 1
6*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(4*a^4*b^3*c^2 - 16*a^5*b*c^3 + x*(32*a^6*
b*c^3 - 8*a^5*b^3*c^2)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2
 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a
^4*b^2*c)))^(1/2)))*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 -
7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*
b^2*c)))^(1/2)*1i + (x*(4*a^4*c^4 - 2*a^3*b^2*c^3) + (-(b^5 - b^2*(-(4*a*c
- b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/
(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(16*a^5*b*c^3 - 4*a^4*b^3*c
^2 + x*(32*a^6*b*c^3 - 8*a^5*b^3*c^2)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2)
 + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 1
6*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)))*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) +
12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a
^5*c^2 - 8*a^4*b^2*c)))^(1/2)*1i)/((x*(4*a^4*c^4 - 2*a^3*b^2*c^3) + (-(b^5
- b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c -
b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(16*a^5*b*c^
3 - 4*a^4*b^3*c^2 + x*(32*a^6*b*c^3 - 8*a^5*b^3*c^2)*(-(b^5 - b^2*(-(4*a*c
- b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/
(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)))*(-(b^5 - b^2*(-(4*a*c - b
^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*
(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) - (x*(4*a^4*c^4 - 2*a^3*b^2*c^
3) + (-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c
*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*
(4*a^4*b^3*c^2 - 16*a^5*b*c^3 + x*(32*a^6*b*c^3 - 8*a^5*b^3*c^2)*(-(b^5 - b
^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2
)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)))*(-(b^5 - b^2*
(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3
)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) + 2*a^3*c^4))*(-(b
^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c
 - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*2i - 1/(a
*x)
```

sympy [A] time = 2.62, size = 148, normalized size = 0.85

$$\mathrm{RootSum}\left(t^4\left(256a^5c^2-128a^4b^2c+16a^3b^4\right)+t^2\left(48a^2bc^2-28ab^3c+4b^5\right)+c^3,\left(t\mapsto t\log\left(x+\frac{-64t^3a^5c^2+48\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**5+b*x**3+a*x),x)
```

```
[Out] RootSum(_t**4*(256*a**5*c**2 - 128*a**4*b**2*c + 16*a**3*b**4) + _t**2*(48*
a**2*b*c**2 - 28*a*b**3*c + 4*b**5) + c**3, Lambda(_t, _t*log(x + (-64*_t**
3*a**5*c**2 + 48*_t**3*a**4*b**2*c - 8*_t**3*a**3*b**4 - 10*_t*a**2*b*c**2
+ 10*_t*a*b**3*c - 2*_t*b**5)/(a*c**3 - b**2*c**2)))) - 1/(a*x)
```

3.88 $\int \frac{1}{x^2(ax+bx^3+cx^5)}\,dx$

Optimal. Leaf size=89

$$-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}}+\frac{b\log\left(a+bx^2+cx^4\right)}{4a^2}-\frac{b\log(x)}{a^2}-\frac{1}{2ax^2}$$

```
[Out] -1/2/a/x^2-b*ln(x)/a^2+1/4*b*ln(c*x^4+b*x^2+a)/a^2-1/2*(-2*a*c+b^2)*arctanh
((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {1585, 1114, 709, 800, 634, 618, 206, 628}

$$-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}}+\frac{b\log\left(a+bx^2+cx^4\right)}{4a^2}-\frac{b\log(x)}{a^2}-\frac{1}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a*x + b*x^3 + c*x^5)),x]
```

```
[Out] -1/(2*a*x^2) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*
a^2*Sqrt[b^2 - 4*a*c]) - (b*Log[x])/a^2 + (b*Log[a + b*x^2 + c*x^4])/(4*a^2
)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
```

```
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^2\left(ax+bx^3+cx^5\right)}\,dx &= \int \frac{1}{x^3\left(a+bx^2+cx^4\right)}\,dx\\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^2\left(a+bx+cx^2\right)}\,dx,x,x^2\right)\\
&= -\frac{1}{2ax^2}+\frac{\text{Subst}\left(\int \frac{-b-cx}{x\left(a+bx+cx^2\right)}\,dx,x,x^2\right)}{2a}\\
&= -\frac{1}{2ax^2}+\frac{\text{Subst}\left(\int \left(-\frac{b}{ax}+\frac{b^2-ac+bcx}{a\left(a+bx+cx^2\right)}\right)dx,x,x^2\right)}{2a}\\
&= -\frac{1}{2ax^2}-\frac{b\log(x)}{a^2}+\frac{\text{Subst}\left(\int \frac{b^2-ac+bcx}{a+bx+cx^2}\,dx,x,x^2\right)}{2a^2}\\
&= -\frac{1}{2ax^2}-\frac{b\log(x)}{a^2}+\frac{b\,\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^2\right)}{4a^2}+\frac{\left(b^2-2ac\right)\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx\right.}{4a^2}\\
&= -\frac{1}{2ax^2}-\frac{b\log(x)}{a^2}+\frac{b\log\left(a+bx^2+cx^4\right)}{4a^2}-\frac{\left(b^2-2ac\right)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+\right.}{2a^2}\\
&= -\frac{1}{2ax^2}-\frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}}-\frac{b\log(x)}{a^2}+\frac{b\log\left(a+bx^2+cx^4\right)}{4a^2}
\end{aligned}
$$

Mathematica [A] time = 0.12, size = 135, normalized size = 1.52

$$
\frac{\frac{\left(b\sqrt{b^2-4ac}-2ac+b^2\right)\log\left(-\sqrt{b^2-4ac}+b+2cx^2\right)}{\sqrt{b^2-4ac}}+\frac{\left(b\sqrt{b^2-4ac}+2ac-b^2\right)\log\left(\sqrt{b^2-4ac}+b+2cx^2\right)}{\sqrt{b^2-4ac}}-\frac{2a}{x^2}-4b\log(x)}{4a^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a*x + b*x^3 + c*x^5)),x]
```

```
[Out] ((-2*a)/x^2 - 4*b*Log[x] + ((b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c])*Log[b - Sqr
t[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c] + ((-b^2 + 2*a*c + b*Sqrt[b^2
- 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c])/(4*a^2)
```

fricas [A] time = 1.05, size = 293, normalized size = 3.29

$$\left[-\frac{\left(b^2-2\,ac\right)\sqrt{b^2-4\,ac}\,x^2\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac+\left(2\,cx^2+b\right)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)-\left(b^3-4\,abc\right)x^2\log\left(cx^4+bx^2+a\right)+4\left(b^{\cdots}\right.}{4\left(a^2b^2-4\,a^3c\right)x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^5+b*x^3+a*x),x, algorithm="fricas")
```

```
[Out] [-1/4*((b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*x^2*log((2*c^2*x^4 + 2*b*c*x^2 + b^2
 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - (b^3 - 4
*a*b*c)*x^2*log(c*x^4 + b*x^2 + a) + 4*(b^3 - 4*a*b*c)*x^2*log(x) + 2*a*b^2
 - 8*a^2*c)/((a^2*b^2 - 4*a^3*c)*x^2), -1/4*(2*(b^2 - 2*a*c)*sqrt(-b^2 + 4*
a*c)*x^2*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (b^3 - 4
*a*b*c)*x^2*log(c*x^4 + b*x^2 + a) + 4*(b^3 - 4*a*b*c)*x^2*log(x) + 2*a*b^2
 - 8*a^2*c)/((a^2*b^2 - 4*a^3*c)*x^2)]
```

giac [A] time = 0.43, size = 94, normalized size = 1.06

$$\frac{b\log\left(cx^4+bx^2+a\right)}{4\,a^2}-\frac{b\log\left(x^2\right)}{2\,a^2}+\frac{\left(b^2-2\,ac\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\sqrt{-b^2+4\,ac}\,a^2}+\frac{bx^2-a}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^5+b*x^3+a*x),x, algorithm="giac")
```

```
[Out] 1/4*b*log(c*x^4 + b*x^2 + a)/a^2 - 1/2*b*log(x^2)/a^2 + 1/2*(b^2 - 2*a*c)*a
rctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a^2) + 1/2*(b*x
^2 - a)/(a^2*x^2)
```

maple [A] time = 0.01, size = 119, normalized size = 1.34

$$-\frac{c\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\,a}+\frac{b^2\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}\,a^2}-\frac{b\ln(x)}{a^2}+\frac{b\ln\left(c\,x^4+b\,x^2+a\right)}{4a^2}-\frac{1}{2a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^5+b*x^3+a*x),x)
```

```
[Out] -1/2/a/x^2-1/a^2*b*ln(x)+1/4*b*ln(c*x^4+b*x^2+a)/a^2-1/a/(4*a*c-b^2)^(1/2)*
arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*c+1/2/a^2/(4*a*c-b^2)^(1/2)*arctan((2
*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{b\log(x)}{a^2}+\frac{\frac{1}{4}\,b\log\left(cx^4+bx^2+a\right)+\frac{\left(b^2-2\,ac\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\sqrt{-b^2+4\,ac}}}{a^2}-\frac{1}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^5+b*x^3+a*x),x, algorithm="maxima")
```

```
[Out] -b*log(x)/a^2 + integrate((b*c*x^3 + (b^2 - a*c)*x)/(c*x^4 + b*x^2 + a), x)
/a^2 - 1/2/(a*x^2)
```

mupad [B] time = 3.91, size = 2033, normalized size = 22.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a*x + b*x^3 + c*x^5)),x)
```

```
[Out] (atan((16*a^6*x^2*(((3*b^4 + a^2*c^2 - 9*a*b^2*c)*(c^5/a^3 + ((2*b^3 - 8*a*
b*c)*((6*b*c^4)/a^2 + ((2*b^3 - 8*a*b*c)*((20*a^3*c^4 + 2*a^2*b^2*c^3)/a^3
+ ((2*b^3 - 8*a*b*c)*(40*a^4*b*c^3 - 12*a^3*b^3*c^2))/(2*a^3*(16*a^3*c - 4*
a^2*b^2))))/(2*(16*a^3*c - 4*a^2*b^2))))/(2*(16*a^3*c - 4*a^2*b^2)) - ((((2
*a*c - b^2)*((20*a^3*c^4 + 2*a^2*b^2*c^3)/a^3 + ((2*b^3 - 8*a*b*c)*(40*a^4*
b*c^3 - 12*a^3*b^3*c^2))/(2*a^3*(16*a^3*c - 4*a^2*b^2))))/(4*a^2*(4*a*c - b
^2)^(1/2)) + ((2*b^3 - 8*a*b*c)*(40*a^4*b*c^3 - 12*a^3*b^3*c^2)*(2*a*c - b^
2))/(8*a^5*(4*a*c - b^2)^(1/2)*(16*a^3*c - 4*a^2*b^2)))*(2*a*c - b^2))/(4*a
^2*(4*a*c - b^2)^(1/2)) - ((2*b^3 - 8*a*b*c)*(40*a^4*b*c^3 - 12*a^3*b^3*c^2
)*(2*a*c - b^2)^2)/(32*a^7*(4*a*c - b^2)*(16*a^3*c - 4*a^2*b^2))))/(8*a^3*c
^2*(a^2*c^2 - 6*b^4 + 24*a*b^2*c)) + ((((2*b^3 - 8*a*b*c)*(((2*a*c - b^2)*(
(20*a^3*c^4 + 2*a^2*b^2*c^3)/a^3 + ((2*b^3 - 8*a*b*c)*(40*a^4*b*c^3 - 12*a^
3*b^3*c^2))/(2*a^3*(16*a^3*c - 4*a^2*b^2))))/(4*a^2*(4*a*c - b^2)^(1/2)) +
((2*b^3 - 8*a*b*c)*(40*a^4*b*c^3 - 12*a^3*b^3*c^2)*(2*a*c - b^2))/(8*a^5*(4
*a*c - b^2)^(1/2)*(16*a^3*c - 4*a^2*b^2))))/(2*(16*a^3*c - 4*a^2*b^2)) - ((
40*a^4*b*c^3 - 12*a^3*b^3*c^2)*(2*a*c - b^2)^3)/(64*a^9*(4*a*c - b^2)^(3/2)
) + (((6*b*c^4)/a^2 + ((2*b^3 - 8*a*b*c)*((20*a^3*c^4 + 2*a^2*b^2*c^3)/a^3
+ ((2*b^3 - 8*a*b*c)*(40*a^4*b*c^3 - 12*a^3*b^3*c^2))/(2*a^3*(16*a^3*c - 4*
a^2*b^2))))/(2*(16*a^3*c - 4*a^2*b^2)))*(2*a*c - b^2))/(4*a^2*(4*a*c - b^2)
^(1/2)))*(3*b^5 + 13*a^2*b*c^2 - 15*a*b^3*c))/(8*a^3*c^2*(4*a*c - b^2)^(1/2
)*(a^2*c^2 - 6*b^4 + 24*a*b^2*c)))*(4*a*c - b^2)^(3/2))/(4*a^2*c^4 + b^4*c^
2 - 4*a*b^2*c^3) - (2*a^3*(4*a*c - b^2)*(3*b^5 + 13*a^2*b*c^2 - 15*a*b^3*c)
*(((2*b^3 - 8*a*b*c)*((((4*a^3*b*c^3 - 4*a^2*b^3*c^2)/a^3 + (2*a*b^2*c^2*(2
*b^3 - 8*a*b*c))/(16*a^3*c - 4*a^2*b^2))*(2*a*c - b^2))/(4*a^2*(4*a*c - b^2
)^(1/2)) + (b^2*c^2*(2*b^3 - 8*a*b*c)*(2*a*c - b^2))/(2*a*(4*a*c - b^2)^(1/
2)*(16*a^3*c - 4*a^2*b^2))))/(2*(16*a^3*c - 4*a^2*b^2)) + ((2*a*c - b^2)*((
a^2*c^4 - 4*a*b^2*c^3)/a^3 + ((2*b^3 - 8*a*b*c)*((4*a^3*b*c^3 - 4*a^2*b^3*c
^2)/a^3 + (2*a*b^2*c^2*(2*b^3 - 8*a*b*c))/(16*a^3*c - 4*a^2*b^2)))/(2*(16*a
^3*c - 4*a^2*b^2))))/(4*a^2*(4*a*c - b^2)^(1/2)) - (b^2*c^2*(2*a*c - b^2)^3
)/(16*a^5*(4*a*c - b^2)^(3/2))))/(c^2*(a^2*c^2 - 6*b^4 + 24*a*b^2*c)*(4*a^2
*c^4 + b^4*c^2 - 4*a*b^2*c^3)) + (2*a^3*(4*a*c - b^2)^(3/2)*(3*b^4 + a^2*c^
2 - 9*a*b^2*c)*((b*c^4)/a^3 - ((2*b^3 - 8*a*b*c)*((a^2*c^4 - 4*a*b^2*c^3)/a
^3 + ((2*b^3 - 8*a*b*c)*((4*a^3*b*c^3 - 4*a^2*b^3*c^2)/a^3 + (2*a*b^2*c^2*(
2*b^3 - 8*a*b*c))/(16*a^3*c - 4*a^2*b^2)))/(2*(16*a^3*c - 4*a^2*b^2))))/(2*
(16*a^3*c - 4*a^2*b^2)) + ((2*a*c - b^2)*((((4*a^3*b*c^3 - 4*a^2*b^3*c^2)/a
^3 + (2*a*b^2*c^2*(2*b^3 - 8*a*b*c))/(16*a^3*c - 4*a^2*b^2))*(2*a*c - b^2))
/(4*a^2*(4*a*c - b^2)^(1/2)) + (b^2*c^2*(2*b^3 - 8*a*b*c)*(2*a*c - b^2))/(2
*a*(4*a*c - b^2)^(1/2)*(16*a^3*c - 4*a^2*b^2))))/(4*a^2*(4*a*c - b^2)^(1/2)
) + (b^2*c^2*(2*b^3 - 8*a*b*c)*(2*a*c - b^2)^2)/(8*a^3*(4*a*c - b^2)*(16*a^
3*c - 4*a^2*b^2))))/(c^2*(a^2*c^2 - 6*b^4 + 24*a*b^2*c)*(4*a^2*c^4 + b^4*c^
2 - 4*a*b^2*c^3)))*(2*a*c - b^2))/(2*a^2*(4*a*c - b^2)^(1/2)) - (b*log(x))/
a^2 - (log(a + b*x^2 + c*x^4)*(2*b^3 - 8*a*b*c))/(2*(16*a^3*c - 4*a^2*b^2))
 - 1/(2*a*x^2)
```

sympy [B] time = 123.75, size = 345, normalized size = 3.88

$$\left(\frac{b}{4a^2} - \frac{\sqrt{-4ac+b^2}\left(2ac-b^2\right)}{4a^2\left(4ac-b^2\right)}\right)\log\left(x^2 + \frac{-8a^3c\left(\frac{b}{4a^2} - \frac{\sqrt{-4ac+b^2}\left(2ac-b^2\right)}{4a^2\left(4ac-b^2\right)}\right) + 2a^2b^2\left(\frac{b}{4a^2} - \frac{\sqrt{-4ac+b^2}\left(2ac-b^2\right)}{4a^2\left(4ac-b^2\right)}\right) + 3abc}{2ac^2 - b^2c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**5+b*x**3+a*x),x)
```

```
[Out] (b/(4*a**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*a**2*(4*a*c - b**2)))*l
og(x**2 + (-8*a**3*c*(b/(4*a**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*a*
*2*(4*a*c - b**2))) + 2*a**2*b**2*(b/(4*a**2) - sqrt(-4*a*c + b**2)*(2*a*c
- b**2)/(4*a**2*(4*a*c - b**2))) + 3*a*b*c - b**3)/(2*a*c**2 - b**2*c)) + (
b/(4*a**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*a**2*(4*a*c - b**2)))*lo
g(x**2 + (-8*a**3*c*(b/(4*a**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*a**
2*(4*a*c - b**2))) + 2*a**2*b**2*(b/(4*a**2) + sqrt(-4*a*c + b**2)*(2*a*c -
 b**2)/(4*a**2*(4*a*c - b**2))) + 3*a*b*c - b**3)/(2*a*c**2 - b**2*c)) - 1/
(2*a*x**2) - b*log(x)/a**2
```

3.89 $\int \frac{x^{11}}{\left(ax+bx^3+cx^5\right)^2}\,dx$

Optimal. Leaf size=166

$$-\frac{\left(6a^2c^2-6ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{c^3\left(b^2-4ac\right)^{3/2}}+\frac{x^2\left(b^2-3ac\right)}{c^2\left(b^2-4ac\right)}-\frac{bx^4}{2c\left(b^2-4ac\right)}+\frac{x^6\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{b\log\left(a+b\right.}{2c}$$

```
[Out] (-3*a*c+b^2)*x^2/c^2/(-4*a*c+b^2)-1/2*b*x^4/c/(-4*a*c+b^2)+1/2*x^6*(b*x^2+2
*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-(6*a^2*c^2-6*a*b^2*c+b^4)*arctanh((2*c*x^2
+b)/(-4*a*c+b^2)^(1/2))/c^3/(-4*a*c+b^2)^(3/2)-1/2*b*ln(c*x^4+b*x^2+a)/c^3
```

Rubi [A] time = 0.22, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {1585, 1114, 738, 800, 634, 618, 206, 628}

$$-\frac{\left(6a^2c^2-6ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{c^3\left(b^2-4ac\right)^{3/2}}+\frac{x^2\left(b^2-3ac\right)}{c^2\left(b^2-4ac\right)}+\frac{x^6\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{bx^4}{2c\left(b^2-4ac\right)}-\frac{b\log\left(a+b\right.}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] ((b^2 - 3*a*c)*x^2)/(c^2*(b^2 - 4*a*c)) - (b*x^4)/(2*c*(b^2 - 4*a*c)) + (x^
6*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((b^4 - 6*a*b^2*c
+ 6*a^2*c^2)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(c^3*(b^2 - 4*a*c)^(
3/2)) - (b*Log[a + b*x^2 + c*x^4])/(2*c^3)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
```

```
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\int \frac{x^{11}}{\left(ax+bx^3+cx^5\right)^2}\,dx = \int \frac{x^9}{\left(a+bx^2+cx^4\right)^2}\,dx$$

$$= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^4}{\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right)$$

$$= \frac{x^6\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int \frac{x^2(6a+2bx)}{a+bx+cx^2}\,dx, x, x^2\right)}{2\left(b^2-4ac\right)}$$

$$= \frac{x^6\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int \left(-\frac{2\left(b^2-3ac\right)}{c^2}+\frac{2bx}{c}+\frac{2\left(a\left(b^2-3ac\right)+b\left(b^2-4ac\right)x\right)}{c^2\left(a+bx+cx^2\right)}\right) \cdots\right.}{2\left(b^2-4ac\right)}$$

$$= \frac{\left(b^2-3ac\right)x^2}{c^2\left(b^2-4ac\right)} - \frac{bx^4}{2c\left(b^2-4ac\right)} + \frac{x^6\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int \frac{a\left(b^2-3ac\right)+b \cdots}{a+bx+ \cdots}\right.}{c^2\left(b^2- \cdots\right.}$$

$$= \frac{\left(b^2-3ac\right)x^2}{c^2\left(b^2-4ac\right)} - \frac{bx^4}{2c\left(b^2-4ac\right)} + \frac{x^6\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{b\,\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2} \cdots\right.}{2c^3}$$

$$= \frac{\left(b^2-3ac\right)x^2}{c^2\left(b^2-4ac\right)} - \frac{bx^4}{2c\left(b^2-4ac\right)} + \frac{x^6\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{b\log\left(a+bx^2+cx^4 \cdots\right.}{2c^3}$$

$$= \frac{\left(b^2-3ac\right)x^2}{c^2\left(b^2-4ac\right)} - \frac{bx^4}{2c\left(b^2-4ac\right)} + \frac{x^6\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(b^4-6ab^2c+6a^2c^2 \cdots\right.}{c^3\left(b^2- \cdots\right.}$$

Mathematica [A] time = 0.19, size = 151, normalized size = 0.91

$$\frac{-\frac{2\left(6a^2c^2-6ab^2c+b^4\right)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{3/2}}+\frac{a^2c\left(3b-2cx^2\right)-ab^2\left(b-4cx^2\right)+b^4\left(-x^2\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-b\log\left(a+bx^2+cx^4\right)+cx^2}{2c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] (c*x^2 + (-(b^4*x^2) - a*b^2*(b - 4*c*x^2) + a^2*c*(3*b - 2*c*x^2))/((b^2 -
 4*a*c)*(a + b*x^2 + c*x^4)) - (2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTan[(b +
 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) - b*Log[a + b*x^2 + c*x
^4])/(2*c^3)
```

fricas [B] time = 0.73, size = 868, normalized size = 5.23

$$\left[\frac{\left(b^4c^2-8\,ab^2c^3+16\,a^2c^4\right)x^6-ab^5+7\,a^2b^3c-12\,a^3bc^2+\left(b^5c-8\,ab^3c^2+16\,a^2bc^3\right)x^4-\left(b^6-9\,ab^4c+26\,a^2b^2c^2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] [1/2*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^6 - a*b^5 + 7*a^2*b^3*c - 12*a
^3*b*c^2 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^4 - (b^6 - 9*a*b^4*c + 26
*a^2*b^2*c^2 - 24*a^3*c^3)*x^2 - (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2 + (b^4*c
- 6*a*b^2*c^2 + 6*a^2*c^3)*x^4 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*x^2)*sqrt(
b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(
b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 +
(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^4 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^
2)*x^2)*log(c*x^4 + b*x^2 + a))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5 + (
b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*x^4 + (b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b
*c^5)*x^2), 1/2*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^6 - a*b^5 + 7*a^2*b
^3*c - 12*a^3*b*c^2 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^4 - (b^6 - 9*a
*b^4*c + 26*a^2*b^2*c^2 - 24*a^3*c^3)*x^2 - 2*(a*b^4 - 6*a^2*b^2*c + 6*a^3*
c^2 + (b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*x^4 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^
2)*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 -
4*a*c)) - (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + (b^5*c - 8*a*b^3*c^2 + 16*a
^2*b*c^3)*x^4 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*x^2)*log(c*x^4 + b*x^2 +
 a))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5 + (b^4*c^4 - 8*a*b^2*c^5 + 16*
a^2*c^6)*x^4 + (b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x^2)]
```

giac [A] time = 1.98, size = 161, normalized size = 0.97

$$\frac{\left(b^4-6\,ab^2c+6\,a^2c^2\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^2c^3-4\,ac^4\right)\sqrt{-b^2+4\,ac}}+\frac{x^2}{2\,c^2}+\frac{b^3x^4-4\,abcx^4-2\,a^2cx^2-a^2b}{2\left(cx^4+bx^2+a\right)\left(b^2c^2-4\,ac^3\right)}-\frac{b\log\left(cx^4+bx^2+a\right)}{2\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] (b^4 - 6*a*b^2*c + 6*a^2*c^2)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^
2*c^3 - 4*a*c^4)*sqrt(-b^2 + 4*a*c)) + 1/2*x^2/c^2 + 1/2*(b^3*x^4 - 4*a*b*c
*x^4 - 2*a^2*c*x^2 - a^2*b)/((c*x^4 + b*x^2 + a)*(b^2*c^2 - 4*a*c^3)) - 1/2
*b*log(c*x^4 + b*x^2 + a)/c^3
```

maple [B] time = 0.01, size = 383, normalized size = 2.31

$$\frac{a^2x^2}{\left(c\,x^4+b\,x^2+a\right)\left(4ac-b^2\right)c}-\frac{2a\,b^2x^2}{\left(c\,x^4+b\,x^2+a\right)\left(4ac-b^2\right)c^2}+\frac{b^4x^2}{2\left(c\,x^4+b\,x^2+a\right)\left(4ac-b^2\right)c^3}-\frac{6a^2\arctan\left(\frac{2c}{\sqrt{4}}\right.}{\left(4ac-b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(c*x^5+b*x^3+a*x)^2,x)
```

```
[Out] 1/2*x^2/c^2+1/c/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^2*a^2-2/c^2/(c*x^4+b*x^2+a)/(
4*a*c-b^2)*x^2*a*b^2+1/2/c^3/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^2*b^4-3/2/c^2/(c
*x^4+b*x^2+a)/(4*a*c-b^2)*a^2*b+1/2/c^3/(c*x^4+b*x^2+a)/(4*a*c-b^2)*a*b^3-2
/c^2/(4*a*c-b^2)*ln(c*x^4+b*x^2+a)*a*b+1/2/c^3/(4*a*c-b^2)*ln(c*x^4+b*x^2+a
)*b^3-6/c/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a^2+6/c^2
/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*b^2-1/c^3/(4*a*c
-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{ab^3-3\,a^2bc+\left(b^4-4\,ab^2c+2\,a^2c^2\right)x^2}{2\left(ab^2c^3-4\,a^2c^4+\left(b^2c^4-4\,ac^5\right)x^4+\left(b^3c^3-4\,abc^4\right)x^2\right)}+\frac{x^2}{2\,c^2}+\frac{-2\int\frac{\left(b^3-4\,abc\right)x^3+\left(ab^2-3\,a^2c\right)x}{cx^4+bx^2+a}\,dx}{b^2c^2-4\,ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(a*b^3 - 3*a^2*b*c + (b^4 - 4*a*b^2*c + 2*a^2*c^2)*x^2)/(a*b^2*c^3 - 4
*a^2*c^4 + (b^2*c^4 - 4*a*c^5)*x^4 + (b^3*c^3 - 4*a*b*c^4)*x^2) + 1/2*x^2/c
^2 + 2*integrate(-((b^3 - 4*a*b*c)*x^3 + (a*b^2 - 3*a^2*c)*x)/(c*x^4 + b*x^
2 + a), x)/(b^2*c^2 - 4*a*c^3)
```

mupad [B] time = 0.53, size = 1473, normalized size = 8.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(a*x + b*x^3 + c*x^5)^2,x)
```

```
[Out] ((a*(b^3 - 3*a*b*c))/(2*c*(4*a*c - b^2)) + (x^2*(b^4 + 2*a^2*c^2 - 4*a*b^2*
c))/(2*c*(4*a*c - b^2)))/(a*c^2 + c^3*x^4 + b*c^2*x^2) + x^2/(2*c^2) + (log
(a + b*x^2 + c*x^4)*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c))/(2*
(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) + (atan(((4*a*c^5*(
4*a*c - b^2)^3 - b^2*c^4*(4*a*c - b^2)^3)*(((((16*a*b)/c + (8*a*c^2*(b^7 -
64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c))/(64*a^3*c^6 - b^6*c^3 + 12*a*b
^4*c^4 - 48*a^2*b^2*c^5))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(2*c^3*(4*a*c - b^
2)^(3/2)) + (4*a*(b^4 + 6*a^2*c^2 - 6*a*b^2*c)*(b^7 - 64*a^3*b*c^3 + 48*a^2
*b^3*c^2 - 12*a*b^5*c))/(c*(4*a*c - b^2)^(3/2)*(64*a^3*c^6 - b^6*c^3 + 12*a
*b^4*c^4 - 48*a^2*b^2*c^5)))/(2*a*(4*a*c - b^2)) - x^2*(((((4*(6*a^2*c^5 +
3*b^4*c^3 - 14*a*b^2*c^4))/(4*a*c^5 - b^2*c^4) + (2*(2*b^3*c^6 - 8*a*b*c^7)
*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c))/((4*a*c^5 - b^2*c^4)*(
64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)))*(b^4 + 6*a^2*c^2 -
6*a*b^2*c))/(2*c^3*(4*a*c - b^2)^(3/2)) + ((2*b^3*c^6 - 8*a*b*c^7)*(b^4 + 6
*a^2*c^2 - 6*a*b^2*c)*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c))/(
c^3*(4*a*c - b^2)^(3/2)*(4*a*c^5 - b^2*c^4)*(64*a^3*c^6 - b^6*c^3 + 12*a*b^
4*c^4 - 48*a^2*b^2*c^5)))/(2*a*(4*a*c - b^2)) + (b*((4*(b^5 + 3*a^2*b*c^2 -
 5*a*b^3*c))/(4*a*c^5 - b^2*c^4) + (((4*(6*a^2*c^5 + 3*b^4*c^3 - 14*a*b^2*c
^4))/(4*a*c^5 - b^2*c^4) + (2*(2*b^3*c^6 - 8*a*b*c^7)*(b^7 - 64*a^3*b*c^3 +
 48*a^2*b^3*c^2 - 12*a*b^5*c))/((4*a*c^5 - b^2*c^4)*(64*a^3*c^6 - b^6*c^3 +
```

```
 12*a*b^4*c^4 - 48*a^2*b^2*c^5)))*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12
*a*b^5*c))/(2*(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) - ((2
*b^3*c^6 - 8*a*b*c^7)*(b^4 + 6*a^2*c^2 - 6*a*b^2*c)^2)/(c^6*(4*a*c - b^2)^3
*(4*a*c^5 - b^2*c^4))))/(2*a*(4*a*c - b^2)^(3/2))) + (b*((4*a*b^2)/c^4 + ((
(16*a*b)/c + (8*a*c^2*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c))/(
64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5))*(b^7 - 64*a^3*b*c^3
+ 48*a^2*b^3*c^2 - 12*a*b^5*c))/(2*(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4 - 4
8*a^2*b^2*c^5)) - (4*a*(b^4 + 6*a^2*c^2 - 6*a*b^2*c)^2)/(c^4*(4*a*c - b^2)^
3)))/(2*a*(4*a*c - b^2)^(3/2))))/(2*b^8 + 72*a^4*c^4 + 96*a^2*b^4*c^2 - 144
*a^3*b^2*c^3 - 24*a*b^6*c))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(c^3*(4*a*c - b^
2)^(3/2))
```

sympy [B] time = 112.28, size = 877, normalized size = 5.28

$$\left(-\frac{b}{2c^3}-\frac{\sqrt{-\left(4ac-b^2\right)^3}\left(6a^2c^2-6ab^2c+b^4\right)}{2c^3\left(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6\right)}\right)\log\left(x^2+\frac{-5a^2bc-16a^2c^4\left(-\frac{b}{2c^3}-\frac{\sqrt{-\left(4ac-b^2\right)^3}\left(6a^2c^2-6ab^2c+b^4\right)}{2c^3\left(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6\right)}\right)\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] (-b/(2*c**3) - sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(
2*c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))*log(x**2 +
 (-5*a**2*b*c - 16*a**2*c**4*(-b/(2*c**3) - sqrt(-(4*a*c - b**2)**3)*(6*a**
2*c**2 - 6*a*b**2*c + b**4)/(2*c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*
a*b**4*c - b**6))) + a*b**3 + 8*a*b**2*c**3*(-b/(2*c**3) - sqrt(-(4*a*c - b
**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(2*c**3*(64*a**3*c**3 - 48*a**2*
b**2*c**2 + 12*a*b**4*c - b**6))) - b**4*c**2*(-b/(2*c**3) - sqrt(-(4*a*c -
 b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(2*c**3*(64*a**3*c**3 - 48*a**
2*b**2*c**2 + 12*a*b**4*c - b**6))))/(6*a**2*c**2 - 6*a*b**2*c + b**4)) + (
-b/(2*c**3) + sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(2
*c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))*log(x**2 +
(-5*a**2*b*c - 16*a**2*c**4*(-b/(2*c**3) + sqrt(-(4*a*c - b**2)**3)*(6*a**2
*c**2 - 6*a*b**2*c + b**4)/(2*c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a
*b**4*c - b**6))) + a*b**3 + 8*a*b**2*c**3*(-b/(2*c**3) + sqrt(-(4*a*c - b*
*2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(2*c**3*(64*a**3*c**3 - 48*a**2*b
**2*c**2 + 12*a*b**4*c - b**6))) - b**4*c**2*(-b/(2*c**3) + sqrt(-(4*a*c -
b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(2*c**3*(64*a**3*c**3 - 48*a**2
*b**2*c**2 + 12*a*b**4*c - b**6))))/(6*a**2*c**2 - 6*a*b**2*c + b**4)) + (-
3*a**2*b*c + a*b**3 + x**2*(2*a**2*c**2 - 4*a*b**2*c + b**4))/(8*a**2*c**4
- 2*a*b**2*c**3 + x**4*(8*a*c**5 - 2*b**2*c**4) + x**2*(8*a*b*c**4 - 2*b**3
*c**3)) + x**2/(2*c**2)
```

3.90 $\int \frac{x^{10}}{\left(ax+bx^3+cx^5\right)^2}\,dx$

Optimal. Leaf size=331

$$-\frac{\left(-\frac{20a^2c^2-19ab^2c+3b^4}{\sqrt{b^2-4ac}}-13abc+3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,c^{5/2}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{20a^2c^2-19ab^2c+3b^4}{\sqrt{b^2-4ac}}-13abc+3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\,c^{5/2}\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[Out] 1/2*(-10*a*c+3*b^2)*x/c^2/(-4*a*c+b^2)-1/2*b*x^3/c/(-4*a*c+b^2)+1/2*x^5*(b*
x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-1/4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a
*c+b^2)^(1/2))^(1/2))*(3*b^3-13*a*b*c+(-20*a^2*c^2+19*a*b^2*c-3*b^4)/(-4*a*
c+b^2)^(1/2))/c^(5/2)/(-4*a*c+b^2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/4
*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(3*b^3-13*a*b*c+(20
*a^2*c^2-19*a*b^2*c+3*b^4)/(-4*a*c+b^2)^(1/2))/c^(5/2)/(-4*a*c+b^2)*2^(1/2)
/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 0.70, antiderivative size = 331, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1585, 1120, 1279, 1166, 205}

$$-\frac{\left(-\frac{20a^2c^2-19ab^2c+3b^4}{\sqrt{b^2-4ac}}-13abc+3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,c^{5/2}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{20a^2c^2-19ab^2c+3b^4}{\sqrt{b^2-4ac}}-13abc+3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\,c^{5/2}\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In] Int[x^10/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] ((3*b^2 - 10*a*c)*x)/(2*c^2*(b^2 - 4*a*c)) - (b*x^3)/(2*c*(b^2 - 4*a*c)) +
(x^5*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((3*b^3 - 13*a*
b*c - (3*b^4 - 19*a*b^2*c + 20*a^2*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*
Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(5/2)*(b^2 - 4*a*c)*S
qrt[b - Sqrt[b^2 - 4*a*c]]) - ((3*b^3 - 13*a*b*c + (3*b^4 - 19*a*b^2*c + 20
*a^2*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 -
 4*a*c]]])/(2*Sqrt[2]*c^(5/2)*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1120

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> -Simp[(d^3*(d*x)^(m - 3)*(2*a + b*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*
(p + 1)*(b^2 - 4*a*c)), x] + Dist[d^4/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^
(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[
m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
```

```
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\int \frac{x^{10}}{\left(ax+bx^3+cx^5\right)^2}\,dx = \int \frac{x^8}{\left(a+bx^2+cx^4\right)^2}\,dx$$

$$= \frac{x^5\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\int \frac{x^4\left(10a+3bx^2\right)}{a+bx^2+cx^4}\,dx}{2\left(b^2-4ac\right)}$$

$$= -\frac{bx^3}{2c\left(b^2-4ac\right)} + \frac{x^5\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\int \frac{x^2\left(9ab+3\left(3b^2-10ac\right)x^2\right)}{a+bx^2+cx^4}\,dx}{6c\left(b^2-4ac\right)}$$

$$= \frac{\left(3b^2-10ac\right)x}{2c^2\left(b^2-4ac\right)} - \frac{bx^3}{2c\left(b^2-4ac\right)} + \frac{x^5\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\int \frac{3a\left(3b^2-10ac\right)+3b\left(3b^2-1\right.}{a+bx^2+cx^4}}{6c^2\left(b^2-4ac\right)}$$

$$= \frac{\left(3b^2-10ac\right)x}{2c^2\left(b^2-4ac\right)} - \frac{bx^3}{2c\left(b^2-4ac\right)} + \frac{x^5\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(3b^3-13abc-\frac{3b^4-19}{\sqrt{}}\right.}{4c}$$

$$= \frac{\left(3b^2-10ac\right)x}{2c^2\left(b^2-4ac\right)} - \frac{bx^3}{2c\left(b^2-4ac\right)} + \frac{x^5\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(3b^3-13abc-\frac{3b^4-19}{\sqrt{}}\right.}{2\sqrt{2}\,c^{5/2}\left(b^2-\right.}$$

Mathematica [A] time = 0.66, size = 327, normalized size = 0.99

$$\frac{-\frac{2\sqrt{c}\,x\left(2a^2c-ab\left(b-3cx^2\right)+b^3\left(-x^2\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\sqrt{2}\left(-20a^2c^2+19ab^2c-13abc\sqrt{b^2-4ac}+3b^3\sqrt{b^2-4ac}-3b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2}\left(20a^2c^2-19ab^2c-13a\right.}{}}{4c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] (4*Sqrt[c]*x - (2*Sqrt[c]*x*(2*a^2*c - b^3*x^2 - a*b*(b - 3*c*x^2)))/((b^2
- 4*a*c)*(a + b*x^2 + c*x^4)) - (Sqrt[2]*(-3*b^4 + 19*a*b^2*c - 20*a^2*c^2
```

```
+ 3*b^3*Sqrt[b^2 - 4*a*c] - 13*a*b*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqr
t[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^
2 - 4*a*c]]) - (Sqrt[2]*(3*b^4 - 19*a*b^2*c + 20*a^2*c^2 + 3*b^3*Sqrt[b^2 -
 4*a*c] - 13*a*b*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + S
qrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*c
^(5/2))
```

fricas [B] time = 0.91, size = 2856, normalized size = 8.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(4*(b^2*c - 4*a*c^2)*x^5 + 2*(3*b^3 - 11*a*b*c)*x^3 + sqrt(1/2)*(a*b^2*
c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2)*sqrt
(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 + (b^6*c^5 - 12*a*
b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^
2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*
a^2*b^2*c^12 - 64*a^3*c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64
*a^3*c^8))*log(-(189*a^2*b^6 - 1971*a^3*b^4*c + 5625*a^4*b^2*c^2 - 2500*a^5
*c^3)*x + 1/2*sqrt(1/2)*(27*b^10 - 459*a*b^8*c + 2961*a^2*b^6*c^2 - 8818*a^
3*b^4*c^3 + 11360*a^4*b^2*c^4 - 4000*a^5*c^5 - (3*b^9*c^5 - 52*a*b^7*c^6 +
336*a^2*b^5*c^7 - 960*a^3*b^3*c^8 + 1024*a^4*b*c^9)*sqrt((81*b^8 - 918*a*b^
6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^10 - 12*a*b
^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(-(9*b^7 - 105*a*b^5*c + 385
*a^2*b^3*c^2 - 420*a^3*b*c^3 + (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 6
4*a^3*c^8)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3
 + 625*a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13))
)/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8))) - sqrt(1/2)*(a*b
^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2)*s
qrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 + (b^6*c^5 - 12
*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*sqrt((81*b^8 - 918*a*b^6*c + 3051
*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 +
48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 -
 64*a^3*c^8))*log(-(189*a^2*b^6 - 1971*a^3*b^4*c + 5625*a^4*b^2*c^2 - 2500*
a^5*c^3)*x - 1/2*sqrt(1/2)*(27*b^10 - 459*a*b^8*c + 2961*a^2*b^6*c^2 - 8818
*a^3*b^4*c^3 + 11360*a^4*b^2*c^4 - 4000*a^5*c^5 - (3*b^9*c^5 - 52*a*b^7*c^6
 + 336*a^2*b^5*c^7 - 960*a^3*b^3*c^8 + 1024*a^4*b*c^9)*sqrt((81*b^8 - 918*a
*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^10 - 12*
a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(-(9*b^7 - 105*a*b^5*c +
385*a^2*b^3*c^2 - 420*a^3*b*c^3 + (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7
- 64*a^3*c^8)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*
c^3 + 625*a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^1
3)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8))) + sqrt(1/2)*(
a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2
)*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 - (b^6*c^5 -
 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*sqrt((81*b^8 - 918*a*b^6*c + 3
051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11
 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^
7 - 64*a^3*c^8))*log(-(189*a^2*b^6 - 1971*a^3*b^4*c + 5625*a^4*b^2*c^2 - 25
00*a^5*c^3)*x + 1/2*sqrt(1/2)*(27*b^10 - 459*a*b^8*c + 2961*a^2*b^6*c^2 - 8
818*a^3*b^4*c^3 + 11360*a^4*b^2*c^4 - 4000*a^5*c^5 + (3*b^9*c^5 - 52*a*b^7*
c^6 + 336*a^2*b^5*c^7 - 960*a^3*b^3*c^8 + 1024*a^4*b*c^9)*sqrt((81*b^8 - 91
8*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^10 -
12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(-(9*b^7 - 105*a*b^5*c
 + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 - (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c
^7 - 64*a^3*c^8)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b
^2*c^3 + 625*a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*
c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8))) - sqrt(1/2
```

```
)*(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*
x^2)*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 - (b^6*c^
5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*sqrt((81*b^8 - 918*a*b^6*c
+ 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^10 - 12*a*b^4*c
^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2
*c^7 - 64*a^3*c^8))*log(-(189*a^2*b^6 - 1971*a^3*b^4*c + 5625*a^4*b^2*c^2 -
 2500*a^5*c^3)*x - 1/2*sqrt(1/2)*(27*b^10 - 459*a*b^8*c + 2961*a^2*b^6*c^2
- 8818*a^3*b^4*c^3 + 11360*a^4*b^2*c^4 - 4000*a^5*c^5 + (3*b^9*c^5 - 52*a*b
^7*c^6 + 336*a^2*b^5*c^7 - 960*a^3*b^3*c^8 + 1024*a^4*b*c^9)*sqrt((81*b^8 -
 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^10
 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(-(9*b^7 - 105*a*b^
5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 - (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^
2*c^7 - 64*a^3*c^8)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^
3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a
^3*c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8))) + 2*(3*
a*b^2 - 10*a^2*c)*x)/(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^
3*c^2 - 4*a*b*c^3)*x^2)
```

giac [B] time = 3.71, size = 3339, normalized size = 10.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^3*x^3 - 3*a*b*c*x^3 + a*b^2*x - 2*a^2*c*x)/((c*x^4 + b*x^2 + a)*(b^2
*c^2 - 4*a*c^3)) + x/c^2 + 1/16*(6*b^9*c^6 - 86*a*b^7*c^7 + 440*a^2*b^5*c^8
 - 928*a^3*b^3*c^9 + 640*a^4*b*c^10 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*b^9*c^4 + 43*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a*b^7*c^5 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*b^8*c^5 - 220*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a^2*b^5*c^6 - 62*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a*b^6*c^6 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*b^7*c^6 + 464*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a^3*b^3*c^7 + 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a^2*b^4*c^7 + 31*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a*b^5*c^7 - 320*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a^4*b*c^8 - 160*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a^3*b^2*c^8 - 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a^2*b^3*c^8 + 80*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a^3*b*c^9 - 6*(b^2 - 4*a*c)*b^7*c^6 + 62*(b^2 - 4*a*c)*a*b^5*c^7 - 19
2*(b^2 - 4*a*c)*a^2*b^3*c^8 + 160*(b^2 - 4*a*c)*a^3*b*c^9 - (6*b^5*c^2 - 50
*a*b^3*c^3 + 104*a^2*b*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*b^5 + 25*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a*b^3*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*b^4*c - 52*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b
*c^2 - 26*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c
^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 +
13*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 6*(b
^2 - 4*a*c)*b^3*c^2 + 26*(b^2 - 4*a*c)*a*b*c^3)*(b^2*c^2 - 4*a*c^3)^2 - 2*(
3*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c^3 - 34*sqrt(2)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^4 - 6*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a*b^5*c^4 + 6*a*b^6*c^4 + 128*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
3*b^2*c^5 + 44*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^5 + 3*sqrt
(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^5 - 68*a^2*b^4*c^5 - 160*sqrt(2
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*c^6 - 80*sqrt(2)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^3*b*c^6 - 22*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*
c^6 + 256*a^3*b^2*c^6 + 40*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^7
- 320*a^4*c^7 - 6*(b^2 - 4*a*c)*a*b^4*c^4 + 44*(b^2 - 4*a*c)*a^2*b^2*c^5 -
80*(b^2 - 4*a*c)*a^3*c^6)*abs(-b^2*c^2 + 4*a*c^3))*arctan(2*sqrt(1/2)*x/sqr
```

```
t((b^3*c^2 - 4*a*b*c^3 + sqrt((b^3*c^2 - 4*a*b*c^3)^2 - 4*(a*b^2*c^2 - 4*a^
2*c^3)*(b^2*c^3 - 4*a*c^4)))/(b^2*c^3 - 4*a*c^4)))/((a*b^6*c^5 - 12*a^2*b^4
*c^6 - 2*a*b^5*c^6 + 48*a^3*b^2*c^7 + 16*a^2*b^3*c^7 + a*b^4*c^7 - 64*a^4*c
^8 - 32*a^3*b*c^8 - 8*a^2*b^2*c^8 + 16*a^3*c^9)*abs(-b^2*c^2 + 4*a*c^3)*abs
(c)) - 1/16*(6*b^9*c^6 - 86*a*b^7*c^7 + 440*a^2*b^5*c^8 - 928*a^3*b^3*c^9 +
 640*a^4*b*c^10 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*b^9*c^4 + 43*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*b^7*c^5 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^8*
c^5 - 220*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5
*c^6 - 62*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c
^6 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7*c^6 +
464*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^7 +
 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^7
+ 31*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^7 -
320*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^8 - 1
60*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^8 -
96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^8 +
80*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^9 - 6*
(b^2 - 4*a*c)*b^7*c^6 + 62*(b^2 - 4*a*c)*a*b^5*c^7 - 192*(b^2 - 4*a*c)*a^2*
b^3*c^8 + 160*(b^2 - 4*a*c)*a^3*b*c^9 - (6*b^5*c^2 - 50*a*b^3*c^3 + 104*a^2
*b*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 +
25*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 6*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 52*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 26*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 3*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 13*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 6*(b^2 - 4*a*c)*b^3*c^2
+ 26*(b^2 - 4*a*c)*a*b*c^3)*(b^2*c^2 - 4*a*c^3)^2 + 2*(3*sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a*b^6*c^3 - 34*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^2*b^4*c^4 - 6*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^4 - 6*a*b
^6*c^4 + 128*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^5 + 44*sqrt(
2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^5 + 3*sqrt(2)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a*b^4*c^5 + 68*a^2*b^4*c^5 - 160*sqrt(2)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^4*c^6 - 80*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^
6 - 22*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^6 - 256*a^3*b^2*c^
6 + 40*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^7 + 320*a^4*c^7 + 6*(b
^2 - 4*a*c)*a*b^4*c^4 - 44*(b^2 - 4*a*c)*a^2*b^2*c^5 + 80*(b^2 - 4*a*c)*a^3
*c^6)*abs(-b^2*c^2 + 4*a*c^3))*arctan(2*sqrt(1/2)*x/sqrt((b^3*c^2 - 4*a*b*c
^3 - sqrt((b^3*c^2 - 4*a*b*c^3)^2 - 4*(a*b^2*c^2 - 4*a^2*c^3)*(b^2*c^3 - 4*
a*c^4)))/(b^2*c^3 - 4*a*c^4)))/((a*b^6*c^5 - 12*a^2*b^4*c^6 - 2*a*b^5*c^6 +
 48*a^3*b^2*c^7 + 16*a^2*b^3*c^7 + a*b^4*c^7 - 64*a^4*c^8 - 32*a^3*b*c^8 -
8*a^2*b^2*c^8 + 16*a^3*c^9)*abs(-b^2*c^2 + 4*a*c^3)*abs(c))
```

maple [B] time = 0.04, size = 844, normalized size = 2.55

$$\frac{3ab\,x^3}{2\left(c\,x^4+b\,x^2+a\right)\left(4ac-b^2\right)c}-\frac{b^3x^3}{2\left(c\,x^4+b\,x^2+a\right)\left(4ac-b^2\right)c^2}+\frac{5\sqrt{2}\,a^2\operatorname{arctanh}\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{\left(4ac-b^2\right)\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(c*x^5+b*x^3+a*x)^2,x)
```

```
[Out] 1/c^2*x+3/2/c/(c*x^4+b*x^2+a)*b/(4*a*c-b^2)*x^3*a-1/2/c^2/(c*x^4+b*x^2+a)*b
^3/(4*a*c-b^2)*x^3+1/c/(c*x^4+b*x^2+a)*a^2/(4*a*c-b^2)*x-1/2/c^2/(c*x^4+b*x
^2+a)*a/(4*a*c-b^2)*x*b^2+13/4/c/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2
))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*b-3/4/
c^2/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/(
```

```
(-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^3+5/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*
2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)
^(1/2))*c)^(1/2)*c*x)*a^2-19/4/c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-
b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^
(1/2)*c*x)*a*b^2+3/4/c^2/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*
c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*
x)*b^4-13/4/c/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2
^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*b+3/4/c^2/(4*a*c-b^2)*2^(1/2
)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*c*x)*b^3+5/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^
(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a^2-19
/4/c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2
)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*b^2+3/4/c^2/(4*a*c
-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^
(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(b^3-3\,abc\right)x^3+\left(ab^2-2\,a^2c\right)x}{2\left(ab^2c^2-4\,a^2c^3+\left(b^2c^3-4\,ac^4\right)x^4+\left(b^3c^2-4\,abc^3\right)x^2\right)}+\frac{-\int\frac{3\,ab^2-10\,a^2c+\left(3\,b^3-13\,abc\right)x^2}{cx^4+bx^2+a}\,dx}{2\left(b^2c^2-4\,ac^3\right)}+\frac{x}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((b^3 - 3*a*b*c)*x^3 + (a*b^2 - 2*a^2*c)*x)/(a*b^2*c^2 - 4*a^2*c^3 + (b
^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2) + 1/2*integrate(-(3*a*b^
2 - 10*a^2*c + (3*b^3 - 13*a*b*c)*x^2)/(c*x^4 + b*x^2 + a), x)/(b^2*c^2 - 4
*a*c^3) + x/c^2
```

mupad [B] time = 3.76, size = 7599, normalized size = 22.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(a*x + b*x^3 + c*x^5)^2,x)
```

```
[Out] ((b*x^3*(3*a*c - b^2))/(2*(4*a*c - b^2)) + (a*x*(2*a*c - b^2))/(2*(4*a*c -
b^2)))/(a*c^2 + c^3*x^4 + b*c^2*x^2) - atan(((((10240*a^5*c^7 + 48*a*b^8*c^
3 - 736*a^2*b^6*c^4 + 4224*a^3*b^4*c^5 - 10752*a^4*b^2*c^6)/(8*(64*a^3*c^6
- b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) - (x*(-(9*b^13 + 9*b^4*(-(4*a*c
 - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 +
 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2
) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11
+ b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*
b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*(16*b^7*c^5 - 192*a*b^5*c^6 - 1024*a^3
*b*c^8 + 768*a^2*b^3*c^7))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*(-(9*b
^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 -
 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(
4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))
/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3
*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2) - (x*(9*b^8 + 200*
a^4*c^4 + 481*a^2*b^4*c^2 - 718*a^3*b^2*c^3 - 114*a*b^6*c))/(2*(16*a^2*c^5
+ b^4*c^3 - 8*a*b^2*c^4)))*(-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 268
80*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 4
4800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*
a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10
*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2
*c^10)))^(1/2)*1i - (((10240*a^5*c^7 + 48*a*b^8*c^3 - 736*a^2*b^6*c^4 + 422
```

```
4*a^3*b^4*c^5 - 10752*a^4*b^2*c^6)/(8*(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4
- 48*a^2*b^2*c^5)) + (x*(-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*
a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 4480
0*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b
^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^
6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^
10)))^(1/2)*(16*b^7*c^5 - 192*a*b^5*c^6 - 1024*a^3*b*c^8 + 768*a^2*b^3*c^7)
)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*(-(9*b^13 + 9*b^4*(-(4*a*c - b^
2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 3024
0*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 2
13*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^1
2*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c
^9 - 6144*a^5*b^2*c^10)))^(1/2) + (x*(9*b^8 + 200*a^4*c^4 + 481*a^2*b^4*c^2
 - 718*a^3*b^2*c^3 - 114*a*b^6*c))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4))
)*(-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b
^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2
*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9
)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 -
1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*1i)/((((10
240*a^5*c^7 + 48*a*b^8*c^3 - 736*a^2*b^6*c^4 + 4224*a^3*b^4*c^5 - 10752*a^4
*b^2*c^6)/(8*(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) - (x*(
-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*
c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^
2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(
1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 128
0*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*(16*b^7*c^5 -
 192*a*b^5*c^6 - 1024*a^3*b*c^8 + 768*a^2*b^3*c^7))/(2*(16*a^2*c^5 + b^4*c^
3 - 8*a*b^2*c^4)))*(-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b
*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5
*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*
(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 2
40*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))
^(1/2) - (x*(9*b^8 + 200*a^4*c^4 + 481*a^2*b^4*c^2 - 718*a^3*b^2*c^3 - 114*
a*b^6*c))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*(-(9*b^13 + 9*b^4*(-(4*
a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^
3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(
1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^
11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a
^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2) + (((10240*a^5*c^7 + 48*a*b^8*c^3 -
 736*a^2*b^6*c^4 + 4224*a^3*b^4*c^5 - 10752*a^4*b^2*c^6)/(8*(64*a^3*c^6 - b
^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) + (x*(-(9*b^13 + 9*b^4*(-(4*a*c -
b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30
240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) -
 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b
^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4
*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*(16*b^7*c^5 - 192*a*b^5*c^6 - 1024*a^3*b*
c^8 + 768*a^2*b^3*c^7))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*(-(9*b^13
 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10
656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a
*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(3
2*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^
6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2) + (x*(9*b^8 + 200*a^4
*c^4 + 481*a^2*b^4*c^2 - 718*a^3*b^2*c^3 - 114*a*b^6*c))/(2*(16*a^2*c^5 + b
^4*c^3 - 8*a*b^2*c^4)))*(-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*
a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 4480
0*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b
^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^
6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^
10)))^(1/2) + (63*a^3*b^5 - 573*a^4*b^3*c + 1300*a^5*b*c^2)/(4*(64*a^3*c^6
```

```
- b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5))))*(-(9*b^13 + 9*b^4*(-(4*a*c -
b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30
240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) -
 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b
^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4
*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*2i - atan(((((10240*a^5*c^7 + 48*a*b^8*c^
3 - 736*a^2*b^6*c^4 + 4224*a^3*b^4*c^5 - 10752*a^4*b^2*c^6)/(8*(64*a^3*c^6
- b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) - (x*(-(9*b^13 - 9*b^4*(-(4*a*c
 - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 +
 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2
) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11
+ b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*
b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*(16*b^7*c^5 - 192*a*b^5*c^6 - 1024*a^3
*b*c^8 + 768*a^2*b^3*c^7))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*(-(9*b
^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 -
 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(
4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))
/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3
*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2) - (x*(9*b^8 + 200*
a^4*c^4 + 481*a^2*b^4*c^2 - 718*a^3*b^2*c^3 - 114*a*b^6*c))/(2*(16*a^2*c^5
+ b^4*c^3 - 8*a*b^2*c^4)))*(-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 268
80*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 4
4800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*
a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10
*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2
*c^10)))^(1/2)*1i - (((10240*a^5*c^7 + 48*a*b^8*c^3 - 736*a^2*b^6*c^4 + 422
4*a^3*b^4*c^5 - 10752*a^4*b^2*c^6)/(8*(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4
- 48*a^2*b^2*c^5)) + (x*(-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*
a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 4480
0*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b
^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^
6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^
10)))^(1/2)*(16*b^7*c^5 - 192*a*b^5*c^6 - 1024*a^3*b*c^8 + 768*a^2*b^3*c^7)
)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*(-(9*b^13 - 9*b^4*(-(4*a*c - b^
2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 3024
0*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 2
13*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^1
2*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c
^9 - 6144*a^5*b^2*c^10)))^(1/2) + (x*(9*b^8 + 200*a^4*c^4 + 481*a^2*b^4*c^2
 - 718*a^3*b^2*c^3 - 114*a*b^6*c))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4))
)*(-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b
^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2
*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9
)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 -
1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*1i)/((((10
240*a^5*c^7 + 48*a*b^8*c^3 - 736*a^2*b^6*c^4 + 4224*a^3*b^4*c^5 - 10752*a^4
*b^2*c^6)/(8*(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) - (x*(
-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*
c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^
2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(
1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 128
0*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*(16*b^7*c^5 -
 192*a*b^5*c^6 - 1024*a^3*b*c^8 + 768*a^2*b^3*c^7))/(2*(16*a^2*c^5 + b^4*c^
3 - 8*a*b^2*c^4)))*(-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b
*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5
*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*
(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 2
40*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))
^(1/2) - (x*(9*b^8 + 200*a^4*c^4 + 481*a^2*b^4*c^2 - 718*a^3*b^2*c^3 - 114*
```

```
a*b^6*c))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*(-(9*b^13 - 9*b^4*(-(4*
a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^
3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(
1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^
11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a
^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2) + (((10240*a^5*c^7 + 48*a*b^8*c^3 -
 736*a^2*b^6*c^4 + 4224*a^3*b^4*c^5 - 10752*a^4*b^2*c^6)/(8*(64*a^3*c^6 - b
^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) + (x*(-(9*b^13 - 9*b^4*(-(4*a*c -
b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30
240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) -
 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b
^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4
*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*(16*b^7*c^5 - 192*a*b^5*c^6 - 1024*a^3*b*
c^8 + 768*a^2*b^3*c^7))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*(-(9*b^13
 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10
656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a
*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(3
2*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^
6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2) + (x*(9*b^8 + 200*a^4
*c^4 + 481*a^2*b^4*c^2 - 718*a^3*b^2*c^3 - 114*a*b^6*c))/(2*(16*a^2*c^5 + b
^4*c^3 - 8*a*b^2*c^4)))*(-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*
a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 4480
0*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b
^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^
6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^
10)))^(1/2) + (63*a^3*b^5 - 573*a^4*b^3*c + 1300*a^5*b*c^2)/(4*(64*a^3*c^6
- b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5))))*(-(9*b^13 - 9*b^4*(-(4*a*c -
b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30
240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) -
 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b
^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4
*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*2i + x/c^2
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10/(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] Timed out
```

3.91 $\int \frac{x^9}{\left(ax+bx^3+cx^5\right)^2}\,dx$

Optimal. Leaf size=132

$$\frac{b\left(b^2-6ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\left(b^2-4ac\right)^{3/2}}-\frac{bx^2}{2c\left(b^2-4ac\right)}+\frac{x^4\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\log\left(a+bx^2+cx^4\right)}{4c^2}$$

```
[Out] -1/2*b*x^2/c/(-4*a*c+b^2)+1/2*x^4*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+
1/2*b*(-6*a*c+b^2)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/c^2/(-4*a*c+b^2)
^(3/2)+1/4*ln(c*x^4+b*x^2+a)/c^2
```

Rubi [A] time = 0.15, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {1585, 1114, 738, 773, 634, 618, 206, 628}

$$\frac{b\left(b^2-6ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\left(b^2-4ac\right)^{3/2}}+\frac{x^4\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{bx^2}{2c\left(b^2-4ac\right)}+\frac{\log\left(a+bx^2+cx^4\right)}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] -(b*x^2)/(2*c*(b^2 - 4*a*c)) + (x^4*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*
x^2 + c*x^4)) + (b*(b^2 - 6*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/
(2*c^2*(b^2 - 4*a*c)^(3/2)) + Log[a + b*x^2 + c*x^4]/(4*c^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
```

```
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 773

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^9}{\left(ax+bx^3+cx^5\right)^2}\,dx &= \int \frac{x^7}{\left(a+bx^2+cx^4\right)^2}\,dx\\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^3}{\left(a+bx+cx^2\right)^2}\,dx,x,x^2\right)\\
&= \frac{x^4\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\text{Subst}\left(\int \frac{x(4a+bx)}{a+bx+cx^2}\,dx,x,x^2\right)}{2\left(b^2-4ac\right)}\\
&= -\frac{bx^2}{2c\left(b^2-4ac\right)}+\frac{x^4\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\text{Subst}\left(\int \frac{-ab+\left(-b^2+4ac\right)x}{a+bx+cx^2}\,dx,x,x^2\right)}{2c\left(b^2-4ac\right)}\\
&= -\frac{bx^2}{2c\left(b^2-4ac\right)}+\frac{x^4\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^2\right)}{4c^2}-\frac{(b\,(}{}\\
&= -\frac{bx^2}{2c\left(b^2-4ac\right)}+\frac{x^4\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\log\left(a+bx^2+cx^4\right)}{4c^2}+\frac{\left(b\left(b^2-6a\right.\right.}{}\\
&= -\frac{bx^2}{2c\left(b^2-4ac\right)}+\frac{x^4\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{b\left(b^2-6ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\left(b^2-4ac\right)^{3/2}}+\frac{\text{l}}{}
\end{aligned}$$

Mathematica [A] time = 0.17, size = 121, normalized size = 0.92

$$\frac{\frac{2\left(-2a^2c+ab\left(b-3cx^2\right)+b^3x^2\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{2b\left(b^2-6ac\right)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{3/2}}+\log\left(a+bx^2+cx^4\right)}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] ((2*(-2*a^2*c + b^3*x^2 + a*b*(b - 3*c*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c
*x^4)) + (2*b*(b^2 - 6*a*c)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2
 + 4*a*c)^(3/2) + Log[a + b*x^2 + c*x^4])/(4*c^2)
```

fricas [B] time = 0.65, size = 663, normalized size = 5.02

$$\left[\frac{2\,ab^4-12\,a^2b^2c+16\,a^3c^2+2\left(b^5-7\,ab^3c+12\,a^2bc^2\right)x^2+\left(\left(b^3c-6\,abc^2\right)x^4+ab^3-6\,a^2bc+\left(b^4-6\,ab^2c\right)x^2\right)\sqrt{\cdot}}{4\left(ab^4c^2-8\,a^2b^2c^3+16\,\cdots\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + 2*(b^5 - 7*a*b^3*c + 12*a^2*b*c
^2)*x^2 + ((b^3*c - 6*a*b*c^2)*x^4 + a*b^3 - 6*a^2*b*c + (b^4 - 6*a*b^2*c)*
x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2
+ b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + (a*b^4 - 8*a^2*b^2*c + 16*a^
3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*
b*c^2)*x^2)*log(c*x^4 + b*x^2 + a))/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4
 + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^4 + (b^5*c^2 - 8*a*b^3*c^3 + 16*a
^2*b*c^4)*x^2), 1/4*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + 2*(b^5 - 7*a*b^3
*c + 12*a^2*b*c^2)*x^2 + 2*((b^3*c - 6*a*b*c^2)*x^4 + a*b^3 - 6*a^2*b*c + (
b^4 - 6*a*b^2*c)*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 +
4*a*c)/(b^2 - 4*a*c)) + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^
2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^2)*log(c*x^4 +
 b*x^2 + a))/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4 + (b^4*c^3 - 8*a*b^2*c
^4 + 16*a^2*c^5)*x^4 + (b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x^2)]
```

giac [A] time = 1.95, size = 152, normalized size = 1.15

$$-\frac{\left(b^3-6\,abc\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\left(b^2c^2-4\,ac^3\right)\sqrt{-b^2+4\,ac}}-\frac{b^2cx^4-4\,ac^2x^4-b^3x^2+2\,abcx^2-ab^2}{4\left(cx^4+bx^2+a\right)\left(b^2c^2-4\,ac^3\right)}+\frac{\log\left(cx^4+bx^2+a\right)}{4\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] -1/2*(b^3 - 6*a*b*c)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^2*c^2 - 4
*a*c^3)*sqrt(-b^2 + 4*a*c)) - 1/4*(b^2*c*x^4 - 4*a*c^2*x^4 - b^3*x^2 + 2*a*
b*c*x^2 - a*b^2)/((c*x^4 + b*x^2 + a)*(b^2*c^2 - 4*a*c^3)) + 1/4*log(c*x^4
+ b*x^2 + a)/c^2
```

maple [A] time = 0.01, size = 222, normalized size = 1.68

$$-\frac{3ab\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}c}+\frac{b^3\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{2\left(4ac-b^2\right)^{\frac{3}{2}}c^2}+\frac{a\ln\left(c\,x^4+b\,x^2+a\right)}{\left(4ac-b^2\right)c}-\frac{b^2\ln\left(c\,x^4+b\,x^2+a\right)}{4\left(4ac-b^2\right)c^2}+\frac{\frac{\left(3ac-b^2\right)b\,x^2}{\left(4ac-b^2\right)c^2}+\frac{\left(2ac-b^2\right)\cdots}{\left(4ac-b^2\right)\cdots}}{2c\,x^4+2b\,x^2+2a\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(c*x^5+b*x^3+a*x)^2,x)
```

```
[Out] 1/2*((3*a*c-b^2)/(4*a*c-b^2)*b/c^2*x^2+(2*a*c-b^2)/(4*a*c-b^2)*a/c^2)/(c*x^
4+b*x^2+a)+1/c/(4*a*c-b^2)*ln(c*x^4+b*x^2+a)*a-1/4/c^2/(4*a*c-b^2)*ln(c*x^4
```

```
+b*x^2+a)*b^2-3/c/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a
*b+1/2/c^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{ab^2 - 2\,a^2c + \left(b^3 - 3\,abc\right)x^2}{2\left(ab^2c^2 - 4\,a^2c^3 + \left(b^2c^3 - 4\,ac^4\right)x^4 + \left(b^3c^2 - 4\,abc^3\right)x^2\right)} - \frac{-\int \frac{\left(b^2-4\,ac\right)x^3+abx}{cx^4+bx^2+a}\,dx}{b^2c - 4\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(a*b^2 - 2*a^2*c + (b^3 - 3*a*b*c)*x^2)/(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c
^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2) - integrate(-((b^2 - 4*a*c)*
x^3 + a*b*x)/(c*x^4 + b*x^2 + a), x)/(b^2*c - 4*a*c^2)
```

mupad [B] time = 2.94, size = 1336, normalized size = 10.12

$$\frac{\frac{a\left(2\,a\,c-b^2\right)}{2\,c^2\left(4\,a\,c-b^2\right)}+\frac{b\,x^2\left(3\,a\,c-b^2\right)}{2\,c^2\left(4\,a\,c-b^2\right)}}{c\,x^4+b\,x^2+a}-\frac{\ln\left(c\,x^4+b\,x^2+a\right)\left(-128\,a^3\,c^3+96\,a^2\,b^2\,c^2-24\,a\,b^4\,c+2\,b^6\right)}{2\left(256\,a^3\,c^5-192\,a^2\,b^2\,c^4+48\,a\,b^4\,c^3-4\,b^6\,c^2\right)}+\frac{b\,\mathrm{atan}\left(\frac{\left(8\,a\,c^3\left(4\,a\,c-\right.\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(a*x + b*x^3 + c*x^5)^2,x)
```

```
[Out] ((a*(2*a*c - b^2))/(2*c^2*(4*a*c - b^2)) + (b*x^2*(3*a*c - b^2))/(2*c^2*(4*
a*c - b^2)))/(a + b*x^2 + c*x^4) - (log(a + b*x^2 + c*x^4)*(2*b^6 - 128*a^3
*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c))/(2*(256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^4
*c^3 - 192*a^2*b^2*c^4)) + (b*atan(((8*a*c^3*(4*a*c - b^2)^3 - 2*b^2*c^2*(4
*a*c - b^2)^3)*(x^2*(((b*((6*b^3*c^2 - 28*a*b*c^3)/(4*a*c^3 - b^2*c^2) + ((
8*b^3*c^4 - 32*a*b*c^5)*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c)
)/(2*(4*a*c^3 - b^2*c^2)*(256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^3 - 192*a^2*
b^2*c^4)))*(6*a*c - b^2))/(8*c^2*(4*a*c - b^2)^(3/2)) + (b*(8*b^3*c^4 - 32*
a*b*c^5)*(6*a*c - b^2)*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c))
/(16*c^2*(4*a*c - b^2)^(3/2)*(4*a*c^3 - b^2*c^2)*(256*a^3*c^5 - 4*b^6*c^2 +
 48*a*b^4*c^3 - 192*a^2*b^2*c^4)))/(a*(4*a*c - b^2)) - (b*((b^3 - 5*a*b*c)/
(4*a*c^3 - b^2*c^2) + (((6*b^3*c^2 - 28*a*b*c^3)/(4*a*c^3 - b^2*c^2) + ((8*
b^3*c^4 - 32*a*b*c^5)*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c))/
(2*(4*a*c^3 - b^2*c^2)*(256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^3 - 192*a^2*b^
2*c^4)))*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c))/(2*(256*a^3*c
^5 - 4*b^6*c^2 + 48*a*b^4*c^3 - 192*a^2*b^2*c^4)) - (b^2*((b^3*c^4)/2 - 2*a
*b*c^5)*(6*a*c - b^2)^2)/(c^4*(4*a*c - b^2)^3*(4*a*c^3 - b^2*c^2))))/(2*a*(
4*a*c - b^2)^(3/2))) - ((b*(6*a*c - b^2)*(8*a + (8*a*c^2*(2*b^6 - 128*a^3*c
^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c))/(256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^3
- 192*a^2*b^2*c^4)))/(8*c^2*(4*a*c - b^2)^(3/2)) + (a*b*(6*a*c - b^2)*(2*b^
6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c))/((4*a*c - b^2)^(3/2)*(256*a
^3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^3 - 192*a^2*b^2*c^4)))/(a*(4*a*c - b^2)) +
```

```
(b*(a/c^2 + ((8*a + (8*a*c^2*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b
^4*c))/(256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^3 - 192*a^2*b^2*c^4))*(2*b^6 -
 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c))/(2*(256*a^3*c^5 - 4*b^6*c^2 +
48*a*b^4*c^3 - 192*a^2*b^2*c^4)) - (a*b^2*(6*a*c - b^2)^2)/(c^2*(4*a*c - b^
2)^3)))/(2*a*(4*a*c - b^2)^(3/2))))/(b^6 + 36*a^2*b^2*c^2 - 12*a*b^4*c))*(6
*a*c - b^2))/(2*c^2*(4*a*c - b^2)^(3/2))
```

sympy [B] time = 19.76, size = 745, normalized size = 5.64

$$\left(-\frac{b\sqrt{-\left(4ac-b^2\right)^3}\left(6ac-b^2\right)}{4c^2\left(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6\right)}+\frac{1}{4c^2}\right)\log\left(x^2+\frac{-32a^2c^3\left(-\frac{b\sqrt{-\left(4ac-b^2\right)^3}\left(6ac-b^2\right)}{4c^2\left(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6\right)}+\frac{1}{4c^2}\right)+8a^2c+\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] (-b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(4*c**2*(64*a**3*c**3 - 48*a**2
*b**2*c**2 + 12*a*b**4*c - b**6)) + 1/(4*c**2))*log(x**2 + (-32*a**2*c**3*(
-b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(4*c**2*(64*a**3*c**3 - 48*a**2*
b**2*c**2 + 12*a*b**4*c - b**6)) + 1/(4*c**2)) + 8*a**2*c + 16*a*b**2*c**2*
(-b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(4*c**2*(64*a**3*c**3 - 48*a**2
*b**2*c**2 + 12*a*b**4*c - b**6)) + 1/(4*c**2)) - a*b**2 - 2*b**4*c*(-b*sqr
t(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(4*c**2*(64*a**3*c**3 - 48*a**2*b**2*c
**2 + 12*a*b**4*c - b**6)) + 1/(4*c**2)))/(6*a*b*c - b**3)) + (b*sqrt(-(4*a
*c - b**2)**3)*(6*a*c - b**2)/(4*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 1
2*a*b**4*c - b**6)) + 1/(4*c**2))*log(x**2 + (-32*a**2*c**3*(b*sqrt(-(4*a*c
 - b**2)**3)*(6*a*c - b**2)/(4*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*
a*b**4*c - b**6)) + 1/(4*c**2)) + 8*a**2*c + 16*a*b**2*c**2*(b*sqrt(-(4*a*c
 - b**2)**3)*(6*a*c - b**2)/(4*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*
a*b**4*c - b**6)) + 1/(4*c**2)) - a*b**2 - 2*b**4*c*(b*sqrt(-(4*a*c - b**2)
**3)*(6*a*c - b**2)/(4*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c
 - b**6)) + 1/(4*c**2)))/(6*a*b*c - b**3)) + (2*a**2*c - a*b**2 + x**2*(3*a
*b*c - b**3))/(8*a**2*c**3 - 2*a*b**2*c**2 + x**4*(8*a*c**4 - 2*b**2*c**3)
+ x**2*(8*a*b*c**3 - 2*b**3*c**2))
```

3.92 $\int \frac{x^8}{\left(ax+bx^3+cx^5\right)^2}\,dx$

Optimal. Leaf size=271

$$\frac{\left(-\frac{b\left(b^2-8ac\right)}{\sqrt{b^2-4ac}}-6ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,c^{3/2}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{b\left(b^2-8ac\right)}{\sqrt{b^2-4ac}}-6ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\,c^{3/2}\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x^3\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2\right.}$$

```
[Out] -1/2*b*x/c/(-4*a*c+b^2)+1/2*x^3*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/
4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b^2-6*a*c-b*(-8*a
*c+b^2)/(-4*a*c+b^2)^(1/2))/c^(3/2)/(-4*a*c+b^2)*2^(1/2)/(b-(-4*a*c+b^2)^(1
/2))^(1/2)+1/4*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b^2-
6*a*c+b*(-8*a*c+b^2)/(-4*a*c+b^2)^(1/2))/c^(3/2)/(-4*a*c+b^2)*2^(1/2)/(b+(-
4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 0.53, antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1585, 1120, 1279, 1166, 205}

$$\frac{\left(-\frac{b\left(b^2-8ac\right)}{\sqrt{b^2-4ac}}-6ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,c^{3/2}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{b\left(b^2-8ac\right)}{\sqrt{b^2-4ac}}-6ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\,c^{3/2}\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x^3\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] -(b*x)/(2*c*(b^2 - 4*a*c)) + (x^3*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^
2 + c*x^4)) + ((b^2 - 6*a*c - (b*(b^2 - 8*a*c))/Sqrt[b^2 - 4*a*c])*ArcTan[(
Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(3/2)*(b^2 -
4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^2 - 6*a*c + (b*(b^2 - 8*a*c))/Sqr
t[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2
*Sqrt[2]*c^(3/2)*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1120

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> -Simp[(d^3*(d*x)^(m - 3)*(2*a + b*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*
(p + 1)*(b^2 - 4*a*c)), x] + Dist[d^4/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^
(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[
m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\int \frac{x^8}{\left(ax+bx^3+cx^5\right)^2}\,dx = \int \frac{x^6}{\left(a+bx^2+cx^4\right)^2}\,dx$$

$$= \frac{x^3\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\int \frac{x^2\left(6a+bx^2\right)}{a+bx^2+cx^4}\,dx}{2\left(b^2-4ac\right)}$$

$$= -\frac{bx}{2c\left(b^2-4ac\right)} + \frac{x^3\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\int \frac{ab+\left(b^2-6ac\right)x^2}{a+bx^2+cx^4}\,dx}{2c\left(b^2-4ac\right)}$$

$$= -\frac{bx}{2c\left(b^2-4ac\right)} + \frac{x^3\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\left(b^2-6ac-\frac{b\left(b^2-8ac\right)}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+c\ldots}}{4c\left(b^2-4ac\right)}$$

$$= -\frac{bx}{2c\left(b^2-4ac\right)} + \frac{x^3\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\left(b^2-6ac-\frac{b\left(b^2-8ac\right)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\ldots}{\sqrt{b-\sqrt{b^2-\ldots}}}\right.}{2\sqrt{2}\,c^{3/2}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4a\ldots}}}$$

Mathematica [A] time = 0.51, size = 282, normalized size = 1.04

$$\frac{-\frac{2\sqrt{c}\,x\left(a\left(b-2cx^2\right)+b^2x^2\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\sqrt{2}\left(b^2\sqrt{b^2-4ac}-6ac\sqrt{b^2-4ac}+8abc-b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\left(b^2\sqrt{b^2-4ac}-6ac\sqrt{b^2-4ac}-8abc+b^3\right)\tan^{-1}\left(\ldots\right.}{\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}}{4c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] ((-2*Sqrt[c]*x*(b^2*x^2 + a*(b - 2*c*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x
^4)) + (Sqrt[2]*(-b^3 + 8*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - 6*a*c*Sqrt[b^2 -
4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*
a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(b^3 - 8*a*b*c + b^2*Sqr
t[b^2 - 4*a*c] - 6*a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
 + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/
(4*c^(3/2))
```

fricas [B] time = 1.06, size = 2257, normalized size = 8.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(2*(b^2 - 2*a*c)*x^3 + 2*a*b*x - sqrt(1/2)*((b^2*c^2 - 4*a*c^3)*x^4 +
a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2)*sqrt(-(b^5 - 15*a*b^3*c + 60
*a^2*b*c^2 + (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((b
^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64
*a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6))*log((5*
a*b^4 - 81*a^2*b^2*c + 324*a^3*c^2)*x + 1/2*sqrt(1/2)*(b^7 - 17*a*b^5*c + 8
8*a^2*b^3*c^2 - 144*a^3*b*c^3 - (b^8*c^3 - 24*a*b^6*c^4 + 192*a^2*b^4*c^5 -
 640*a^3*b^2*c^6 + 768*a^4*c^7)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c
^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))*sqrt(-(b^5 - 15*a*b^3*c
+ 60*a^2*b*c^2 + (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqr
t((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8
- 64*a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6))) +
sqrt(1/2)*((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c
^2)*x^2)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (b^6*c^3 - 12*a*b^4*c^4 +
 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c^6
 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^4 +
48*a^2*b^2*c^5 - 64*a^3*c^6))*log((5*a*b^4 - 81*a^2*b^2*c + 324*a^3*c^2)*x
- 1/2*sqrt(1/2)*(b^7 - 17*a*b^5*c + 88*a^2*b^3*c^2 - 144*a^3*b*c^3 - (b^8*c
^3 - 24*a*b^6*c^4 + 192*a^2*b^4*c^5 - 640*a^3*b^2*c^6 + 768*a^4*c^7)*sqrt((
b^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 6
4*a^3*c^9)))*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (b^6*c^3 - 12*a*b^4*c
^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6
*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^
4 + 48*a^2*b^2*c^5 - 64*a^3*c^6))) - sqrt(1/2)*((b^2*c^2 - 4*a*c^3)*x^4 + a
*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2)*sqrt(-(b^5 - 15*a*b^3*c + 60*
a^2*b*c^2 - (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((b^
4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*
a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6))*log((5*a
*b^4 - 81*a^2*b^2*c + 324*a^3*c^2)*x + 1/2*sqrt(1/2)*(b^7 - 17*a*b^5*c + 88
*a^2*b^3*c^2 - 144*a^3*b*c^3 + (b^8*c^3 - 24*a*b^6*c^4 + 192*a^2*b^4*c^5 -
640*a^3*b^2*c^6 + 768*a^4*c^7)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c^
6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))*sqrt(-(b^5 - 15*a*b^3*c +
 60*a^2*b*c^2 - (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt
((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 -
 64*a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6))) + s
qrt(1/2)*((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^
2)*x^2)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 - (b^6*c^3 - 12*a*b^4*c^4 +
48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c^6
- 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^4 + 4
8*a^2*b^2*c^5 - 64*a^3*c^6))*log((5*a*b^4 - 81*a^2*b^2*c + 324*a^3*c^2)*x -
 1/2*sqrt(1/2)*(b^7 - 17*a*b^5*c + 88*a^2*b^3*c^2 - 144*a^3*b*c^3 + (b^8*c^
3 - 24*a*b^6*c^4 + 192*a^2*b^4*c^5 - 640*a^3*b^2*c^6 + 768*a^4*c^7)*sqrt((b
^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64
*a^3*c^9)))*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 - (b^6*c^3 - 12*a*b^4*c^
4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*
c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^4
 + 48*a^2*b^2*c^5 - 64*a^3*c^6))))/((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a
^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2)
```

giac [B] time = 4.02, size = 2736, normalized size = 10.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] -1/2*(b^2*x^3 - 2*a*c*x^3 + a*b*x)/((c*x^4 + b*x^2 + a)*(b^2*c - 4*a*c^2))
- 1/16*(2*b^8*c^4 - 32*a*b^6*c^5 + 160*a^2*b^4*c^6 - 256*a^3*b^2*c^7 - sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^8*c^2 + 16*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c^3 + 2*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7*c^3 - 80*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^4 - 24*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^4 - sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c^4 + 128*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^5 + 64*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^5 + 12*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^5 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^6 - 2*(b^2 - 4*a*c)*b^6*c^4 + 24
*(b^2 - 4*a*c)*a*b^4*c^5 - 64*(b^2 - 4*a*c)*a^2*b^2*c^6 - (2*b^4*c^2 - 20*a
*b^2*c^3 + 48*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*b^4 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c
 - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 12
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 6*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 - 2*(b^2 - 4*a*c)*b^2*
c^2 + 12*(b^2 - 4*a*c)*a*c^3)*(b^2*c - 4*a*c^2)^2 - 2*(sqrt(2)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*b^5*c^2 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^2*b^3*c^3 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 - 2*a*b^5*
c^3 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 + 8*sqrt(2)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a*b^3*c^4 + 16*a^2*b^3*c^4 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^2*b*c^5 - 32*a^3*b*c^5 + 2*(b^2 - 4*a*c)*a*b^3*c^3 - 8*(b^2 - 4*a*c)*a
^2*b*c^4)*abs(b^2*c - 4*a*c^2))*arctan(2*sqrt(1/2)*x/sqrt((b^3*c - 4*a*b*c^
2 + sqrt((b^3*c - 4*a*b*c^2)^2 - 4*(a*b^2*c - 4*a^2*c^2)*(b^2*c^2 - 4*a*c^3
)))/(b^2*c^2 - 4*a*c^3)))/((a*b^6*c^3 - 12*a^2*b^4*c^4 - 2*a*b^5*c^4 + 48*a
^3*b^2*c^5 + 16*a^2*b^3*c^5 + a*b^4*c^5 - 64*a^4*c^6 - 32*a^3*b*c^6 - 8*a^2
*b^2*c^6 + 16*a^3*c^7)*abs(b^2*c - 4*a*c^2)*abs(c)) + 1/16*(2*b^8*c^4 - 32*
a*b^6*c^5 + 160*a^2*b^4*c^6 - 256*a^3*b^2*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^8*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*b^7*c^3 - 80*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^2*b^4*c^4 - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a*b^5*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*b^6*c^4 + 128*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^3*b^2*c^5 + 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^2*b^3*c^5 + 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a*b^4*c^5 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*a^2*b^2*c^6 - 2*(b^2 - 4*a*c)*b^6*c^4 + 24*(b^2 - 4*a*c)*a*b^4*c^
5 - 64*(b^2 - 4*a*c)*a^2*b^2*c^6 - (2*b^4*c^2 - 20*a*b^2*c^3 + 48*a^2*c^4 -
 sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4 + 10*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c - 24*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 12*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2 + 12*(b^2 - 4*a*c)*
a*c^3)*(b^2*c - 4*a*c^2)^2 + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b
^5*c^2 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 - 2*sqrt(2)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 + 2*a*b^5*c^3 + 16*sqrt(2)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^2*b^2*c^4 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 - 16*a
```

```
^2*b^3*c^4 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 + 32*a^3*b
*c^5 - 2*(b^2 - 4*a*c)*a*b^3*c^3 + 8*(b^2 - 4*a*c)*a^2*b*c^4)*abs(b^2*c - 4
*a*c^2))*arctan(2*sqrt(1/2)*x/sqrt((b^3*c - 4*a*b*c^2 - sqrt((b^3*c - 4*a*b
*c^2)^2 - 4*(a*b^2*c - 4*a^2*c^2)*(b^2*c^2 - 4*a*c^3)))/(b^2*c^2 - 4*a*c^3)
))/((a*b^6*c^3 - 12*a^2*b^4*c^4 - 2*a*b^5*c^4 + 48*a^3*b^2*c^5 + 16*a^2*b^3
*c^5 + a*b^4*c^5 - 64*a^4*c^6 - 32*a^3*b*c^6 - 8*a^2*b^2*c^6 + 16*a^3*c^7)*
abs(b^2*c - 4*a*c^2)*abs(c))
```

maple [B] time = 0.03, size = 602, normalized size = 2.22

$$\frac{2\sqrt{2}\,ab\operatorname{arctanh}\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{\left(4ac-b^2\right)\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\frac{2\sqrt{2}\,ab\arctan\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{\left(4ac-b^2\right)\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\sqrt{2}}{4\left(4ac-b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(c*x^5+b*x^3+a*x)^2,x)
```

```
[Out] (-1/2*(2*a*c-b^2)/(4*a*c-b^2)/c*x^3+1/2/(4*a*c-b^2)*a*b/c*x)/(c*x^4+b*x^2+a
)-3/2/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)
/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a+1/4/(4*a*c-b^2)/c*2^(1/2)/((-b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2
)*c*x)*b^2+2/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*b-1/4/(
4*a*c-b^2)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*a
rctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^3+3/2/(4*a*c-b^2)*2
^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/
2))*c)^(1/2)*c*x)*a-1/4/(4*a*c-b^2)/c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2+2/(4*a*c-b^2)/
(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/
((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*b-1/4/(4*a*c-b^2)/c/(-4*a*c+b^2)^(1
/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2
)^(1/2))*c)^(1/2)*c*x)*b^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(b^2-2\,ac\right)x^3+abx}{2\left(\left(b^2c^2-4\,ac^3\right)x^4+ab^2c-4\,a^2c^2+\left(b^3c-4\,abc^2\right)x^2\right)}-\frac{-\int\frac{\left(b^2-6\,ac\right)x^2+ab}{cx^4+bx^2+a}\,dx}{2\left(b^2c-4\,ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((b^2 - 2*a*c)*x^3 + a*b*x)/((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2
*c^2 + (b^3*c - 4*a*b*c^2)*x^2) - 1/2*integrate(-((b^2 - 6*a*c)*x^2 + a*b)/
(c*x^4 + b*x^2 + a), x)/(b^2*c - 4*a*c^2)
```

mupad [B] time = 3.86, size = 6293, normalized size = 23.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(a*x + b*x^3 + c*x^5)^2,x)
```

```
[Out] - ((x^3*(2*a*c - b^2))/(2*c*(4*a*c - b^2)) - (a*b*x)/(2*c*(4*a*c - b^2)))/(
a + b*x^2 + c*x^4) - atan(((((16*a*b^7*c^2 - 1024*a^4*b*c^5 - 192*a^2*b^5*c
^3 + 768*a^3*b^3*c^4)/(8*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^
```

```
3)) - (x*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*
b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c
 - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^
8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2)*(16
*b^7*c^3 - 192*a*b^5*c^4 - 1024*a^3*b*c^6 + 768*a^2*b^3*c^5))/(2*(b^4*c + 1
6*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^
5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*
c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^1
0*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^
2*c^8)))^(1/2) - (x*(b^6 - 72*a^3*c^3 + 74*a^2*b^2*c^2 - 16*a*b^4*c))/(2*(b
^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) -
 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 2
7*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3 -
24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 614
4*a^5*b^2*c^8)))^(1/2)*1i - (((16*a*b^7*c^2 - 1024*a^4*b*c^5 - 192*a^2*b^5*
c^3 + 768*a^3*b^3*c^4)/(8*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c
^3)) + (x*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2
*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*
c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b
^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2)*(1
6*b^7*c^3 - 192*a*b^5*c^4 - 1024*a^3*b*c^6 + 768*a^2*b^3*c^5))/(2*(b^4*c +
16*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a
^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9
*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^
10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b
^2*c^8)))^(1/2) + (x*(b^6 - 72*a^3*c^3 + 74*a^2*b^2*c^2 - 16*a*b^4*c))/(2*(
b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2)
- 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 -
27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3 -
 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 61
44*a^5*b^2*c^8)))^(1/2)*1i)/((((16*a*b^7*c^2 - 1024*a^4*b*c^5 - 192*a^2*b^5
*c^3 + 768*a^3*b^3*c^4)/(8*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*
c^3)) - (x*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^
2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a
*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*
b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2)*(
16*b^7*c^3 - 192*a*b^5*c^4 - 1024*a^3*b*c^6 + 768*a^2*b^3*c^5))/(2*(b^4*c +
 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*
a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^
9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b
^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*
b^2*c^8)))^(1/2) - (x*(b^6 - 72*a^3*c^3 + 74*a^2*b^2*c^2 - 16*a*b^4*c))/(2*
(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2)
 - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 -
 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3
- 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6
144*a^5*b^2*c^8)))^(1/2) + (((16*a*b^7*c^2 - 1024*a^4*b*c^5 - 192*a^2*b^5*c
^3 + 768*a^3*b^3*c^4)/(8*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^
3)) + (x*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*
b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c
 - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^
8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2)*(16
*b^7*c^3 - 192*a*b^5*c^4 - 1024*a^3*b*c^6 + 768*a^2*b^3*c^5))/(2*(b^4*c + 1
6*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^
5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*
c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^1
0*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^
2*c^8)))^(1/2) + (x*(b^6 - 72*a^3*c^3 + 74*a^2*b^2*c^2 - 16*a*b^4*c))/(2*(b
^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) -
```

3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 2
7*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3 -
24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 614
4*a^5*b^2*c^8)))^(1/2) + (5*a^2*b^4 + 216*a^4*c^2 - 66*a^3*b^2*c)/(4*(b^6*c
 - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3))))*(-(b^11 + b^2*(-(4*a*c -
b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*
a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^
9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^
4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2)*2i - atan(((((16*a*b^7*c^2 - 1024*a^4
*b*c^5 - 192*a^2*b^5*c^3 + 768*a^3*b^3*c^4)/(8*(b^6*c - 64*a^3*c^4 - 12*a*b
^4*c^2 + 48*a^2*b^2*c^3)) - (x*(-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 384
0*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*
b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a
*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^
5*b^2*c^8)))^(1/2)*(16*b^7*c^3 - 192*a*b^5*c^4 - 1024*a^3*b*c^6 + 768*a^2*b
^3*c^5))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^11 - b^2*(-(4*a*c - b
^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a
^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9
 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4
*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2) - (x*(b^6 - 72*a^3*c^3 + 74*a^2*b^2*c^
2 - 16*a*b^4*c))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^11 - b^2*(-(4
*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3
 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096
*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 +
3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2)*1i - (((16*a*b^7*c^2 - 1024*a^
4*b*c^5 - 192*a^2*b^5*c^3 + 768*a^3*b^3*c^4)/(8*(b^6*c - 64*a^3*c^4 - 12*a*
b^4*c^2 + 48*a^2*b^2*c^3)) + (x*(-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 38
40*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a
*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3 - 24*
a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a
^5*b^2*c^8)))^(1/2)*(16*b^7*c^3 - 192*a*b^5*c^4 - 1024*a^3*b*c^6 + 768*a^2*
b^3*c^5))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^11 - b^2*(-(4*a*c -
b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*
a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^
9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^
4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2) + (x*(b^6 - 72*a^3*c^3 + 74*a^2*b^2*c
^2 - 16*a*b^4*c))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^11 - b^2*(-(
4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3
 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(409
6*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 +
 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2)*1i)/((((16*a*b^7*c^2 - 1024*a
^4*b*c^5 - 192*a^2*b^5*c^3 + 768*a^3*b^3*c^4)/(8*(b^6*c - 64*a^3*c^4 - 12*a
*b^4*c^2 + 48*a^2*b^2*c^3)) - (x*(-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 3
840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*
a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3 - 24
*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*
a^5*b^2*c^8)))^(1/2)*(16*b^7*c^3 - 192*a*b^5*c^4 - 1024*a^3*b*c^6 + 768*a^2
*b^3*c^5))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^11 - b^2*(-(4*a*c -
 b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840
*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c
^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a
^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2) - (x*(b^6 - 72*a^3*c^3 + 74*a^2*b^2*
c^2 - 16*a*b^4*c))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^11 - b^2*(-
(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^
3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(40
96*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6
 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2) + (((16*a*b^7*c^2 - 1024*a^4
*b*c^5 - 192*a^2*b^5*c^3 + 768*a^3*b^3*c^4)/(8*(b^6*c - 64*a^3*c^4 - 12*a*b
^4*c^2 + 48*a^2*b^2*c^3)) + (x*(-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 384

```
0*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*
b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a
*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^
5*b^2*c^8)))^(1/2)*(16*b^7*c^3 - 192*a*b^5*c^4 - 1024*a^3*b*c^6 + 768*a^2*b
^3*c^5))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^11 - b^2*(-(4*a*c - b
^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a
^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9
 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4
*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2) + (x*(b^6 - 72*a^3*c^3 + 74*a^2*b^2*c^
2 - 16*a*b^4*c))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^11 - b^2*(-(4
*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3
+ 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096
*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 +
3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2) + (5*a^2*b^4 + 216*a^4*c^2 - 6
6*a^3*b^2*c)/(4*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3))))*(-(
b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 15
04*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(
1/2))/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280
*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2)*2i
```

sympy [A] time = 33.31, size = 379, normalized size = 1.40

$$\frac{abx + x^3\left(-2ac + b^2\right)}{8a^2c^2 - 2ab^2c + x^4\left(8ac^3 - 2b^2c^2\right) + x^2\left(8abc^2 - 2b^3c\right)} + \text{RootSum}\left(t^4\left(1048576a^6c^9 - 1572864a^5b^2c^8 + 983040a\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] (a*b*x + x**3*(-2*a*c + b**2))/(8*a**2*c**2 - 2*a*b**2*c + x**4*(8*a*c**3 -
 2*b**2*c**2) + x**2*(8*a*b*c**2 - 2*b**3*c)) + RootSum(_t**4*(1048576*a**6
*c**9 - 1572864*a**5*b**2*c**8 + 983040*a**4*b**4*c**7 - 327680*a**3*b**6*c
**6 + 61440*a**2*b**8*c**5 - 6144*a*b**10*c**4 + 256*b**12*c**3) + _t**2*(-
61440*a**5*b*c**5 + 61440*a**4*b**3*c**4 - 24064*a**3*b**5*c**3 + 4608*a**2
*b**7*c**2 - 432*a*b**9*c + 16*b**11) + 1296*a**5*c**2 - 360*a**4*b**2*c +
25*a**3*b**4, Lambda(_t, _t*log(x + (49152*_t**3*a**4*c**7 - 40960*_t**3*a*
*3*b**2*c**6 + 12288*_t**3*a**2*b**4*c**5 - 1536*_t**3*a*b**6*c**4 + 64*_t*
*3*b**8*c**3 - 1728*_t*a**3*b*c**3 + 656*_t*a**2*b**3*c**2 - 88*_t*a*b**5*c
 + 4*_t*b**7)/(324*a**3*c**2 - 81*a**2*b**2*c + 5*a*b**4))))
```

3.93 $\int \frac{x^7}{\left(ax+bx^3+cx^5\right)^2} dx$

Optimal. Leaf size=78

$$\frac{2a\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}+\frac{x^2\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

```
[Out] 1/2*x^2*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+2*a*arctanh((2*c*x^2+b)/(-
4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)
```

Rubi [A] time = 0.07, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1585, 1114, 722, 618, 206}

$$\frac{x^2\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{2a\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] (x^2*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (2*a*ArcTanh[(b
 + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 722

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*(2*p + 3)*(c*d^2 -
b*d*e + a*e^2))/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c
*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
0] && LtQ[p, -1]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
```

```
Q[r - p]
```

Rubi steps

$$\int \frac{x^7}{\left(ax+bx^3+cx^5\right)^2}\,dx=\int \frac{x^5}{\left(a+bx^2+cx^4\right)^2}\,dx$$
$$=\frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2}{\left(a+bx+cx^2\right)^2}\,dx,x,x^2\right)$$
$$=\frac{x^2\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{a\operatorname{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,x^2\right)}{b^2-4ac}$$
$$=\frac{x^2\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{(2a)\operatorname{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx^2\right)}{b^2-4ac}$$
$$=\frac{x^2\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{2a\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}$$

Mathematica [A] time = 0.09, size = 93, normalized size = 1.19

$$\frac{2a\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{3/2}}+\frac{a\left(b-2cx^2\right)+b^2x^2}{2c\left(4ac-b^2\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] (b^2*x^2 + a*(b - 2*c*x^2))/(2*c*(-b^2 + 4*a*c)*(a + b*x^2 + c*x^4)) + (2*a
*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2)
```

fricas [B] time = 0.87, size = 407, normalized size = 5.22

$$\left[-\frac{ab^3-4a^2bc+\left(b^4-6ab^2c+8a^2c^2\right)x^2+2\left(ac^2x^4+abcx^2+a^2c\right)\sqrt{b^2-4ac}\log\left(\frac{2c^2x^4+2bcx^2+b^2-2ac-\left(2cx^2+b\right)\sqrt{b^2}}{cx^4+bx^2+a}\right.}{2\left(ab^4c-8a^2b^2c^2+16a^3c^3+\left(b^4c^2-8ab^2c^3+16a^2c^4\right)x^4+\left(b^5c-8ab^3c^2+16a^2bc^3\right)x^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(a*b^3 - 4*a^2*b*c + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*x^2 + 2*(a*c^2*x^4
 + a*b*c*x^2 + a^2*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 -
2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)))/(a*b^4*c - 8
*a^2*b^2*c^2 + 16*a^3*c^3 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^4 + (b^5
*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2), -1/2*(a*b^3 - 4*a^2*b*c + (b^4 - 6*a
*b^2*c + 8*a^2*c^2)*x^2 - 4*(a*c^2*x^4 + a*b*c*x^2 + a^2*c)*sqrt(-b^2 + 4*a
*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)))/(a*b^4*c - 8*a
^2*b^2*c^2 + 16*a^3*c^3 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^4 + (b^5*c
 - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2)]
```

giac [A] time = 2.00, size = 96, normalized size = 1.23

$$-\frac{2a\arctan\left(\frac{2cx^2+b}{\sqrt{-b^2+4ac}}\right)}{\left(b^2-4ac\right)\sqrt{-b^2+4ac}}-\frac{b^2x^2-2acx^2+ab}{2\left(cx^4+bx^2+a\right)\left(b^2c-4ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] -2*a*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-b^2 + 4*
a*c)) - 1/2*(b^2*x^2 - 2*a*c*x^2 + a*b)/((c*x^4 + b*x^2 + a)*(b^2*c - 4*a*c
^2))
```

maple [A] time = 0.01, size = 104, normalized size = 1.33

$$\frac{2a\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}}+\frac{\frac{ab}{\left(4ac-b^2\right)c}-\frac{\left(2ac-b^2\right)x^2}{\left(4ac-b^2\right)c}}{2c\,x^4+2b\,x^2+2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(c*x^5+b*x^3+a*x)^2,x)
```

```
[Out] 1/2*(-(2*a*c-b^2)/(4*a*c-b^2)/c*x^2+1/(4*a*c-b^2)*a*b/c)/(c*x^4+b*x^2+a)+2*
a/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 2.20, size = 187, normalized size = 2.40

$$-\frac{\frac{x^2\left(2\,a\,c-b^2\right)}{2\,c\left(4\,a\,c-b^2\right)}-\frac{a\,b}{2\,c\left(4\,a\,c-b^2\right)}}{c\,x^4+b\,x^2+a}-\frac{2\,a\,\mathrm{atan}\left(\frac{b^3-4\,a\,b\,c}{\left(4\,a\,c-b^2\right)^{3/2}}-\frac{x^2\left(\frac{4\,a\,c^2}{\left(4\,a\,c-b^2\right)^{7/2}}+\frac{4\,a\left(b^3\,c^2-4\,a\,b\,c^3\right)\left(b^3-4\,a\,b\,c\right)}{\left(4\,a\,c-b^2\right)^{13/2}}\right)\left(4\,a\,c-b^2\right)^4}{8\,a^2\,c^2}\right)}{\left(4\,a\,c-b^2\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a*x + b*x^3 + c*x^5)^2,x)
```

```
[Out] - ((x^2*(2*a*c - b^2))/(2*c*(4*a*c - b^2)) - (a*b)/(2*c*(4*a*c - b^2)))/(a
+ b*x^2 + c*x^4) - (2*a*atan((b^3 - 4*a*b*c)/(4*a*c - b^2)^(3/2) - (x^2*((4
*a*c^2)/(4*a*c - b^2)^(7/2) + (4*a*(b^3*c^2 - 4*a*b*c^3)*(b^3 - 4*a*b*c))/(
4*a*c - b^2)^(13/2))*(4*a*c - b^2)^4)/(8*a^2*c^2)))/(4*a*c - b^2)^(3/2)
```

sympy [B] time = 1.52, size = 282, normalized size = 3.62

$$-a\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\log\left(x^2+\frac{-16a^3c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+8a^2b^2c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}-ab^4\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+ab}{2ac}\right)+a\sqrt{-\frac{1}{\left(4ac-\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] -a*sqrt(-1/(4*a*c - b**2)**3)*log(x**2 + (-16*a**3*c**2*sqrt(-1/(4*a*c - b*
*2)**3) + 8*a**2*b**2*c*sqrt(-1/(4*a*c - b**2)**3) - a*b**4*sqrt(-1/(4*a*c
- b**2)**3) + a*b)/(2*a*c)) + a*sqrt(-1/(4*a*c - b**2)**3)*log(x**2 + (16*a
**3*c**2*sqrt(-1/(4*a*c - b**2)**3) - 8*a**2*b**2*c*sqrt(-1/(4*a*c - b**2)*
*3) + a*b**4*sqrt(-1/(4*a*c - b**2)**3) + a*b)/(2*a*c)) + (a*b + x**2*(-2*a
*c + b**2))/(8*a**2*c**2 - 2*a*b**2*c + x**4*(8*a*c**3 - 2*b**2*c**2) + x**
2*(8*a*b*c**2 - 2*b**3*c))
```

3.94 $\int \frac{x^6}{\left(ax+bx^3+cx^5\right)^2}\,dx$

Optimal. Leaf size=237

$$\frac{x\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\left(b-\frac{4ac+b^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(b\sqrt{b^2-4ac}+4ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[Out] 1/2*x*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/4*arctan(x*2^(1/2)*c^(1/2)
/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b+(-4*a*c-b^2)/(-4*a*c+b^2)^(1/2))/(-4*a*c+
b^2)*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/4*arctan(x*2^(1/2)*c^(1
/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b^2+4*a*c+b*(-4*a*c+b^2)^(1/2))/(-4*a*c+
b^2)^(3/2)*2^(1/2)/c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 0.36, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1585, 1120, 1166, 205}

$$\frac{x\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\left(b-\frac{4ac+b^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(b\sqrt{b^2-4ac}+4ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] (x*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((b - (b^2 + 4*a*
c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]
]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^2 +
 4*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2
- 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*
c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1120

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> -Simp[(d^3*(d*x)^(m - 3)*(2*a + b*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*
(p + 1)*(b^2 - 4*a*c)), x] + Dist[d^4/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^
(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[
m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\int \frac{x^6}{\left(ax+bx^3+cx^5\right)^2}\,dx = \int \frac{x^4}{\left(a+bx^2+cx^4\right)^2}\,dx$$

$$= \frac{x\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\int \frac{2a-bx^2}{a+bx^2+cx^4}\,dx}{2\left(b^2-4ac\right)}$$

$$= \frac{x\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(b^2+4ac-b\sqrt{b^2-4ac}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{4\left(b^2-4ac\right)^{3/2}} + \frac{\left(b^2+\right.}{}$$

$$= \frac{x\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(b^2+4ac-b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(b^2+\right.}{2\sqrt{}}$$

Mathematica [A] time = 0.41, size = 235, normalized size = 0.99

$$\frac{1}{4}\left(\frac{2\left(2ax+bx^3\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\sqrt{2}\left(b\sqrt{b^2-4ac}-4ac-b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\left(b\sqrt{b^2-4ac}+4ac+b^2\right.}{\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{}}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] ((2*(2*a*x + b*x^3))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*(-b^2 -
 4*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2
- 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sq
rt[2]*(b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
 + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*
a*c]]))/4
```

fricas [B] time = 0.95, size = 1668, normalized size = 7.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b*x^3 + sqrt(1/2)*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 -
4*a*b*c)*x^2)*sqrt(-(b^3 + 12*a*b*c + (b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^
3 - 64*a^3*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))
/(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4))*log((3*b^2 + 4*a*c)*
x + sqrt(1/2)*(b^4 - 8*a*b^2*c + 16*a^2*c^2 + 2*(b^7*c - 12*a*b^5*c^2 + 48*
a^2*b^3*c^3 - 64*a^3*b*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 -
64*a^3*c^5))*sqrt(-(b^3 + 12*a*b*c + (b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3
 - 64*a^3*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/
```

```
(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4))) - sqrt(1/2)*((b^2*c
- 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(-(b^3 + 12*a*b
*c + (b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)/sqrt(b^6*c^2 - 12
*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^6*c - 12*a*b^4*c^2 + 48*a^2*b
^2*c^3 - 64*a^3*c^4))*log((3*b^2 + 4*a*c)*x - sqrt(1/2)*(b^4 - 8*a*b^2*c +
16*a^2*c^2 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)/sqrt(
b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))*sqrt(-(b^3 + 12*a*b*
c + (b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)/sqrt(b^6*c^2 - 12*
a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^
2*c^3 - 64*a^3*c^4))) + sqrt(1/2)*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c
+ (b^3 - 4*a*b*c)*x^2)*sqrt(-(b^3 + 12*a*b*c - (b^6*c - 12*a*b^4*c^2 + 48*a
^2*b^2*c^3 - 64*a^3*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*
a^3*c^5))/(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4))*log((3*b^2
+ 4*a*c)*x + sqrt(1/2)*(b^4 - 8*a*b^2*c + 16*a^2*c^2 - 2*(b^7*c - 12*a*b^5*
c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b
^2*c^4 - 64*a^3*c^5))*sqrt(-(b^3 + 12*a*b*c - (b^6*c - 12*a*b^4*c^2 + 48*a^
2*b^2*c^3 - 64*a^3*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a
^3*c^5))/(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4))) - sqrt(1/2)
*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(-(b^3
 + 12*a*b*c - (b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)/sqrt(b^6
*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^6*c - 12*a*b^4*c^2 +
 48*a^2*b^2*c^3 - 64*a^3*c^4))*log((3*b^2 + 4*a*c)*x - sqrt(1/2)*(b^4 - 8*a
*b^2*c + 16*a^2*c^2 - 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c
^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))*sqrt(-(b^3
+ 12*a*b*c - (b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)/sqrt(b^6*
c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^6*c - 12*a*b^4*c^2 +
48*a^2*b^2*c^3 - 64*a^3*c^4))) + 4*a*x)/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*
a^2*c + (b^3 - 4*a*b*c)*x^2)
```

giac [B] time = 3.39, size = 2132, normalized size = 9.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b*x^3 + 2*a*x)/((c*x^4 + b*x^2 + a)*(b^2 - 4*a*c)) - 1/16*(2*b^7*c^2 -
 8*a*b^5*c^3 - 32*a^2*b^3*c^4 + 128*a^3*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*b^6*c + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a^2*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*b^5*c^2 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^3*b*c^3 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^2*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^5*c^2 + 32*(b^2 - 4*a*c)*a^2*b*c^4 - (2*b^3
*c^2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c +
 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)
*b*c^2)*(b^2 - 4*a*c)^2 + 4*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*
c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 2*sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*a*b^4*c^2 + 16*sqrt(2)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
2*b*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 16*a^2*b^2*c^
3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 32*a^3*c^4 + 2*(b^2
 - 4*a*c)*a*b^2*c^2 - 8*(b^2 - 4*a*c)*a^2*c^3)*abs(b^2 - 4*a*c))*arctan(2*s
qrt(1/2)*x/sqrt((b^3 - 4*a*b*c + sqrt((b^3 - 4*a*b*c)^2 - 4*(a*b^2 - 4*a^2*
c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c^2)))/((a*b^6*c - 12*a^2*b^4*c^2 - 2*a
*b^5*c^2 + 48*a^3*b^2*c^3 + 16*a^2*b^3*c^3 + a*b^4*c^3 - 64*a^4*c^4 - 32*a^
```

```
3*b*c^4 - 8*a^2*b^2*c^4 + 16*a^3*c^5)*abs(b^2 - 4*a*c)*abs(c)) + 1/16*(2*b^
7*c^2 - 8*a*b^5*c^3 - 32*a^2*b^3*c^4 + 128*a^3*b*c^5 - sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*b^6*c + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*b^5*c^2 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a^3*b*c^3 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^2*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^5*c^2 + 32*(b^2 - 4*a*c)*a^2*b*c^4 -
 (2*b^3*c^2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c -
 sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 -
 4*a*c)*b*c^2)*(b^2 - 4*a*c)^2 - 4*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a*b^4*c - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 2*sqrt(2
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 2*a*b^4*c^2 + 16*sqrt(2)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^2*b*c^3 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 16*a^2
*b^2*c^3 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^4 + 32*a^3*c^4 -
 2*(b^2 - 4*a*c)*a*b^2*c^2 + 8*(b^2 - 4*a*c)*a^2*c^3)*abs(b^2 - 4*a*c))*arc
tan(2*sqrt(1/2)*x/sqrt((b^3 - 4*a*b*c - sqrt((b^3 - 4*a*b*c)^2 - 4*(a*b^2 -
 4*a^2*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c^2)))/((a*b^6*c - 12*a^2*b^4*c^
2 - 2*a*b^5*c^2 + 48*a^3*b^2*c^3 + 16*a^2*b^3*c^3 + a*b^4*c^3 - 64*a^4*c^4
- 32*a^3*b*c^4 - 8*a^2*b^2*c^4 + 16*a^3*c^5)*abs(b^2 - 4*a*c)*abs(c))
```

maple [B] time = 0.03, size = 452, normalized size = 1.91

$$-\frac{\sqrt{2}\, ac \arctan\!\mathrm{h}\left(\frac{\sqrt{2}\, cx}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{\left(4ac-b^2\right)\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}-\frac{\sqrt{2}\, ac \arctan\left(\frac{\sqrt{2}\, cx}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{\left(4ac-b^2\right)\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\sqrt{2}\, b^2 \cdots}{4\left(4ac-b^2\right)\sqrt{\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(c*x^5+b*x^3+a*x)^2,x)
```

```
[Out] (-1/2/(4*a*c-b^2)*b*x^3-a/(4*a*c-b^2)*x)/(c*x^4+b*x^2+a)+1/4/(4*a*c-b^2)*2^
(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(
1/2))*c)^(1/2)*c*x)*b-c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x
)*a-1/4/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^
(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2-1/4/(4*a*c
-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b
^2)^(1/2))*c)^(1/2)*c*x)*b-c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4
*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c
*x)*a-1/4/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)
^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{bx^3+2\,ax}{2\left(\left(b^2c-4\,ac^2\right)x^4+ab^2-4\,a^2c+\left(b^3-4\,abc\right)x^2\right)}+\frac{-\frac{\left(\sqrt{2}\,\sqrt{bc+\sqrt{b^2-4\,ac}\,c}\,b^4-8\,\sqrt{2}\,\sqrt{bc+\sqrt{b^2-4\,ac}\,c}\,ab^2c-2\,\sqrt{2}\,\sqrt{bc+\sqrt{b^2-4\,ac}\,c}\,b^3c\cdots}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b*x^3 + 2*a*x)/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b
*c)*x^2) + 1/2*integrate((b*x^2 - 2*a)/(c*x^4 + b*x^2 + a), x)/(b^2 - 4*a*c
)
```

mupad [B] time = 3.64, size = 4973, normalized size = 20.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a*x + b*x^3 + c*x^5)^2,x)
```

```
[Out] - atan(((((2048*a^4*c^5 - 32*a*b^6*c^2 + 384*a^2*b^4*c^3 - 1536*a^3*b^2*c^4
)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - (x*(((-(4*a*c - b^
2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(
b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4
+ 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2)*(16*b^7*c^2 - 192*a*b^5*c^3
- 1024*a^3*b*c^5 + 768*a^2*b^3*c^4))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*((
(-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b
^3*c^3)/(32*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280
*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2) - (x*(b^4*c + 8
*a^2*c^3 + 2*a*b^2*c^2))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(((-(4*a*c - b
^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*
(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4
 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2)*1i - (((2048*a^4*c^5 - 32*a
*b^6*c^2 + 384*a^2*b^4*c^3 - 1536*a^3*b^2*c^4)/(8*(b^6 - 64*a^3*c^3 + 48*a^
2*b^2*c^2 - 12*a*b^4*c)) + (x*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*
c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(b^12*c + 4096*a^6*c^7 - 24*a*b
^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*
b^2*c^6)))^(1/2)*(16*b^7*c^2 - 192*a*b^5*c^3 - 1024*a^3*b*c^5 + 768*a^2*b^3
*c^4))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(((-(4*a*c - b^2)^9)^(1/2) - b^9
 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(b^12*c + 4096*a^6
*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^
5 - 6144*a^5*b^2*c^6)))^(1/2) + (x*(b^4*c + 8*a^2*c^3 + 2*a*b^2*c^2))/(2*(b
^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b
*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(b^12*c + 4096*a^6*c^7 - 24*a*
b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5
*b^2*c^6)))^(1/2)*1i)/((((2048*a^4*c^5 - 32*a*b^6*c^2 + 384*a^2*b^4*c^3 - 1
536*a^3*b^2*c^4)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - (x*
(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3
*b^3*c^3)/(32*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 12
80*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2)*(16*b^7*c^2 -
 192*a*b^5*c^3 - 1024*a^3*b*c^5 + 768*a^2*b^3*c^4))/(2*(b^4 + 16*a^2*c^2 -
8*a*b^2*c)))*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*
c^2 - 512*a^3*b^3*c^3)/(32*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2
*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2)
- (x*(b^4*c + 8*a^2*c^3 + 2*a*b^2*c^2))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))
*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^
3*b^3*c^3)/(32*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1
280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2) - (4*a^2*b*c
^2 + 3*a*b^3*c)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (((2
048*a^4*c^5 - 32*a*b^6*c^2 + 384*a^2*b^4*c^3 - 1536*a^3*b^2*c^4)/(8*(b^6 -
64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(((-(4*a*c - b^2)^9)^(1/2)
- b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(b^12*c + 409
6*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b
^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2)*(16*b^7*c^2 - 192*a*b^5*c^3 - 1024*a^3*b
*c^5 + 768*a^2*b^3*c^4))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(((-(4*a*c - b
^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*
(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4
```

```
 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2) + (x*(b^4*c + 8*a^2*c^3 + 2
*a*b^2*c^2))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(((-(4*a*c - b^2)^9)^(1/2)
 - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(b^12*c + 40
96*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*
b^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2)))*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 76
8*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(b^12*c + 4096*a^6*c^7
 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6
144*a^5*b^2*c^6)))^(1/2)*2i - atan(((((2048*a^4*c^5 - 32*a*b^6*c^2 + 384*a^
2*b^4*c^3 - 1536*a^3*b^2*c^4)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*
b^4*c)) - (x*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5
*c^2 + 512*a^3*b^3*c^3)/(32*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^
2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2)
*(16*b^7*c^2 - 192*a*b^5*c^3 - 1024*a^3*b*c^5 + 768*a^2*b^3*c^4))/(2*(b^4 +
 16*a^2*c^2 - 8*a*b^2*c)))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^
4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(b^12*c + 4096*a^6*c^7 - 24*a*b^1
0*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^
2*c^6)))^(1/2) - (x*(b^4*c + 8*a^2*c^3 + 2*a*b^2*c^2))/(2*(b^4 + 16*a^2*c^2
 - 8*a*b^2*c)))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*
b^5*c^2 + 512*a^3*b^3*c^3)/(32*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240
*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^(1
/2)*1i - (((2048*a^4*c^5 - 32*a*b^6*c^2 + 384*a^2*b^4*c^3 - 1536*a^3*b^2*c^
4)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(-(b^9 + (-(4*
a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32
*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^
4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2)*(16*b^7*c^2 - 192*a*b^5*c^
3 - 1024*a^3*b*c^5 + 768*a^2*b^3*c^4))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*
(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^
3*b^3*c^3)/(32*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1
280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2) + (x*(b^4*c
 + 8*a^2*c^3 + 2*a*b^2*c^2))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(-(b^9 + (-
(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/
(32*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6
*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2)*1i)/((((2048*a^4*c^5 -
32*a*b^6*c^2 + 384*a^2*b^4*c^3 - 1536*a^3*b^2*c^4)/(8*(b^6 - 64*a^3*c^3 + 4
8*a^2*b^2*c^2 - 12*a*b^4*c)) - (x*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a
^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(b^12*c + 4096*a^6*c^7 - 2
4*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144
*a^5*b^2*c^6)))^(1/2)*(16*b^7*c^2 - 192*a*b^5*c^3 - 1024*a^3*b*c^5 + 768*a^
2*b^3*c^4))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(-(b^9 + (-(4*a*c - b^2)^9)
^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(b^12*c + 40
96*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*
b^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2) - (x*(b^4*c + 8*a^2*c^3 + 2*a*b^2*c^2))
/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 76
8*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(b^12*c + 4096*a^6*c^7
 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6
144*a^5*b^2*c^6)))^(1/2) + (((2048*a^4*c^5 - 32*a*b^6*c^2 + 384*a^2*b^4*c^3
 - 1536*a^3*b^2*c^4)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) +
 (x*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 51
2*a^3*b^3*c^3)/(32*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3
 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2)*(16*b^7*
c^2 - 192*a*b^5*c^3 - 1024*a^3*b*c^5 + 768*a^2*b^3*c^4))/(2*(b^4 + 16*a^2*c
^2 - 8*a*b^2*c)))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^
2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 2
40*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^
(1/2) + (x*(b^4*c + 8*a^2*c^3 + 2*a*b^2*c^2))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^
2*c)))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 +
 512*a^3*b^3*c^3)/(32*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*
c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2) - (4*
a^2*b*c^2 + 3*a*b^3*c)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c))
```

```
))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512
*a^3*b^3*c^3)/(32*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3
- 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2)*2i - ((a*
x)/(4*a*c - b^2) + (b*x^3)/(2*(4*a*c - b^2)))/(a + b*x^2 + c*x^4)
```

sympy [A] time = 4.55, size = 296, normalized size = 1.25

$$\frac{-2ax-bx^3}{8a^2c-2ab^2+x^4\left(8ac^2-2b^2c\right)+x^2\left(8abc-2b^3\right)}+\text{RootSum}\left(t^4\left(1048576a^6c^7-1572864a^5b^2c^6+983040a^4b\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] (-2*a*x - b*x**3)/(8*a**2*c - 2*a*b**2 + x**4*(8*a*c**2 - 2*b**2*c) + x**2*
(8*a*b*c - 2*b**3)) + RootSum(_t**4*(1048576*a**6*c**7 - 1572864*a**5*b**2*
c**6 + 983040*a**4*b**4*c**5 - 327680*a**3*b**6*c**4 + 61440*a**2*b**8*c**3
 - 6144*a*b**10*c**2 + 256*b**12*c) + _t**2*(-12288*a**4*b*c**4 + 8192*a**3
*b**3*c**3 - 1536*a**2*b**5*c**2 + 16*b**9) + 16*a**3*c**2 + 24*a**2*b**2*c
 + 9*a*b**4, Lambda(_t, _t*log(x + (16384*_t**3*a**3*b*c**4 - 12288*_t**3*a
**2*b**3*c**3 + 3072*_t**3*a*b**5*c**2 - 256*_t**3*b**7*c + 64*_t*a**2*c**2
 - 128*_t*a*b**2*c - 4*_t*b**4)/(4*a*c + 3*b**2))))
```

3.95 $\int \frac{x^5}{\left(ax+bx^3+cx^5\right)^2}\,dx$

Optimal. Leaf size=75

$$\frac{2a+bx^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{b\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}$$

```
[Out] 1/2*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-b*arctanh((2*c*x^2+b)/(-4*a*c+
b^2)^(1/2))/(-4*a*c+b^2)^(3/2)
```

Rubi [A] time = 0.07, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1585, 1114, 638, 618, 206}

$$\frac{2a+bx^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{b\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] (2*a + b*x^2)/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (b*ArcTanh[(b + 2*c*x
^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5}{\left(ax+bx^3+cx^5\right)^2}\,dx &= \int \frac{x^3}{\left(a+bx^2+cx^4\right)^2}\,dx \\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right) \\
&= \frac{2a+bx^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{b\operatorname{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^2\right)}{2\left(b^2-4ac\right)} \\
&= \frac{2a+bx^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{b\operatorname{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^2\right)}{b^2-4ac} \\
&= \frac{2a+bx^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{b\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}
\end{aligned}
$$

Mathematica [A]   time = 0.07, size = 79, normalized size = 1.05

$$
\frac{2a+bx^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{b\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] (2*a + b*x^2)/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (b*ArcTan[(b + 2*c*x^
2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2)
```

fricas [B]   time = 0.87, size = 360, normalized size = 4.80

$$
\left[\frac{2\,ab^2-8\,a^2c+\left(b^3-4\,abc\right)x^2-\left(bcx^4+b^2x^2+ab\right)\sqrt{b^2-4\,ac}\,\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac+\left(2\,cx^2+b\right)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)}{2\left(ab^4-8\,a^2b^2c+16\,a^3c^2+\left(b^4c-8\,ab^2c^2+16\,a^2c^3\right)x^4+\left(b^5-8\,ab^3c+16\,a^2bc^2\right)x^2\right)}, \frac{2\,ab^2}{2}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*b^2 - 8*a^2*c + (b^3 - 4*a*b*c)*x^2 - (b*c*x^4 + b^2*x^2 + a*b)*s
qrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*s
qrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 +
 (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*
x^2), 1/2*(2*a*b^2 - 8*a^2*c + (b^3 - 4*a*b*c)*x^2 - 2*(b*c*x^4 + b^2*x^2 +
 a*b)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*
a*c)))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^
3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^2)]
```

giac [A]   time = 2.05, size = 82, normalized size = 1.09

$$
\frac{b\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^2-4\,ac\right)\sqrt{-b^2+4\,ac}} + \frac{bx^2+2\,a}{2\left(cx^4+bx^2+a\right)\left(b^2-4\,ac\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] b*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-b^2 + 4*a*c
)) + 1/2*(b*x^2 + 2*a)/((c*x^4 + b*x^2 + a)*(b^2 - 4*a*c))
```

maple [A] time = 0.01, size = 77, normalized size = 1.03

$$-\frac{b \arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}}+\frac{-b\,x^2-2a}{2\left(4ac-b^2\right)\left(c\,x^4+b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(c*x^5+b*x^3+a*x)^2,x)
```

```
[Out] 1/2*(-b*x^2-2*a)/(4*a*c-b^2)/(c*x^4+b*x^2+a)-b/(4*a*c-b^2)^(3/2)*arctan((2*
c*x^2+b)/(4*a*c-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 0.14, size = 178, normalized size = 2.37

$$\frac{b\,\mathrm{atan}\left(\frac{b^3-4\,a\,b\,c}{{\left(4\,a\,c-b^2\right)}^{3/2}}-\frac{x^2\,{\left(4\,a\,c-b^2\right)}^4\,\left(\frac{b^2\,c^2}{a\,{\left(4\,a\,c-b^2\right)}^{7/2}}+\frac{b^2\,\left(2\,b^3\,c^2-8\,a\,b\,c^3\right)\,\left(b^3-4\,a\,b\,c\right)}{2\,a\,{\left(4\,a\,c-b^2\right)}^{13/2}}\right)}{2\,b^2\,c^2}\right)}{{\left(4\,a\,c-b^2\right)}^{3/2}}-\frac{\frac{a}{4\,a\,c-b^2}+\frac{b\,x^2}{2\,\left(4\,a\,c-b^2\right)}}{c\,x^4+b\,x^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a*x + b*x^3 + c*x^5)^2,x)
```

```
[Out] (b*atan((b^3 - 4*a*b*c)/(4*a*c - b^2)^(3/2) - (x^2*(4*a*c - b^2)^4*((b^2*c^
2)/(a*(4*a*c - b^2)^(7/2)) + (b^2*(2*b^3*c^2 - 8*a*b*c^3)*(b^3 - 4*a*b*c))/
(2*a*(4*a*c - b^2)^(13/2))))/(2*b^2*c^2)))/(4*a*c - b^2)^(3/2) - (a/(4*a*c
- b^2) + (b*x^2)/(2*(4*a*c - b^2)))/(a + b*x^2 + c*x^4)
```

sympy [B] time = 1.36, size = 269, normalized size = 3.59

$$\frac{b\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\log\left(x^2+\frac{-16a^2bc^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+8ab^3c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}-b^5\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+b^2}{2bc}\right)}{2}-\frac{b\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\log\left(x^2+\frac{16a^2bc^2\sqrt{-\frac{1}{(4ac}}}{}\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] b*sqrt(-1/(4*a*c - b**2)**3)*log(x**2 + (-16*a**2*b*c**2*sqrt(-1/(4*a*c - b
**2)**3) + 8*a*b**3*c*sqrt(-1/(4*a*c - b**2)**3) - b**5*sqrt(-1/(4*a*c - b*
*2)**3) + b**2)/(2*b*c))/2 - b*sqrt(-1/(4*a*c - b**2)**3)*log(x**2 + (16*a*
*2*b*c**2*sqrt(-1/(4*a*c - b**2)**3) - 8*a*b**3*c*sqrt(-1/(4*a*c - b**2)**3
) + b**5*sqrt(-1/(4*a*c - b**2)**3) + b**2)/(2*b*c))/2 + (-2*a - b*x**2)/(8
*a**2*c - 2*a*b**2 + x**4*(8*a*c**2 - 2*b**2*c) + x**2*(8*a*b*c - 2*b**3))
```

3.96 $\int \frac{x^4}{\left(ax+bx^3+cx^5\right)^2}\,dx$

Optimal. Leaf size=221

$$-\frac{x\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(2b-\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(\sqrt{b^2-4ac}+2b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[Out] -1/2*x*(2*c*x^2+b)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/2*arctan(x*2^(1/2)*c^(1/2
)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(2*b-(-4*a*c+b^2)^(1/2))/(-4*a*c+b^
2)^(3/2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/2*arctan(x*2^(1/2)*c^(1/2)/
(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(2*b+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)
^(3/2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1585, 1119, 1166, 205}

$$-\frac{x\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(2b-\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(\sqrt{b^2-4ac}+2b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] -(x*(b + 2*c*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[c]*(2*b -
Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])
/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(2*b
+ Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]
])/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1119

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d*(d*x)^(m - 1)*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*(p
+ 1)*(b^2 - 4*a*c)), x] - Dist[d^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^(m
- 2)*(b*(m - 1) + 2*c*(m + 4*p + 5)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x
] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[m,
1] && LeQ[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\int \frac{x^4}{\left(ax+bx^3+cx^5\right)^2}\,dx = \int \frac{x^2}{\left(a+bx^2+cx^4\right)^2}\,dx$$

$$= -\frac{x\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\int \frac{b-2cx^2}{a+bx^2+cx^4}\,dx}{2\left(b^2-4ac\right)}$$

$$= -\frac{x\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(c\left(1+\frac{2b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2\left(b^2-4ac\right)} + \frac{\left(c\left(2b-\cdots\right.\right.}{\cdots}$$

$$= -\frac{x\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\sqrt{c}\left(2b-\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c}\,\left(\cdots\right.}{\cdots}$$

Mathematica [A] time = 0.43, size = 222, normalized size = 1.00

$$\frac{-bx-2cx^3}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\sqrt{c}\left(\sqrt{b^2-4ac}-2b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c}\left(\sqrt{b^2-4ac}+2b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\cdots}}{\sqrt{\sqrt{b^2-4\cdots}}}\right)}{\sqrt{2}\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+\cdots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] (-(b*x) - 2*c*x^3)/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (Sqrt[c]*(-2*b +
 Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]
)/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(2*b
 + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]
]])/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

fricas [B] time = 0.90, size = 1680, normalized size = 7.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(4*c*x^3 + sqrt(1/2)*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 -
 4*a*b*c)*x^2)*sqrt(-(b^3 + 12*a*b*c + (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c
^2 - 64*a^4*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)
)/(a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3))*log((3*b^2*c + 4*a*
c^2)*x + 1/2*sqrt(1/2)*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2 - (a*b^8 - 8*a^2*b^6
*c + 128*a^4*b^2*c^3 - 256*a^5*c^4)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^
2*c^2 - 64*a^5*c^3))*sqrt(-(b^3 + 12*a*b*c + (a*b^6 - 12*a^2*b^4*c + 48*a^3
*b^2*c^2 - 64*a^4*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^
5*c^3))/(a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3))) - sqrt(1/2)*
((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(-(b^3
```

```
+ 12*a*b*c + (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)/sqrt(a^2*
b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^6 - 12*a^2*b^4*c +
48*a^3*b^2*c^2 - 64*a^4*c^3))*log((3*b^2*c + 4*a*c^2)*x - 1/2*sqrt(1/2)*(b^
5 - 8*a*b^3*c + 16*a^2*b*c^2 - (a*b^8 - 8*a^2*b^6*c + 128*a^4*b^2*c^3 - 256
*a^5*c^4)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))*sqrt(
-(b^3 + 12*a*b*c + (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)/sqr
t(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^6 - 12*a^2*b^
4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3))) + sqrt(1/2)*((b^2*c - 4*a*c^2)*x^4 + a
*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(-(b^3 + 12*a*b*c - (a*b^6 - 12*a
^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^
4*b^2*c^2 - 64*a^5*c^3))/(a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^
3))*log((3*b^2*c + 4*a*c^2)*x + 1/2*sqrt(1/2)*(b^5 - 8*a*b^3*c + 16*a^2*b*c
^2 + (a*b^8 - 8*a^2*b^6*c + 128*a^4*b^2*c^3 - 256*a^5*c^4)/sqrt(a^2*b^6 - 1
2*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))*sqrt(-(b^3 + 12*a*b*c - (a*b^6
- 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c +
 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*
a^4*c^3))) - sqrt(1/2)*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*
a*b*c)*x^2)*sqrt(-(b^3 + 12*a*b*c - (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2
- 64*a^4*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(
a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3))*log((3*b^2*c + 4*a*c^2
)*x - 1/2*sqrt(1/2)*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2 + (a*b^8 - 8*a^2*b^6*c
+ 128*a^4*b^2*c^3 - 256*a^5*c^4)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c
^2 - 64*a^5*c^3))*sqrt(-(b^3 + 12*a*b*c - (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^
2*c^2 - 64*a^4*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c
^3))/(a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3))) + 2*b*x)/((b^2*
c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)
```

giac [B] time = 3.08, size = 1970, normalized size = 8.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*c*x^3 + b*x)/((c*x^4 + b*x^2 + a)*(b^2 - 4*a*c)) + 1/8*(4*b^6*c^2 -
 32*a*b^4*c^3 + 64*a^2*b^2*c^4 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*b^6 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a*b^4*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*b^5*c - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^2*b^2*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*b^3*c^2 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*
c^2 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3
 - 4*(b^2 - 4*a*c)*b^4*c^2 + 16*(b^2 - 4*a*c)*a*b^2*c^3 - (2*b^2*c^2 - 8*a*
c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2 + 4*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c + 2*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c - sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*c^2 - 2*(b^2 - 4*a*c)*c^2)*(b^2 - 4*a*c)
^2 + (sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 - 8*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^3*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*
c - 2*b^5*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 8*sqrt
(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*b^3*c^2 + 16*a*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a*b*c^3 - 32*a^2*b*c^3 + 2*(b^2 - 4*a*c)*b^3*c - 8*(b^2 - 4*a*c)*a*b*c
^2)*abs(b^2 - 4*a*c))*arctan(2*sqrt(1/2)*x/sqrt((b^3 - 4*a*b*c + sqrt((b^3
- 4*a*b*c)^2 - 4*(a*b^2 - 4*a^2*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c^2)))/
((a*b^6 - 12*a^2*b^4*c - 2*a*b^5*c + 48*a^3*b^2*c^2 + 16*a^2*b^3*c^2 + a*b^
4*c^2 - 64*a^4*c^3 - 32*a^3*b*c^3 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*abs(b^2 - 4
*a*c)*abs(c)) - 1/8*(4*b^6*c^2 - 32*a*b^4*c^3 + 64*a^2*b^2*c^4 - 2*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6 + 16*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 4*sqrt(2)*sqrt(b^2 - 4*
```

```
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 4*(b^2 - 4*a*c)*b^4*c^2 + 16*(b^2 - 4*a*
c)*a*b^2*c^3 - (2*b^2*c^2 - 8*a*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*b*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*c^2 - 2*
(b^2 - 4*a*c)*c^2)*(b^2 - 4*a*c)^2 - (sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*b^5 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 2*sqrt(2)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c + 2*b^5*c + 16*sqrt(2)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a^2*b*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c
^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 16*a*b^3*c^2 - 4*sqr
t(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 + 32*a^2*b*c^3 - 2*(b^2 - 4*a*
c)*b^3*c + 8*(b^2 - 4*a*c)*a*b*c^2)*abs(b^2 - 4*a*c))*arctan(2*sqrt(1/2)*x/
sqrt((b^3 - 4*a*b*c - sqrt((b^3 - 4*a*b*c)^2 - 4*(a*b^2 - 4*a^2*c)*(b^2*c -
 4*a*c^2)))/(b^2*c - 4*a*c^2)))/((a*b^6 - 12*a^2*b^4*c - 2*a*b^5*c + 48*a^3
*b^2*c^2 + 16*a^2*b^3*c^2 + a*b^4*c^2 - 64*a^4*c^3 - 32*a^3*b*c^3 - 8*a^2*b
^2*c^3 + 16*a^3*c^4)*abs(b^2 - 4*a*c)*abs(c))
```

maple [A] time = 0.07, size = 342, normalized size = 1.55

$$\frac{\sqrt{2}\, bc \operatorname{arctanh}\left(\frac{\sqrt{2}\, cx}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{\left(4ac-b^2\right)\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\frac{\sqrt{2}\, bc \arctan\left(\frac{\sqrt{2}\, cx}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{\left(4ac-b^2\right)\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\sqrt{2}\, c \operatorname{arcta}}{2\left(4ac-b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^5+b*x^3+a*x)^2,x)
```

```
[Out] 1/2/(4*a*c-b^2)*x/(x^2+1/2*b/c+1/2*(-4*a*c+b^2)^(1/2)/c)+1/2*c/(4*a*c-b^2)*
2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1
/2))*c)^(1/2)*c*x)+c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2
)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b+1/
2/(4*a*c-b^2)*x/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)-1/2*c/(4*a*c-b^2)*2^
(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(
1/2))*c)^(1/2)*c*x)+c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b
^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*
b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*c*x^3 + b*x)/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*
b*c)*x^2) - 1/2*integrate((2*c*x^2 - b)/(c*x^4 + b*x^2 + a), x)/(b^2 - 4*a*
c)
```

mupad [B] time = 3.36, size = 4854, normalized size = 21.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a*x + b*x^3 + c*x^5)^2,x)
```

```
[Out] atan(((((8*b^7*c^2 - 96*a*b^5*c^3 - 512*a^3*b*c^5 + 384*a^2*b^3*c^4)/(4*(b^
6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(((-(4*a*c - b^2)^9)^(1
/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*b^12 +
 4096*a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a
^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2)*(8*b^7*c^2 - 96*a*b^5*c^3 - 512*a^3*
b*c^5 + 384*a^2*b^3*c^4))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(((-(4*a*c - b^2)
^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*
b^12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 +
3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2) - (x*(4*a*c^4 - 5*b^2*c^3))/(b
^4 + 16*a^2*c^2 - 8*a*b^2*c))*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*
c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*b^12 + 4096*a^7*c^6 - 24*a^2
*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*
b^2*c^5)))^(1/2)*1i - (((8*b^7*c^2 - 96*a*b^5*c^3 - 512*a^3*b*c^5 + 384*a^2
*b^3*c^4)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - (x*(((-(4*
a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^
3)/(32*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*
b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2)*(8*b^7*c^2 - 96*a*b^
5*c^3 - 512*a^3*b*c^5 + 384*a^2*b^3*c^4))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(
((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*
b^3*c^3)/(32*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 128
0*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2) + (x*(4*a*c^4
- 5*b^2*c^3))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(((-(4*a*c - b^2)^9)^(1/2) -
b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*b^12 + 4096*
a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4
*c^4 - 6144*a^6*b^2*c^5)))^(1/2)*1i)/((4*a*c^4 + 3*b^2*c^3)/(2*(b^6 - 64*a^
3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (((8*b^7*c^2 - 96*a*b^5*c^3 - 512*a
^3*b*c^5 + 384*a^2*b^3*c^4)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^
4*c)) + (x*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^
2 - 512*a^3*b^3*c^3)/(32*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b
^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2)*(8
*b^7*c^2 - 96*a*b^5*c^3 - 512*a^3*b*c^5 + 384*a^2*b^3*c^4))/(b^4 + 16*a^2*c
^2 - 8*a*b^2*c))*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*
b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240
*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1
/2) - (x*(4*a*c^4 - 5*b^2*c^3))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(((-(4*a*c
- b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(
32*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*
c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2) + (((8*b^7*c^2 - 96*a*b^
5*c^3 - 512*a^3*b*c^5 + 384*a^2*b^3*c^4)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*
c^2 - 12*a*b^4*c)) - (x*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 +
96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*
c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^
5)))^(1/2)*(8*b^7*c^2 - 96*a*b^5*c^3 - 512*a^3*b*c^5 + 384*a^2*b^3*c^4))/(b
^4 + 16*a^2*c^2 - 8*a*b^2*c))*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*
c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*b^12 + 4096*a^7*c^6 - 24*a^2
*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*
b^2*c^5)))^(1/2) + (x*(4*a*c^4 - 5*b^2*c^3))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c)
)*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a
^3*b^3*c^3)/(32*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 -
1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2)))*(((-(4*a*
c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)
/(32*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^
6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2)*2i + atan(((((8*b^7*c^
2 - 96*a*b^5*c^3 - 512*a^3*b*c^5 + 384*a^2*b^3*c^4)/(4*(b^6 - 64*a^3*c^3 +
48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*
a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(a*b^12 + 4096*a^7*c^6 -
24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 614
4*a^6*b^2*c^5)))^(1/2)*(8*b^7*c^2 - 96*a*b^5*c^3 - 512*a^3*b*c^5 + 384*a^2*
b^3*c^4))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2)
```

```
 - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(a*b^12 + 4096*a^7
*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^
4 - 6144*a^6*b^2*c^5)))^(1/2) - (x*(4*a*c^4 - 5*b^2*c^3))/(b^4 + 16*a^2*c^2
 - 8*a*b^2*c))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b
^5*c^2 + 512*a^3*b^3*c^3)/(32*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240*
a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/
2)*1i - (((8*b^7*c^2 - 96*a*b^5*c^3 - 512*a^3*b*c^5 + 384*a^2*b^3*c^4)/(4*(
b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - (x*(-(b^9 + (-(4*a*c - b
^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(a*b^1
2 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 384
0*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2)*(8*b^7*c^2 - 96*a*b^5*c^3 - 512*a
^3*b*c^5 + 384*a^2*b^3*c^4))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(-(b^9 + (-(4*
a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32
*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^
3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2) + (x*(4*a*c^4 - 5*b^2*c^3)
)/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a
^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(a*b^12 + 4096*a^7*c^6 - 2
4*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144
*a^6*b^2*c^5)))^(1/2)*1i)/((4*a*c^4 + 3*b^2*c^3)/(2*(b^6 - 64*a^3*c^3 + 48*
a^2*b^2*c^2 - 12*a*b^4*c)) + (((8*b^7*c^2 - 96*a*b^5*c^3 - 512*a^3*b*c^5 +
384*a^2*b^3*c^4)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*
(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^
3*b^3*c^3)/(32*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1
280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2)*(8*b^7*c^2 -
 96*a*b^5*c^3 - 512*a^3*b*c^5 + 384*a^2*b^3*c^4))/(b^4 + 16*a^2*c^2 - 8*a*b
^2*c))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 +
 512*a^3*b^3*c^3)/(32*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*
c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2) - (x*
(4*a*c^4 - 5*b^2*c^3))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(-(b^9 + (-(4*a*c -
b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(a*b^
12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 38
40*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2) + (((8*b^7*c^2 - 96*a*b^5*c^3 -
512*a^3*b*c^5 + 384*a^2*b^3*c^4)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12
*a*b^4*c)) - (x*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*
b^5*c^2 + 512*a^3*b^3*c^3)/(32*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240
*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1
/2)*(8*b^7*c^2 - 96*a*b^5*c^3 - 512*a^3*b*c^5 + 384*a^2*b^3*c^4))/(b^4 + 16
*a^2*c^2 - 8*a*b^2*c))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 -
96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*
c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^
5)))^(1/2) + (x*(4*a*c^4 - 5*b^2*c^3))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(-(b
^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^
3*c^3)/(32*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*
a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2)))*(-(b^9 + (-(4*
a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32
*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^
3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2)*2i + ((b*x)/(2*(4*a*c - b^
2)) + (c*x^3)/(4*a*c - b^2))/(a + b*x^2 + c*x^4)
```

sympy [A] time = 13.17, size = 298, normalized size = 1.35

$$\frac{bx+2cx^3}{8a^2c-2ab^2+x^4\left(8ac^2-2b^2c\right)+x^2\left(8abc-2b^3\right)}+\text{RootSum}\left(t^4\left(1048576a^7c^6-1572864a^6b^2c^5+983040a^5b\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] (b*x + 2*c*x**3)/(8*a**2*c - 2*a*b**2 + x**4*(8*a*c**2 - 2*b**2*c) + x**2*(
8*a*b*c - 2*b**3)) + RootSum(_t**4*(1048576*a**7*c**6 - 1572864*a**6*b**2*c
```

```
**5 + 983040*a**5*b**4*c**4 - 327680*a**4*b**6*c**3 + 61440*a**3*b**8*c**2
- 6144*a**2*b**10*c + 256*a*b**12) + _t**2*(-12288*a**4*b*c**4 + 8192*a**3*
b**3*c**3 - 1536*a**2*b**5*c**2 + 16*b**9) + 16*a**2*c**3 + 24*a*b**2*c**2
+ 9*b**4*c, Lambda(_t, _t*log(x + (16384*_t**3*a**5*c**4 - 8192*_t**3*a**4*
b**2*c**3 + 512*_t**3*a**2*b**6*c - 64*_t**3*a*b**8 - 128*_t*a**2*b*c**2 -
16*_t*a*b**3*c - 4*_t*b**5)/(4*a*c**2 + 3*b**2*c))))
```

3.97 $\int \frac{x^3}{\left(ax+bx^3+cx^5\right)^2} \, dx$

Optimal. Leaf size=74

$$\frac{2c\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}-\frac{b+2cx^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

```
[Out] 1/2*(-2*c*x^2-b)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+2*c*arctanh((2*c*x^2+b)/(-4*a
*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)
```

Rubi [A] time = 0.06, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1585, 1107, 614, 618, 206}

$$\frac{2c\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}-\frac{b+2cx^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] -(b + 2*c*x^2)/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (2*c*ArcTanh[(b + 2*
c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\left(ax+bx^3+cx^5\right)^2}\,dx &= \int \frac{x}{\left(a+bx^2+cx^4\right)^2}\,dx\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{\left(a+bx+cx^2\right)^2}\,dx,x,x^2\right)\\ &= -\frac{b+2cx^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{c\operatorname{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,x^2\right)}{b^2-4ac}\\ &= -\frac{b+2cx^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{(2c)\operatorname{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx^2\right)}{b^2-4ac}\\ &= -\frac{b+2cx^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{2c\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.08, size = 79, normalized size = 1.07

$$-\frac{\frac{4c\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+\frac{b+2cx^2}{a+bx^2+cx^4}}{2\left(b^2-4ac\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] -1/2*((b + 2*c*x^2)/(a + b*x^2 + c*x^4) + (4*c*ArcTan[(b + 2*c*x^2)/Sqrt[-b
^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c])/(b^2 - 4*a*c)
```

fricas [B] time = 0.80, size = 361, normalized size = 4.88

$$\left[-\frac{b^3-4\,abc+2\left(b^2c-4\,ac^2\right)x^2+2\left(c^2x^4+bcx^2+ac\right)\sqrt{b^2-4\,ac}\,\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac-\left(2\,cx^2+b\right)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)}{2\left(ab^4-8\,a^2b^2c+16\,a^3c^2+\left(b^4c-8\,ab^2c^2+16\,a^2c^3\right)x^4+\left(b^5-8\,ab^3c+16\,a^2bc^2\right)x^2\right)},-\frac{b^3-\cdots}{2\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*x^2 + 2*(c^2*x^4 + b*c*x^2 + a*c
)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b
)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^
2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^
2)*x^2), -1/2*(b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*x^2 - 4*(c^2*x^4 + b*c*x
^2 + a*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2
- 4*a*c)))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^
2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^2)]
```

giac [A] time = 2.13, size = 82, normalized size = 1.11

$$-\frac{2\,c\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^2-4\,ac\right)\sqrt{-b^2+4\,ac}}-\frac{2\,cx^2+b}{2\left(cx^4+bx^2+a\right)\left(b^2-4\,ac\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] -2*c*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-b^2 + 4*
a*c)) - 1/2*(2*c*x^2 + b)/((c*x^4 + b*x^2 + a)*(b^2 - 4*a*c))
```

maple [A] time = 0.01, size = 75, normalized size = 1.01

$$\frac{2c\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}}+\frac{2c\,x^2+b}{2\left(4ac-b^2\right)\left(c\,x^4+b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^5+b*x^3+a*x)^2,x)
```

```
[Out] 1/2*(2*c*x^2+b)/(4*a*c-b^2)/(c*x^4+b*x^2+a)+2*c/(4*a*c-b^2)^(3/2)*arctan((2
*c*x^2+b)/(4*a*c-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 2.16, size = 172, normalized size = 2.32

$$\frac{\frac{b}{2\left(4\,a\,c-b^2\right)}+\frac{c\,x^2}{4\,a\,c-b^2}}{c\,x^4+b\,x^2+a}-\frac{2\,c\,\mathrm{atan}\left(\frac{b^3-4\,a\,b\,c}{\left(4\,a\,c-b^2\right)^{3/2}}-\frac{x^2\left(4\,a\,c-b^2\right)^4\left(\frac{4\,c^4}{a\left(4\,a\,c-b^2\right)^{7/2}}+\frac{4\,c^2\left(b^3\,c^2-4\,a\,b\,c^3\right)\left(b^3-4\,a\,b\,c\right)}{a\left(4\,a\,c-b^2\right)^{13/2}}\right)}{8\,c^4}\right)}{\left(4\,a\,c-b^2\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*x + b*x^3 + c*x^5)^2,x)
```

```
[Out] (b/(2*(4*a*c - b^2)) + (c*x^2)/(4*a*c - b^2))/(a + b*x^2 + c*x^4) - (2*c*at
an((b^3 - 4*a*b*c)/(4*a*c - b^2)^(3/2) - (x^2*(4*a*c - b^2)^4*((4*c^4)/(a*(
4*a*c - b^2)^(7/2)) + (4*c^2*(b^3*c^2 - 4*a*b*c^3)*(b^3 - 4*a*b*c))/(a*(4*a
*c - b^2)^(13/2))))/(8*c^4)))/(4*a*c - b^2)^(3/2)
```

sympy [B] time = 1.28, size = 267, normalized size = 3.61

$$-c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\log\left(x^2+\frac{-16a^2c^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+8ab^2c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}-b^4c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+bc}{2c^2}\right)+c\sqrt{-\frac{1}{\left(4ac-\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] -c*sqrt(-1/(4*a*c - b**2)**3)*log(x**2 + (-16*a**2*c**3*sqrt(-1/(4*a*c - b*
*2)**3) + 8*a*b**2*c**2*sqrt(-1/(4*a*c - b**2)**3) - b**4*c*sqrt(-1/(4*a*c
- b**2)**3) + b*c)/(2*c**2)) + c*sqrt(-1/(4*a*c - b**2)**3)*log(x**2 + (16*
a**2*c**3*sqrt(-1/(4*a*c - b**2)**3) - 8*a*b**2*c**2*sqrt(-1/(4*a*c - b**2)
**3) + b**4*c*sqrt(-1/(4*a*c - b**2)**3) + b*c)/(2*c**2)) + (b + 2*c*x**2)/
(8*a**2*c - 2*a*b**2 + x**4*(8*a*c**2 - 2*b**2*c) + x**2*(8*a*b*c - 2*b**3)
)
```

3.98 $\int \frac{x^2}{\left(ax+bx^3+cx^5\right)^2}\,dx$

Optimal. Leaf size=252

$$\frac{x\left(-2ac+b^2+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(b\sqrt{b^2-4ac}-12ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,a\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(-b\sqrt{b^2-4ac}-12ac+\right.}{2\sqrt{2}\,a\left(b^2-4ac\right)^{3/2}}$$

```
[Out] 1/2*x*(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/4*arctan(x*2^(1/
2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b^2-12*a*c+b*(-4*a*c+b^2)
^(1/2))/a/(-4*a*c+b^2)^(3/2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/4*arcta
n(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b^2-12*a*c-b*(-4
*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)^(3/2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 0.46, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1585, 1092, 1166, 205}

$$\frac{x\left(-2ac+b^2+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(b\sqrt{b^2-4ac}-12ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,a\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(-b\sqrt{b^2-4ac}-12ac+\right.}{2\sqrt{2}\,a\left(b^2-4ac\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] (x*(b^2 - 2*a*c + b*c*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt
[c]*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
- Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 -
 4*a*c]]) - (Sqrt[c]*(b^2 - 12*a*c - b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*S
qrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)^(3/2)*Sq
rt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1092

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x*(b^2 -
 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2
 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\int \frac{x^2}{\left(ax+bx^3+cx^5\right)^2}\,dx = \int \frac{1}{\left(a+bx^2+cx^4\right)^2}\,dx$$

$$= \frac{x\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\int \frac{b^2-2ac-2\left(b^2-4ac\right)-bcx^2}{a+bx^2+cx^4}\,dx}{2a\left(b^2-4ac\right)}$$

$$= \frac{x\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(c\left(b^2-12ac-b\sqrt{b^2-4ac}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{4a\left(b^2-4ac\right)^{3/2}} + \cdots$$

$$= \frac{x\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\sqrt{c}\left(b^2-12ac+b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,a\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \cdots$$

Mathematica [A] time = 0.42, size = 243, normalized size = 0.96

$$\frac{\frac{2x\left(-2ac+b^2+bcx^2\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\sqrt{2}\sqrt{c}\left(b\sqrt{b^2-4ac}-12ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\sqrt{c}\left(b\sqrt{b^2-4ac}+12ac-b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] ((2*x*(b^2 - 2*a*c + b*c*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[
2]*Sqrt[c]*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/
Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*
c]]) + (Sqrt[2]*Sqrt[c]*(-b^2 + 12*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[
2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + S
qrt[b^2 - 4*a*c]]))/(4*a)
```

fricas [B] time = 1.06, size = 2309, normalized size = 9.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b*c*x^3 + sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c +
 (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (a^3*b^6
 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c + 81*
a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6
- 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))*log((5*b^4*c^2 - 81*a*b^2*c^
3 + 324*a^2*c^4)*x + 1/2*sqrt(1/2)*(b^8 - 23*a*b^6*c + 190*a^2*b^4*c^2 - 67
2*a^3*b^2*c^3 + 864*a^4*c^4 - (a^3*b^9 - 20*a^4*b^7*c + 144*a^5*b^5*c^2 - 4
48*a^6*b^3*c^3 + 512*a^7*b*c^4)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b
^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(-(b^5 - 15*a*b^3*c
```

```
+ 60*a^2*b*c^2 + (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqr
t((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2
- 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))) -
sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b
*c)*x^2)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (a^3*b^6 - 12*a^4*b^4*c +
 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6
 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c +
48*a^5*b^2*c^2 - 64*a^6*c^3))*log((5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*
x - 1/2*sqrt(1/2)*(b^8 - 23*a*b^6*c + 190*a^2*b^4*c^2 - 672*a^3*b^2*c^3 + 8
64*a^4*c^4 - (a^3*b^9 - 20*a^4*b^7*c + 144*a^5*b^5*c^2 - 448*a^6*b^3*c^3 +
512*a^7*b*c^4)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c
 + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 +
(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2
*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(
a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))) + sqrt(1/2)*((a*b^2
*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-(b
^5 - 15*a*b^3*c + 60*a^2*b*c^2 - (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 -
 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c +
 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 -
64*a^6*c^3))*log((5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*x + 1/2*sqrt(1/2)
*(b^8 - 23*a*b^6*c + 190*a^2*b^4*c^2 - 672*a^3*b^2*c^3 + 864*a^4*c^4 + (a^3
*b^9 - 20*a^4*b^7*c + 144*a^5*b^5*c^2 - 448*a^6*b^3*c^3 + 512*a^7*b*c^4)*sq
rt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2
 - 64*a^9*c^3)))*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 - (a^3*b^6 - 12*a^4
*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/
(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*
b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))) - sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x
^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-(b^5 - 15*a*b^3*c +
 60*a^2*b*c^2 - (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt
((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 -
 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))*log(
(5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*x - 1/2*sqrt(1/2)*(b^8 - 23*a*b^6*
c + 190*a^2*b^4*c^2 - 672*a^3*b^2*c^3 + 864*a^4*c^4 + (a^3*b^9 - 20*a^4*b^7
*c + 144*a^5*b^5*c^2 - 448*a^6*b^3*c^3 + 512*a^7*b*c^4)*sqrt((b^4 - 18*a*b^
2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*
sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 - (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b
^2*c^2 - 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7
*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^
2*c^2 - 64*a^6*c^3))) + 2*(b^2 - 2*a*c)*x)/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2
*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)
```

giac [B] time = 3.82, size = 2682, normalized size = 10.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b*c*x^3 + b^2*x - 2*a*c*x)/((c*x^4 + b*x^2 + a)*(a*b^2 - 4*a^2*c)) + 1
/16*(2*a^2*b^7*c^2 - 40*a^3*b^5*c^3 + 224*a^4*b^3*c^4 - 384*a^5*b*c^5 - sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^7 + 20*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c + 2*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c - 112*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^2 - 32*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^2 - sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^2 + 192*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^3 + 96*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^3 + 16*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^3 - 48*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^4 - 2*(b^2 - 4*a*c)*a^2*
```

```
b^5*c^2 + 32*(b^2 - 4*a*c)*a^3*b^3*c^3 - 96*(b^2 - 4*a*c)*a^4*b*c^4 + (2*b^
3*c^2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c
+ 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c
)*b*c^2)*(a*b^2 - 4*a^2*c)^2 + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*b^6 - 14*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c - 2*sqrt(2)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c - 2*a*b^6*c + 64*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 + 20*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^2*b^3*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + 28*a^2*b
^4*c^2 - 96*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^3 - 48*sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 10*sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^2*b^2*c^3 - 128*a^3*b^2*c^3 + 24*sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^3*c^4 + 192*a^4*c^4 + 2*(b^2 - 4*a*c)*a*b^4*c - 20*(b^2 - 4*a*c
)*a^2*b^2*c^2 + 48*(b^2 - 4*a*c)*a^3*c^3)*abs(a*b^2 - 4*a^2*c))*arctan(2*sq
rt(1/2)*x/sqrt((a*b^3 - 4*a^2*b*c + sqrt((a*b^3 - 4*a^2*b*c)^2 - 4*(a^2*b^2
 - 4*a^3*c)*(a*b^2*c - 4*a^2*c^2)))/(a*b^2*c - 4*a^2*c^2)))/((a^3*b^6 - 12*
a^4*b^4*c - 2*a^3*b^5*c + 48*a^5*b^2*c^2 + 16*a^4*b^3*c^2 + a^3*b^4*c^2 - 6
4*a^6*c^3 - 32*a^5*b*c^3 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*abs(a*b^2 - 4*a^2*c)
*abs(c)) - 1/16*(2*a^2*b^7*c^2 - 40*a^3*b^5*c^3 + 224*a^4*b^3*c^4 - 384*a^5
*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^7
+ 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c +
2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c - 112
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^2 - 32
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^2 - sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^2 + 192*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b*c^3 + 96*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^3 + 16*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^3 - 48*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^4 - 2*(b^2 -
 4*a*c)*a^2*b^5*c^2 + 32*(b^2 - 4*a*c)*a^3*b^3*c^3 - 96*(b^2 - 4*a*c)*a^4*b
*c^4 + (2*b^3*c^2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b
^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*
(b^2 - 4*a*c)*b*c^2)*(a*b^2 - 4*a^2*c)^2 - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a*b^6 - 14*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c - 2
*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*a*b^6*c + 64*sqrt(2)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 + 20*sqrt(2)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a^2*b^3*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^
2 - 28*a^2*b^4*c^2 - 96*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*c^3 - 4
8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 10*sqrt(2)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + 128*a^3*b^2*c^3 + 24*sqrt(2)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a^3*c^4 - 192*a^4*c^4 - 2*(b^2 - 4*a*c)*a*b^4*c + 20*
(b^2 - 4*a*c)*a^2*b^2*c^2 - 48*(b^2 - 4*a*c)*a^3*c^3)*abs(a*b^2 - 4*a^2*c))
*arctan(2*sqrt(1/2)*x/sqrt((a*b^3 - 4*a^2*b*c - sqrt((a*b^3 - 4*a^2*b*c)^2
- 4*(a^2*b^2 - 4*a^3*c)*(a*b^2*c - 4*a^2*c^2)))/(a*b^2*c - 4*a^2*c^2)))/((a
^3*b^6 - 12*a^4*b^4*c - 2*a^3*b^5*c + 48*a^5*b^2*c^2 + 16*a^4*b^3*c^2 + a^3
*b^4*c^2 - 64*a^6*c^3 - 32*a^5*b*c^3 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*abs(a*b^
2 - 4*a^2*c)*abs(c))
```

maple [B] time = 0.06, size = 733, normalized size = 2.91

$$\frac{\sqrt{2}\, b^{2} c \,\mathrm{arctanh}\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right) c}}\right)}{4\sqrt{-4ac+b^{2}}\,\left(4ac-b^{2}\right)\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right) c\, a}}+\frac{\sqrt{2}\, b^{2} c \arctan\left(\frac{\sqrt{2}\, c x}{\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right) c}}\right)}{4\sqrt{-4ac+b^{2}}\,\left(4ac-b^{2}\right)\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right) c\, a}}-\frac{3\cdots}{\sqrt{-4ac+\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^5+b*x^3+a*x)^2,x)
```

```
[Out] -1/4/(4*a*c-b^2)/a*x/(x^2+1/2*b/c+1/2*(-4*a*c+b^2)^(1/2)/c)*b-c/(-4*a*c+b^2
)^(1/2)/(4*a*c-b^2)*x/(x^2+1/2*b/c+1/2*(-4*a*c+b^2)^(1/2)/c)+1/4/(-4*a*c+b^
2)^(1/2)/(4*a*c-b^2)/a*x/(x^2+1/2*b/c+1/2*(-4*a*c+b^2)^(1/2)/c)*b^2-1/4*c/(
4*a*c-b^2)/a*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b-3*c^2/(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*2^(1
/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))
*c)^(1/2)*c*x)+1/4*c/(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)/a*2^(1/2)/((b+(-4*a*c+b
^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^
2-1/4/(4*a*c-b^2)/a*x/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)*b+c/(-4*a*c+b^
2)^(1/2)/(4*a*c-b^2)*x/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)-1/4/(-4*a*c+b
^2)^(1/2)/(4*a*c-b^2)/a*x/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)*b^2+1/4*c/
(4*a*c-b^2)/a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-
b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b-3*c^2/(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*
2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)
^(1/2))*c)^(1/2)*c*x)+1/4*c/(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)/a*2^(1/2)/((-b+(
-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/
2)*c*x)*b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b*c*x^3 + (b^2 - 2*a*c)*x)/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^
3*c + (a*b^3 - 4*a^2*b*c)*x^2) + 1/2*integrate((b*c*x^2 + b^2 - 6*a*c)/(c*x
^4 + b*x^2 + a), x)/(a*b^2 - 4*a^2*c)
```

mupad [B] time = 3.85, size = 6404, normalized size = 25.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x + b*x^3 + c*x^5)^2,x)
```

```
[Out] ((x*(2*a*c - b^2))/(2*a*(4*a*c - b^2)) - (b*c*x^3)/(2*a*(4*a*c - b^2)))/(a
+ b*x^2 + c*x^4) + atan(((((6144*a^5*c^6 + 16*a*b^8*c^2 - 288*a^2*b^6*c^3 +
 1920*a^3*b^4*c^4 - 5632*a^4*b^2*c^5)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4
*c + 48*a^4*b^2*c^2)) - (x*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^
5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*
c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b
^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^
2*c^5)))^(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4
*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 + b^2*(-(4*a*c
 - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 38
40*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12
 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840
*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2) + (x*(72*a^2*c^5 + b^4*c^3 - 14*a*
b^2*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 + b^2*(-(4*a*c
- b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 384
0*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12
+ 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*
a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*1i - (((6144*a^5*c^6 + 16*a*b^8*c^2
 - 288*a^2*b^6*c^3 + 1920*a^3*b^4*c^4 - 5632*a^4*b^2*c^5)/(8*(a^2*b^6 - 64*
a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) + (x*(-(b^11 + b^2*(-(4*a*c - b^2
)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4
*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 409
```

```
6*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b
^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a
^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-
(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1
504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^
(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 128
0*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2) - (x*(72*a^2*c
^5 + b^4*c^3 - 14*a*b^2*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(
b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 15
04*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(
1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280
*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*1i)/((((6144*a^
5*c^6 + 16*a*b^8*c^2 - 288*a^2*b^6*c^3 + 1920*a^3*b^4*c^4 - 5632*a^4*b^2*c^
5)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - (x*(-(b^11
+ b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^
3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))
/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*
b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*(1024*a^5*b*c^5 - 16
*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2
 - 8*a^3*b^2*c)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 +
 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c
*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 2
40*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^
(1/2) + (x*(72*a^2*c^5 + b^4*c^3 - 14*a*b^2*c^4))/(2*(a^2*b^4 + 16*a^4*c^2
- 8*a^3*b^2*c)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 +
288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*
(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 24
0*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(
1/2) + (((6144*a^5*c^6 + 16*a*b^8*c^2 - 288*a^2*b^6*c^3 + 1920*a^3*b^4*c^4
- 5632*a^4*b^2*c^5)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^
2)) + (x*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*
b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c
 - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^
8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*(10
24*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2
*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) -
 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 2
7*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 -
24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 614
4*a^8*b^2*c^5)))^(1/2) - (x*(72*a^2*c^5 + b^4*c^3 - 14*a*b^2*c^4))/(2*(a^2*
b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) -
3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27
*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 2
4*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144
*a^8*b^2*c^5)))^(1/2) + (5*b^3*c^4 - 36*a*b*c^5)/(4*(a^2*b^6 - 64*a^5*c^3 -
 12*a^3*b^4*c + 48*a^4*b^2*c^2))))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) -
 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 2
7*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 -
24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 614
4*a^8*b^2*c^5)))^(1/2)*2i + atan(((((6144*a^5*c^6 + 16*a*b^8*c^2 - 288*a^2*
b^6*c^3 + 1920*a^3*b^4*c^4 - 5632*a^4*b^2*c^5)/(8*(a^2*b^6 - 64*a^5*c^3 - 1
2*a^3*b^4*c + 48*a^4*b^2*c^2)) - (x*(-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2)
- 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 -
27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 -
 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 61
44*a^8*b^2*c^5)))^(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3
- 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 - b^2
*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5
*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*
```

```
(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c
^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2) + (x*(72*a^2*c^5 + b^4*c^
3 - 14*a*b^2*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 - b^2*
(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*
c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(
a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^
3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*1i - (((6144*a^5*c^6 + 16*
a*b^8*c^2 - 288*a^2*b^6*c^3 + 1920*a^3*b^4*c^4 - 5632*a^4*b^2*c^5)/(8*(a^2*
b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) + (x*(-(b^11 - b^2*(-(4*
a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 +
 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b
^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3
840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c^
2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^
2*c)))*(-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^
7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c -
 b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*
c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2) - (x*
(72*a^2*c^5 + b^4*c^3 - 14*a*b^2*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2
*c)))*(-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7
*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c -
b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c
^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*1i)/((
((6144*a^5*c^6 + 16*a*b^8*c^2 - 288*a^2*b^6*c^3 + 1920*a^3*b^4*c^4 - 5632*a
^4*b^2*c^5)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - (x
*(-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2
- 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^
9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 -
1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*(1024*a^5*b
*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 1
6*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^
5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*
c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b
^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^
2*c^5)))^(1/2) + (x*(72*a^2*c^5 + b^4*c^3 - 14*a*b^2*c^4))/(2*(a^2*b^4 + 16
*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5
*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c
 + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^
10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2
*c^5)))^(1/2) + (((6144*a^5*c^6 + 16*a*b^8*c^2 - 288*a^2*b^6*c^3 + 1920*a^3
*b^4*c^4 - 5632*a^4*b^2*c^5)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a
^4*b^2*c^2)) + (x*(-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 +
 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c
*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 2
40*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^
(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4)
)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 - b^2*(-(4*a*c - b^2)^9
)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^
3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a
^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*
c^4 - 6144*a^8*b^2*c^5)))^(1/2) - (x*(72*a^2*c^5 + b^4*c^3 - 14*a*b^2*c^4))
/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 - b^2*(-(4*a*c - b^2)^9)
^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3
*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^
9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c
^4 - 6144*a^8*b^2*c^5)))^(1/2) + (5*b^3*c^4 - 36*a*b*c^5)/(4*(a^2*b^6 - 64*
a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2))))*(-(b^11 - b^2*(-(4*a*c - b^2)^9
)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^
3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a
```

```
^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*
c^4 - 6144*a^8*b^2*c^5)))^(1/2)*2i
```

sympy [A] time = 165.89, size = 394, normalized size = 1.56

$$\frac{-bcx^3 + x\left(2ac - b^2\right)}{8a^3c - 2a^2b^2 + x^4\left(8a^2c^2 - 2ab^2c\right) + x^2\left(8a^2bc - 2ab^3\right)} + \mathrm{RootSum}\left(t^4\left(1048576a^9c^6 - 1572864a^8b^2c^5 + 983040a\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] (-b*c*x**3 + x*(2*a*c - b**2))/(8*a**3*c - 2*a**2*b**2 + x**4*(8*a**2*c**2
- 2*a*b**2*c) + x**2*(8*a**2*b*c - 2*a*b**3)) + RootSum(_t**4*(1048576*a**9
*c**6 - 1572864*a**8*b**2*c**5 + 983040*a**7*b**4*c**4 - 327680*a**6*b**6*c
**3 + 61440*a**5*b**8*c**2 - 6144*a**4*b**10*c + 256*a**3*b**12) + _t**2*(-
61440*a**5*b*c**5 + 61440*a**4*b**3*c**4 - 24064*a**3*b**5*c**3 + 4608*a**2
*b**7*c**2 - 432*a*b**9*c + 16*b**11) + 1296*a**2*c**5 - 360*a*b**2*c**4 +
25*b**4*c**3, Lambda(_t, _t*log(x + (32768*_t**3*a**7*b*c**4 - 28672*_t**3*
a**6*b**3*c**3 + 9216*_t**3*a**5*b**5*c**2 - 1280*_t**3*a**4*b**7*c + 64*_t
**3*a**3*b**9 + 1728*_t*a**4*c**4 - 2304*_t*a**3*b**2*c**3 + 740*_t*a**2*b*
*4*c**2 - 92*_t*a*b**6*c + 4*_t*b**8)/(324*a**2*c**4 - 81*a*b**2*c**3 + 5*b
**4*c**2))))
```

3.99 $\int \frac{x}{\left(ax+bx^3+cx^5\right)^2} \, dx$

Optimal. Leaf size=122

$$\frac{b\left(b^2-6ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\left(b^2-4ac\right)^{3/2}}-\frac{\log\left(a+bx^2+cx^4\right)}{4a^2}+\frac{\log(x)}{a^2}+\frac{-2ac+b^2+bcx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

```
[Out] 1/2*(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/2*b*(-6*a*c+b^2)*a
rctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^(3/2)+ln(x)/a^2-1/4
*ln(c*x^4+b*x^2+a)/a^2
```

Rubi [A] time = 0.19, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {1585, 1114, 740, 800, 634, 618, 206, 628}

$$\frac{b\left(b^2-6ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\left(b^2-4ac\right)^{3/2}}-\frac{\log\left(a+bx^2+cx^4\right)}{4a^2}+\frac{\log(x)}{a^2}+\frac{-2ac+b^2+bcx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] (b^2 - 2*a*c + b*c*x^2)/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (b*(b^2 -
 6*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^2*(b^2 - 4*a*c)^(3/2
)) + Log[x]/a^2 - Log[a + b*x^2 + c*x^4]/(4*a^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
```

```
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\int \frac{x}{\left(ax+bx^3+cx^5\right)^2}\,dx = \int \frac{1}{x\left(a+bx^2+cx^4\right)^2}\,dx$$

$$= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right)$$

$$= \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int \frac{-b^2+4ac-bcx}{x\left(a+bx+cx^2\right)}\,dx, x, x^2\right)}{2a\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int \left(\frac{-b^2+4ac}{ax} + \frac{b\left(b^2-5ac\right)+c\left(b^2-4ac\right)x}{a\left(a+bx+cx^2\right)}\right) dx, x, x^2\right)}{2a\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\log(x)}{a^2} - \frac{\text{Subst}\left(\int \frac{b\left(b^2-5ac\right)+c\left(b^2-4ac\right)x}{a+bx+cx^2}\,dx, x, x^2\right)}{2a^2\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\log(x)}{a^2} - \frac{\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)}{4a^2} - \frac{\left(b\left(b^2-6ac\right)\right)}{}$$

$$= \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\log(x)}{a^2} - \frac{\log\left(a+bx^2+cx^4\right)}{4a^2} + \frac{\left(b\left(b^2-6ac\right)\right)\text{Subst}\left(\right.}{2a^2\left(\right.}$$

$$= \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{b\left(b^2-6ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\left(b^2-4ac\right)^{3/2}} + \frac{\log(x)}{a^2} - \frac{\log\left(a+bx\right.}{4a^2}$$

Mathematica [A] time = 0.33, size = 207, normalized size = 1.70

$$\frac{\frac{2a\left(-2ac+b^2+bcx^2\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\left(b^2\sqrt{b^2-4ac}-4ac\sqrt{b^2-4ac}-6abc+b^3\right)\log\left(-\sqrt{b^2-4ac}+b+2cx^2\right)}{\left(b^2-4ac\right)^{3/2}}+\frac{\left(-b^2\sqrt{b^2-4ac}+4ac\sqrt{b^2-4ac}-6abc+b^3\right)\log\left(\sqrt{b^2-4a\ldots\right.}{\left(b^2-4ac\right)^{3/2}}}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] ((2*a*(b^2 - 2*a*c + b*c*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + 4*Log[
x] - ((b^3 - 6*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - 4*a*c*Sqrt[b^2 - 4*a*c])*Log
[b - Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2) + ((b^3 - 6*a*b*c -
b^2*Sqrt[b^2 - 4*a*c] + 4*a*c*Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c]
+ 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/(4*a^2)
```

fricas [B] time = 1.02, size = 813, normalized size = 6.66

$$\left[\frac{2\,ab^4-12\,a^2b^2c+16\,a^3c^2+2\left(ab^3c-4\,a^2bc^2\right)x^2+\left(\left(b^3c-6\,abc^2\right)x^4+ab^3-6\,a^2bc+\left(b^4-6\,ab^2c\right)x^2\right)\sqrt{b^2-\ldots}}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + 2*(a*b^3*c - 4*a^2*b*c^2)*x^2 +
 ((b^3*c - 6*a*b*c^2)*x^4 + a*b^3 - 6*a^2*b*c + (b^4 - 6*a*b^2*c)*x^2)*sqrt
(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt
(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (
b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^
2)*log(c*x^4 + b*x^2 + a) + 4*(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c -
8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^2)*log(x
))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^
4*c^3)*x^4 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2), 1/4*(2*a*b^4 - 12
*a^2*b^2*c + 16*a^3*c^2 + 2*(a*b^3*c - 4*a^2*b*c^2)*x^2 + 2*((b^3*c - 6*a*b
*c^2)*x^4 + a*b^3 - 6*a^2*b*c + (b^4 - 6*a*b^2*c)*x^2)*sqrt(-b^2 + 4*a*c)*a
rctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (a*b^4 - 8*a^2*b^2
*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c
 + 16*a^2*b*c^2)*x^2)*log(c*x^4 + b*x^2 + a) + 4*(a*b^4 - 8*a^2*b^2*c + 16*
a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^
2*b*c^2)*x^2)*log(x))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*
a^3*b^2*c^2 + 16*a^4*c^3)*x^4 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2)
]
```

giac [A] time = 2.32, size = 166, normalized size = 1.36

$$-\frac{\left(b^3-6\,abc\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\left(a^2b^2-4\,a^3c\right)\sqrt{-b^2+4\,ac}}+\frac{b^2cx^4-4\,ac^2x^4+b^3x^2-2\,abcx^2+3\,ab^2-8\,a^2c}{4\left(cx^4+bx^2+a\right)\left(a^2b^2-4\,a^3c\right)}-\frac{\log\left(cx^4+bx^2+a\right)}{4\,a^2}+\frac{\log\ldots}{2\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] -1/2*(b^3 - 6*a*b*c)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((a^2*b^2 - 4
*a^3*c)*sqrt(-b^2 + 4*a*c)) + 1/4*(b^2*c*x^4 - 4*a*c^2*x^4 + b^3*x^2 - 2*a*
b*c*x^2 + 3*a*b^2 - 8*a^2*c)/((c*x^4 + b*x^2 + a)*(a^2*b^2 - 4*a^3*c)) - 1/
4*log(c*x^4 + b*x^2 + a)/a^2 + 1/2*log(x^2)/a^2
```

maple [B] time = 0.02, size = 253, normalized size = 2.07

$$-\frac{bc\,x^2}{2\left(c\,x^4+b\,x^2+a\right)\left(4ac-b^2\right)a}-\frac{3bc\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}a}+\frac{b^3\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{2\left(4ac-b^2\right)^{\frac{3}{2}}a^2}-\frac{b^2}{2\left(c\,x^4+b\,x^2+a\right)\left(4ac-b^2\right)a}-\frac{c\,1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^5+b*x^3+a*x)^2,x)
```

```
[Out] 1/a^2*ln(x)-1/2/a/(c*x^4+b*x^2+a)*b*c/(4*a*c-b^2)*x^2+1/(c*x^4+b*x^2+a)/(4*
a*c-b^2)*c-1/2/a/(c*x^4+b*x^2+a)/(4*a*c-b^2)*b^2-1/a/(4*a*c-b^2)*c*ln(c*x^4
+b*x^2+a)+1/4/a^2/(4*a*c-b^2)*ln(c*x^4+b*x^2+a)*b^2-3/a/(4*a*c-b^2)^(3/2)*a
rctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b*c+1/2/a^2/(4*a*c-b^2)^(3/2)*arctan((
2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{bcx^2+b^2-2\,ac}{2\left(\left(ab^2c-4\,a^2c^2\right)x^4+a^2b^2-4\,a^3c+\left(ab^3-4\,a^2bc\right)x^2\right)}+\frac{-\int\frac{\left(b^2c-4\,ac^2\right)x^3+\left(b^3-5\,abc\right)x}{cx^4+bx^2+a}\,dx}{a^2b^2-4\,a^3c}+\frac{\log(x)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b*c*x^2 + b^2 - 2*a*c)/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c
+ (a*b^3 - 4*a^2*b*c)*x^2) + integrate(-((b^2*c - 4*a*c^2)*x^3 + (b^3 - 5*a
*b*c)*x)/(c*x^4 + b*x^2 + a), x)/(a^2*b^2 - 4*a^3*c) + log(x)/a^2
```

mupad [B] time = 6.31, size = 5048, normalized size = 41.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x + b*x^3 + c*x^5)^2,x)
```

```
[Out] log(x)/a^2 + ((2*a*c - b^2)/(2*a*(4*a*c - b^2)) - (b*c*x^2)/(2*a*(4*a*c - b
^2)))/(a + b*x^2 + c*x^4) - (log(a + b*x^2 + c*x^4)*(2*b^6 - 128*a^3*c^3 +
96*a^2*b^2*c^2 - 24*a*b^4*c))/(2*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c +
192*a^4*b^2*c^2)) + (b*atan((x^2*((((((b*((320*a^5*b*c^6 - 2*a^2*b^7*c^3 +
36*a^3*b^5*c^4 - 192*a^4*b^3*c^5)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48
*a^5*b^2*c^2) - ((2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c)*(2560*
a^7*b*c^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056*a^5*b^5*c^4 - 2688*a^6*
b^3*c^5))/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*
b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(6*a*c - b^2))/(4*a^2
*(4*a*c - b^2)^(3/2)) - (b*(6*a*c - b^2)*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*
c^2 - 24*a*b^4*c)*(2560*a^7*b*c^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056
*a^5*b^5*c^4 - 2688*a^6*b^3*c^5))/(8*a^2*(4*a*c - b^2)^(3/2)*(a^3*b^6 - 64*
a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*
b^4*c + 192*a^4*b^2*c^2)))*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4
*c))/(2*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)) + (b*((
6*a*b^5*c^4 + 80*a^3*b*c^6 - 44*a^2*b^3*c^5)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4
*b^4*c + 48*a^5*b^2*c^2) + (((320*a^5*b*c^6 - 2*a^2*b^7*c^3 + 36*a^3*b^5*c^
4 - 192*a^4*b^3*c^5)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)
 - ((2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c)*(2560*a^7*b*c^6 + 1
2*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056*a^5*b^5*c^4 - 2688*a^6*b^3*c^5))/(2*
(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*b^6 - 256*a^5
*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*
c^2 - 24*a*b^4*c))/(2*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2
```

```
*c^2)))*(6*a*c - b^2))/(4*a^2*(4*a*c - b^2)^(3/2)) + (b^3*(6*a*c - b^2)^3*(
2560*a^7*b*c^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056*a^5*b^5*c^4 - 2688
*a^6*b^3*c^5))/(64*a^6*(4*a*c - b^2)^(9/2)*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b
^4*c + 48*a^5*b^2*c^2)))*(3*b^6 - 40*a^3*c^3 + 69*a^2*b^2*c^2 - 27*a*b^4*c)
)/(8*a^3*c^2*(4*a*c - b^2)^(7/2)*(6*b^6 - 400*a^3*c^3 + 291*a^2*b^2*c^2 - 7
2*a*b^4*c)) + (3*b*(b^4 + 11*a^2*c^2 - 7*a*b^2*c)*((((6*a*b^5*c^4 + 80*a^3*
b*c^6 - 44*a^2*b^3*c^5)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c
^2) + (((320*a^5*b*c^6 - 2*a^2*b^7*c^3 + 36*a^3*b^5*c^4 - 192*a^4*b^3*c^5)/
(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) - ((2*b^6 - 128*a^3*
c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c)*(2560*a^7*b*c^6 + 12*a^3*b^9*c^2 - 184*a
^4*b^7*c^3 + 1056*a^5*b^5*c^4 - 2688*a^6*b^3*c^5))/(2*(a^3*b^6 - 64*a^6*c^3
 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c +
 192*a^4*b^2*c^2)))*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c))/(2
*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(2*b^6 - 128*
a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c))/(2*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^
3*b^4*c + 192*a^4*b^2*c^2)) - (b^3*c^5)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*
c + 48*a^5*b^2*c^2) - (b*(6*a*c - b^2)*((b*((320*a^5*b*c^6 - 2*a^2*b^7*c^3
+ 36*a^3*b^5*c^4 - 192*a^4*b^3*c^5)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c +
48*a^5*b^2*c^2) - ((2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c)*(256
0*a^7*b*c^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056*a^5*b^5*c^4 - 2688*a^
6*b^3*c^5))/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^
2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(6*a*c - b^2))/(4*a
^2*(4*a*c - b^2)^(3/2)) - (b*(6*a*c - b^2)*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^
2*c^2 - 24*a*b^4*c)*(2560*a^7*b*c^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 10
56*a^5*b^5*c^4 - 2688*a^6*b^3*c^5))/(8*a^2*(4*a*c - b^2)^(3/2)*(a^3*b^6 - 6
4*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^
3*b^4*c + 192*a^4*b^2*c^2))))/(4*a^2*(4*a*c - b^2)^(3/2)) + (b^2*(6*a*c - b
^2)^2*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c)*(2560*a^7*b*c^6 +
 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056*a^5*b^5*c^4 - 2688*a^6*b^3*c^5))/(
32*a^4*(4*a*c - b^2)^3*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^
2)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2))))/(8*a^3*c^2
*(4*a*c - b^2)^3*(6*b^6 - 400*a^3*c^3 + 291*a^2*b^2*c^2 - 72*a*b^4*c)))*(16
*a^6*b^6*(4*a*c - b^2)^(9/2) - 1024*a^9*c^3*(4*a*c - b^2)^(9/2) - 192*a^7*b
^4*c*(4*a*c - b^2)^(9/2) + 768*a^8*b^2*c^2*(4*a*c - b^2)^(9/2)))/(b^6*c^2 -
 12*a*b^4*c^3 + 36*a^2*b^2*c^4) + (((b*((4*a*b^4*c^3 - 17*a^2*b^2*c^4)/(a^3
*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c) - (((4*a^2*b^6*c^2 - 36*a^3*b^4*c^3 + 80*a
^4*b^2*c^4)/(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c) + ((4*a^4*b^6*c^2 - 32*a^5
*b^4*c^3 + 64*a^6*b^2*c^4)*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4
*c))/(2*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)*(4*a^2*b^6 - 256*a^5*c^3 - 48*
a^3*b^4*c + 192*a^4*b^2*c^2)))*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a
*b^4*c))/(2*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(6
*a*c - b^2))/(4*a^2*(4*a*c - b^2)^(3/2)) - (((b*((4*a^2*b^6*c^2 - 36*a^3*b^
4*c^3 + 80*a^4*b^2*c^4)/(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c) + ((4*a^4*b^6*
c^2 - 32*a^5*b^4*c^3 + 64*a^6*b^2*c^4)*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^
2 - 24*a*b^4*c))/(2*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)*(4*a^2*b^6 - 256*a
^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(6*a*c - b^2))/(4*a^2*(4*a*c - b
^2)^(3/2)) + (b*(6*a*c - b^2)*(4*a^4*b^6*c^2 - 32*a^5*b^4*c^3 + 64*a^6*b^2*
c^4)*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c))/(8*a^2*(4*a*c - b
^2)^(3/2)*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)*(4*a^2*b^6 - 256*a^5*c^3 - 4
8*a^3*b^4*c + 192*a^4*b^2*c^2)))*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24
*a*b^4*c))/(2*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)) +
 (b^3*(6*a*c - b^2)^3*(4*a^4*b^6*c^2 - 32*a^5*b^4*c^3 + 64*a^6*b^2*c^4))/(6
4*a^6*(4*a*c - b^2)^(9/2)*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))*(16*a^6*b^
6*(4*a*c - b^2)^(9/2) - 1024*a^9*c^3*(4*a*c - b^2)^(9/2) - 192*a^7*b^4*c*(4
*a*c - b^2)^(9/2) + 768*a^8*b^2*c^2*(4*a*c - b^2)^(9/2))*(3*b^6 - 40*a^3*c^
3 + 69*a^2*b^2*c^2 - 27*a*b^4*c))/(8*a^3*c^2*(4*a*c - b^2)^(7/2)*(b^6*c^2 -
 12*a*b^4*c^3 + 36*a^2*b^2*c^4)*(6*b^6 - 400*a^3*c^3 + 291*a^2*b^2*c^2 - 72
*a*b^4*c)) + (3*b*(b^4 + 11*a^2*c^2 - 7*a*b^2*c)*(16*a^6*b^6*(4*a*c - b^2)^
(9/2) - 1024*a^9*c^3*(4*a*c - b^2)^(9/2) - 192*a^7*b^4*c*(4*a*c - b^2)^(9/2
```

```
) + 768*a^8*b^2*c^2*(4*a*c - b^2)^(9/2))*((((4*a*b^4*c^3 - 17*a^2*b^2*c^4)/
(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c) - (((4*a^2*b^6*c^2 - 36*a^3*b^4*c^3 +
80*a^4*b^2*c^4)/(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c) + ((4*a^4*b^6*c^2 - 32
*a^5*b^4*c^3 + 64*a^6*b^2*c^4)*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a
*b^4*c))/(2*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)*(4*a^2*b^6 - 256*a^5*c^3 -
 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 -
24*a*b^4*c))/(2*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2))
)*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c))/(2*(4*a^2*b^6 - 256*
a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)) - (b^2*c^4)/(a^3*b^4 + 16*a^5*c^
2 - 8*a^4*b^2*c) + (b*(6*a*c - b^2)*((b*((4*a^2*b^6*c^2 - 36*a^3*b^4*c^3 +
80*a^4*b^2*c^4)/(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c) + ((4*a^4*b^6*c^2 - 32
*a^5*b^4*c^3 + 64*a^6*b^2*c^4)*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a
*b^4*c))/(2*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)*(4*a^2*b^6 - 256*a^5*c^3 -
 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(6*a*c - b^2))/(4*a^2*(4*a*c - b^2)^(3/2
)) + (b*(6*a*c - b^2)*(4*a^4*b^6*c^2 - 32*a^5*b^4*c^3 + 64*a^6*b^2*c^4)*(2*
b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c))/(8*a^2*(4*a*c - b^2)^(3/2
)*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^
4*c + 192*a^4*b^2*c^2))))/(4*a^2*(4*a*c - b^2)^(3/2)) + (b^2*(6*a*c - b^2)^
2*(4*a^4*b^6*c^2 - 32*a^5*b^4*c^3 + 64*a^6*b^2*c^4)*(2*b^6 - 128*a^3*c^3 +
96*a^2*b^2*c^2 - 24*a*b^4*c))/(32*a^4*(4*a*c - b^2)^3*(a^3*b^4 + 16*a^5*c^2
 - 8*a^4*b^2*c)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2))
))/(8*a^3*c^2*(4*a*c - b^2)^3*(b^6*c^2 - 12*a*b^4*c^3 + 36*a^2*b^2*c^4)*(6*
b^6 - 400*a^3*c^3 + 291*a^2*b^2*c^2 - 72*a*b^4*c)))*(6*a*c - b^2))/(2*a^2*(
4*a*c - b^2)^(3/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] Timed out
```

3.100 $\int \frac{1}{\left(ax+bx^3+cx^5\right)^2} \, dx$

Optimal. Leaf size=308

$$-\frac{3b^2-10ac}{2a^2x\left(b^2-4ac\right)}-\frac{\sqrt{c}\left(\left(3b^2-10ac\right)\sqrt{b^2-4ac}-16abc+3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^2\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(-\left(3b^2-10ac\right)\sqrt{b^2\right.}{2\sqrt{2}a^2\left(b^2\right.}$$

```
[Out] 1/2*(10*a*c-3*b^2)/a^2/(-4*a*c+b^2)/x+1/2*(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2
)/x/(c*x^4+b*x^2+a)-1/4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/
2))*c^(1/2)*(3*b^3-16*a*b*c+(-10*a*c+3*b^2)*(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c
+b^2)^(3/2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/4*arctan(x*2^(1/2)*c^(1/
2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(3*b^3-16*a*b*c-(-10*a*c+3*b^2)*(-
4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^(3/2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/
2)
```

Rubi [A] time = 1.35, antiderivative size = 308, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1594, 1121, 1281, 1166, 205}

$$-\frac{3b^2-10ac}{2a^2x\left(b^2-4ac\right)}-\frac{\sqrt{c}\left(\left(3b^2-10ac\right)\sqrt{b^2-4ac}-16abc+3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^2\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(-\left(3b^2-10ac\right)\sqrt{b^2\right.}{2\sqrt{2}a^2\left(b^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^3 + c*x^5)^(-2),x]
```

```
[Out] -(3*b^2 - 10*a*c)/(2*a^2*(b^2 - 4*a*c)*x) + (b^2 - 2*a*c + b*c*x^2)/(2*a*(b
^2 - 4*a*c)*x*(a + b*x^2 + c*x^4)) - (Sqrt[c]*(3*b^3 - 16*a*b*c + (3*b^2 -
10*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4
*a*c]]])/(2*Sqrt[2]*a^2*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) +
(Sqrt[c]*(3*b^3 - 16*a*b*c - (3*b^2 - 10*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sq
rt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^2*(b^2 - 4*a*c)
^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1121

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> -Simp[((d*x)^(m + 1)*(b^2 - 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1
))/(2*a*d*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)),
Int[(d*x)^m*(a + b*x^2 + c*x^4)^(p + 1)*Simp[b^2*(m + 2*p + 3) - 2*a*c*(m +
 4*p + 5) + b*c*(m + 4*p + 7)*x^2, x], x], x] /; FreeQ[{a, b, c, d, m}, x]
&& NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] || In
tegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
```

```
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rubi steps

$$\int \frac{1}{\left(ax+bx^3+cx^5\right)^2}\,dx = \int \frac{1}{x^2\left(a+bx^2+cx^4\right)^2}\,dx$$

$$= \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)} - \frac{\int \frac{-3b^2+10ac-3bcx^2}{x^2\left(a+bx^2+cx^4\right)}\,dx}{2a\left(b^2-4ac\right)}$$

$$= -\frac{3b^2-10ac}{2a^2\left(b^2-4ac\right)x} + \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)} + \frac{\int \frac{-b\left(3b^2-13ac\right)-c\left(3b^2-10ac\right)x^2}{a+bx^2+cx^4}\,dx}{2a^2\left(b^2-4ac\right)}$$

$$= -\frac{3b^2-10ac}{2a^2\left(b^2-4ac\right)x} + \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)} - \frac{\left(c\left(3b^2-10ac+\frac{3b^3}{\sqrt{b^2-4ac}}-\frac{16a \cdots}{\sqrt{b^2- \cdots}}\right.\right.}{4a^2\left(b^2- \cdots\right.}$$

$$= -\frac{3b^2-10ac}{2a^2\left(b^2-4ac\right)x} + \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)} - \frac{\sqrt{c}\left(3b^2-10ac+\frac{3b^3}{\sqrt{b^2-4ac}}-\frac{1 \cdots}{\sqrt{b^{\cdots}}}\right.}{2\sqrt{2}\,a^2\left(b^2-4ac\right)\sqrt{\cdots}}$$

Mathematica [A] time = 0.62, size = 302, normalized size = 0.98

$$\frac{-\frac{2x\left(-3abc-2ac^2x^2+b^3+b^2cx^2\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\sqrt{2}\sqrt{c}\left(-3b^2\sqrt{b^2-4ac}+10ac\sqrt{b^2-4ac}+16abc-3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\sqrt{c}\left(-3b^2\sqrt{b^2-4ac}+10ac\sqrt{b^2-4a \cdots}\right.}{\left(b^2-4ac\right)^{3/2}\sqrt{\cdots}}}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^3 + c*x^5)^(-2),x]
```

```
[Out] (-4/x - (2*x*(b^3 - 3*a*b*c + b^2*c*x^2 - 2*a*c^2*x^2))/((b^2 - 4*a*c)*(a +
 b*x^2 + c*x^4)) + (Sqrt[2]*Sqrt[c]*(-3*b^3 + 16*a*b*c - 3*b^2*Sqrt[b^2 - 4
*a*c] + 10*a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[
b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2
]*Sqrt[c]*(3*b^3 - 16*a*b*c - 3*b^2*Sqrt[b^2 - 4*a*c] + 10*a*c*Sqrt[b^2 - 4
*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a
*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*a^2)
```

fricas [B] time = 1.24, size = 2912, normalized size = 9.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(2*(3*b^2*c - 10*a*c^2)*x^4 + 4*a*b^2 - 16*a^2*c + 2*(3*b^3 - 11*a*b*c
)*x^2 - sqrt(1/2)*((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3
+ (a^3*b^2 - 4*a^4*c)*x)*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420
*a^3*b*c^3 + (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*sqrt((8
1*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(a
^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^6 - 12*a^
6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))*log(-(189*b^6*c^3 - 1971*a*b^4*c^4
+ 5625*a^2*b^2*c^5 - 2500*a^3*c^6)*x + 1/2*sqrt(1/2)*(27*b^11 - 486*a*b^9*c
 + 3330*a^2*b^7*c^2 - 10549*a^3*b^5*c^3 + 14408*a^4*b^3*c^4 - 5200*a^5*b*c^
5 - (3*a^5*b^10 - 55*a^6*b^8*c + 392*a^7*b^6*c^2 - 1344*a^8*b^4*c^3 + 2176*
a^9*b^2*c^4 - 1280*a^10*c^5)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2
- 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c
^2 - 64*a^13*c^3)))*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*
b*c^3 + (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*sqrt((81*b^8
 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(a^10*b
^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^6 - 12*a^6*b^4
*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))) + sqrt(1/2)*((a^2*b^2*c - 4*a^3*c^2)*x^
5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x)*sqrt(-(9*b^7 - 105*a
*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 + (a^5*b^6 - 12*a^6*b^4*c + 48*a^7
*b^2*c^2 - 64*a^8*c^3)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550
*a^3*b^2*c^3 + 625*a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 6
4*a^13*c^3)))/(a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))*log(-
(189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 - 2500*a^3*c^6)*x - 1/2*sq
rt(1/2)*(27*b^11 - 486*a*b^9*c + 3330*a^2*b^7*c^2 - 10549*a^3*b^5*c^3 + 144
08*a^4*b^3*c^4 - 5200*a^5*b*c^5 - (3*a^5*b^10 - 55*a^6*b^8*c + 392*a^7*b^6*
c^2 - 1344*a^8*b^4*c^3 + 2176*a^9*b^2*c^4 - 1280*a^10*c^5)*sqrt((81*b^8 - 9
18*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(a^10*b^6 -
 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt(-(9*b^7 - 105*a*b^5*
c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 + (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*
c^2 - 64*a^8*c^3)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*
b^2*c^3 + 625*a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^1
3*c^3)))/(a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))) - sqrt(1/
2)*((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*
a^4*c)*x)*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 - (a
^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*sqrt((81*b^8 - 918*a*b
^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(a^10*b^6 - 12*a^
11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^6 - 12*a^6*b^4*c + 48*a^
7*b^2*c^2 - 64*a^8*c^3))*log(-(189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*
c^5 - 2500*a^3*c^6)*x + 1/2*sqrt(1/2)*(27*b^11 - 486*a*b^9*c + 3330*a^2*b^7
*c^2 - 10549*a^3*b^5*c^3 + 14408*a^4*b^3*c^4 - 5200*a^5*b*c^5 + (3*a^5*b^10
 - 55*a^6*b^8*c + 392*a^7*b^6*c^2 - 1344*a^8*b^4*c^3 + 2176*a^9*b^2*c^4 - 1
280*a^10*c^5)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*
c^3 + 625*a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^
3)))*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 - (a^5*b^
6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*sqrt((81*b^8 - 918*a*b^6*c
+ 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(a^10*b^6 - 12*a^11*b^
4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2
*c^2 - 64*a^8*c^3))) + sqrt(1/2)*((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 -
4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x)*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^
2*b^3*c^2 - 420*a^3*b*c^3 - (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a
^8*c^3)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 +
625*a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(
```

```
a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))*log(-(189*b^6*c^3 -
1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 - 2500*a^3*c^6)*x - 1/2*sqrt(1/2)*(27*b^1
1 - 486*a*b^9*c + 3330*a^2*b^7*c^2 - 10549*a^3*b^5*c^3 + 14408*a^4*b^3*c^4
- 5200*a^5*b*c^5 + (3*a^5*b^10 - 55*a^6*b^8*c + 392*a^7*b^6*c^2 - 1344*a^8*
b^4*c^3 + 2176*a^9*b^2*c^4 - 1280*a^10*c^5)*sqrt((81*b^8 - 918*a*b^6*c + 30
51*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c
+ 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3
*c^2 - 420*a^3*b*c^3 - (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^
3)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a
^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b
^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))))/((a^2*b^2*c - 4*a^3*c^2
)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x)
```

giac [B] time = 2.43, size = 3087, normalized size = 10.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] -1/2*(3*b^2*c*x^4 - 10*a*c^2*x^4 + 3*b^3*x^2 - 11*a*b*c*x^2 + 2*a*b^2 - 8*a
^2*c)/((c*x^5 + b*x^3 + a*x)*(a^2*b^2 - 4*a^3*c)) - 1/16*(6*a^4*b^8*c^2 - 8
0*a^5*b^6*c^3 + 352*a^6*b^4*c^4 - 512*a^7*b^2*c^5 - 3*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^8 + 40*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^6*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^7*c - 176*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c^2 - 56*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c^2 + 256*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^3 + 128*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^3 + 28*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^3 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^4 - 6*(b^2 - 4*a*c)*a^4*b^6*c^2 + 5
6*(b^2 - 4*a*c)*a^5*b^4*c^3 - 128*(b^2 - 4*a*c)*a^6*b^2*c^4 + (6*b^4*c^2 -
44*a*b^2*c^3 + 80*a^2*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*b^4 + 22*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a*b^2*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
b^3*c - 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^
2 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 -
3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 10*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 - 6*(b^2 - 4*
a*c)*b^2*c^2 + 20*(b^2 - 4*a*c)*a*c^3)*(a^2*b^2 - 4*a^3*c)^2 + 2*(3*sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^7 - 37*sqrt(2)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a^3*b^5*c - 6*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c
- 6*a^2*b^7*c + 152*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^2 + 5
0*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^2 + 3*sqrt(2)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^2 + 74*a^3*b^5*c^2 - 208*sqrt(2)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^5*b*c^3 - 104*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^4*b^2*c^3 - 25*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^3 -
304*a^4*b^3*c^3 + 52*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^4 + 41
6*a^5*b*c^4 + 6*(b^2 - 4*a*c)*a^2*b^5*c - 50*(b^2 - 4*a*c)*a^3*b^3*c^2 + 10
4*(b^2 - 4*a*c)*a^4*b*c^3)*abs(a^2*b^2 - 4*a^3*c))*arctan(2*sqrt(1/2)*x/sqr
t((a^2*b^3 - 4*a^3*b*c + sqrt((a^2*b^3 - 4*a^3*b*c)^2 - 4*(a^3*b^2 - 4*a^4*
c)*(a^2*b^2*c - 4*a^3*c^2)))/(a^2*b^2*c - 4*a^3*c^2)))/((a^5*b^6 - 12*a^6*b
^4*c - 2*a^5*b^5*c + 48*a^7*b^2*c^2 + 16*a^6*b^3*c^2 + a^5*b^4*c^2 - 64*a^8
*c^3 - 32*a^7*b*c^3 - 8*a^6*b^2*c^3 + 16*a^7*c^4)*abs(a^2*b^2 - 4*a^3*c)*ab
s(c)) + 1/16*(6*a^4*b^8*c^2 - 80*a^5*b^6*c^3 + 352*a^6*b^4*c^4 - 512*a^7*b^
2*c^5 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^8
 + 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^6*c +
 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^7*c - 17
```

```
6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c^2 - 5
6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c^2 - 3
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c^2 + 25
6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^3 + 1
28*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^3 +
28*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^3 -
64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^4 -
6*(b^2 - 4*a*c)*a^4*b^6*c^2 + 56*(b^2 - 4*a*c)*a^5*b^4*c^3 - 128*(b^2 - 4*a
*c)*a^6*b^2*c^4 + (6*b^4*c^2 - 44*a*b^2*c^3 + 80*a^2*c^4 - 3*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4 + 22*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c - 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*b^2*c^2 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a*c^3 - 6*(b^2 - 4*a*c)*b^2*c^2 + 20*(b^2 - 4*a*c)*a*c^3)*(a^2*
b^2 - 4*a^3*c)^2 - 2*(3*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^7 - 3
7*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c - 6*sqrt(2)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c + 6*a^2*b^7*c + 152*sqrt(2)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^4*b^3*c^2 + 50*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
3*b^4*c^2 + 3*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^2 - 74*a^3*
b^5*c^2 - 208*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b*c^3 - 104*sqrt(
2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^3 - 25*sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^3*b^3*c^3 + 304*a^4*b^3*c^3 + 52*sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^4*b*c^4 - 416*a^5*b*c^4 - 6*(b^2 - 4*a*c)*a^2*b^5*c + 50
*(b^2 - 4*a*c)*a^3*b^3*c^2 - 104*(b^2 - 4*a*c)*a^4*b*c^3)*abs(a^2*b^2 - 4*a
^3*c))*arctan(2*sqrt(1/2)*x/sqrt((a^2*b^3 - 4*a^3*b*c - sqrt((a^2*b^3 - 4*a
^3*b*c)^2 - 4*(a^3*b^2 - 4*a^4*c)*(a^2*b^2*c - 4*a^3*c^2)))/(a^2*b^2*c - 4*
a^3*c^2)))/((a^5*b^6 - 12*a^6*b^4*c - 2*a^5*b^5*c + 48*a^7*b^2*c^2 + 16*a^6
*b^3*c^2 + a^5*b^4*c^2 - 64*a^8*c^3 - 32*a^7*b*c^3 - 8*a^6*b^2*c^3 + 16*a^7
*c^4)*abs(a^2*b^2 - 4*a^3*c)*abs(c))
```

maple [B] time = 0.03, size = 712, normalized size = 2.31

$$-\frac{c^{2}x^{3}}{\left(c\,x^{4}+b\,x^{2}+a\right)\left(4ac-b^{2}\right)a}+\frac{b^{2}c\,x^{3}}{2\left(c\,x^{4}+b\,x^{2}+a\right)\left(4ac-b^{2}\right)a^{2}}+\frac{4\sqrt{2}\;b\,c^{2}\operatorname{arctanh}\left(\frac{\sqrt{2}\;cx}{\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{\left(4ac-b^{2}\right)\sqrt{-4ac+b^{2}}\;\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^5+b*x^3+a*x)^2,x)
```

```
[Out] -1/a^2/x-1/a/(c*x^4+b*x^2+a)*c^2/(4*a*c-b^2)*x^3+1/2/a^2/(c*x^4+b*x^2+a)*c/
(4*a*c-b^2)*x^3*b^2-3/2/a/(c*x^4+b*x^2+a)*b*c/(4*a*c-b^2)*x+1/2/a^2/(c*x^4+
b*x^2+a)*b^3/(4*a*c-b^2)*x+5/2/a*c^2/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^
(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)-3/4/
a^2*c/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)
/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2+4/a*c^2/(4*a*c-b^2)/(-4*a*c+b^2
)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*
a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b-3/4/a^2*c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^
(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(
1/2))*c)^(1/2)*c*x)*b^3-5/2/a*c^2/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2
))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)+3/4/a^2*c/
(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4
*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2+4/a*c^2/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2
^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/
2))*c)^(1/2)*c*x)*b-3/4/a^2*c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-
```

```
4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*
c*x)*b^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(3\,b^2c-10\,ac^2\right)x^4+2\,ab^2-8\,a^2c+\left(3\,b^3-11\,abc\right)x^2}{2\left(\left(a^2b^2c-4\,a^3c^2\right)x^5+\left(a^2b^3-4\,a^3bc\right)x^3+\left(a^3b^2-4\,a^4c\right)x\right)}+\frac{-\int\frac{3\,b^3-13\,abc+\left(3\,b^2c-10\,ac^2\right)x^2}{cx^4+bx^2+a}\,dx}{2\left(a^2b^2-4\,a^3c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((3*b^2*c - 10*a*c^2)*x^4 + 2*a*b^2 - 8*a^2*c + (3*b^3 - 11*a*b*c)*x^2
)/((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a
^4*c)*x) + 1/2*integrate(-(3*b^3 - 13*a*b*c + (3*b^2*c - 10*a*c^2)*x^2)/(c*
x^4 + b*x^2 + a), x)/(a^2*b^2 - 4*a^3*c)
```

mupad [B] time = 2.64, size = 7555, normalized size = 24.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x + b*x^3 + c*x^5)^2,x)
```

```
[Out] - atan((((-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 207
7*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 -
 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c -
 b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8
*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(85
1968*a^14*b*c^8 + 192*a^8*b^13*c^2 - 4672*a^9*b^11*c^3 + 47360*a^10*b^9*c^4
 - 256000*a^11*b^7*c^5 + 778240*a^12*b^5*c^6 - 1261568*a^13*b^3*c^7 + x*(-(
9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^
2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*
(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/
2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*
a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(1048576*a^16*b
*c^8 + 256*a^10*b^13*c^2 - 6144*a^11*b^11*c^3 + 61440*a^12*b^9*c^4 - 327680
*a^13*b^7*c^5 + 983040*a^14*b^5*c^6 - 1572864*a^15*b^3*c^7)) + x*(204800*a^
12*c^9 + 144*a^6*b^12*c^3 - 3264*a^7*b^10*c^4 + 30112*a^8*b^8*c^5 - 143360*
a^9*b^6*c^6 + 365568*a^10*b^4*c^7 - 458752*a^11*b^2*c^8))*(-(9*b^13 - 9*b^4
*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*
b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2
)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b
^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 +
3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*1i - ((-(9*b^13 - 9*b^4*(-(4*
a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^
3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(
1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 +
4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a
^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(851968*a^14*b*c^8 + 192*a^8*b^13*c
^2 - 4672*a^9*b^11*c^3 + 47360*a^10*b^9*c^4 - 256000*a^11*b^7*c^5 + 778240*
a^12*b^5*c^6 - 1261568*a^13*b^3*c^7 - x*(-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9
)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^
4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a
*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^
6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 -
 6144*a^10*b^2*c^5)))^(1/2)*(1048576*a^16*b*c^8 + 256*a^10*b^13*c^2 - 6144*
a^11*b^11*c^3 + 61440*a^12*b^9*c^4 - 327680*a^13*b^7*c^5 + 983040*a^14*b^5*
c^6 - 1572864*a^15*b^3*c^7)) - x*(204800*a^12*c^9 + 144*a^6*b^12*c^3 - 3264
```

```
*a^7*b^10*c^4 + 30112*a^8*b^8*c^5 - 143360*a^9*b^6*c^6 + 365568*a^10*b^4*c^
7 - 458752*a^11*b^2*c^8))*(-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 2688
0*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44
800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a
*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^1
0*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2
*c^5)))^(1/2)*1i)/(((-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*
b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^
5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c
*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c +
240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5))
)^(1/2)*(851968*a^14*b*c^8 + 192*a^8*b^13*c^2 - 4672*a^9*b^11*c^3 + 47360*a
^10*b^9*c^4 - 256000*a^11*b^7*c^5 + 778240*a^12*b^5*c^6 - 1261568*a^13*b^3*
c^7 + x*(-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077
*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 -
25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c -
b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*
c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(104
8576*a^16*b*c^8 + 256*a^10*b^13*c^2 - 6144*a^11*b^11*c^3 + 61440*a^12*b^9*c
^4 - 327680*a^13*b^7*c^5 + 983040*a^14*b^5*c^6 - 1572864*a^15*b^3*c^7)) + x
*(204800*a^12*c^9 + 144*a^6*b^12*c^3 - 3264*a^7*b^10*c^4 + 30112*a^8*b^8*c^
5 - 143360*a^9*b^6*c^6 + 365568*a^10*b^4*c^7 - 458752*a^11*b^2*c^8))*(-(9*b
^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 -
 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(
4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))
/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8
*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2) + ((-(9*b^13 - 9*b
^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^
3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b
^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5
*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3
+ 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(851968*a^14*b*c^8 + 192*a^
8*b^13*c^2 - 4672*a^9*b^11*c^3 + 47360*a^10*b^9*c^4 - 256000*a^11*b^7*c^5 +
 778240*a^12*b^5*c^6 - 1261568*a^13*b^3*c^7 - x*(-(9*b^13 - 9*b^4*(-(4*a*c
- b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 +
30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2)
 - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096
*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b
^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(1048576*a^16*b*c^8 + 256*a^10*b^13*c^2
 - 6144*a^11*b^11*c^3 + 61440*a^12*b^9*c^4 - 327680*a^13*b^7*c^5 + 983040*a
^14*b^5*c^6 - 1572864*a^15*b^3*c^7)) - x*(204800*a^12*c^9 + 144*a^6*b^12*c^
3 - 3264*a^7*b^10*c^4 + 30112*a^8*b^8*c^5 - 143360*a^9*b^6*c^6 + 365568*a^1
0*b^4*c^7 - 458752*a^11*b^2*c^8))*(-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2
) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*
c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*
c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24
*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*
a^10*b^2*c^5)))^(1/2) + 128000*a^10*c^9 + 504*a^6*b^8*c^5 - 8112*a^7*b^6*c^
6 + 48704*a^8*b^4*c^7 - 129280*a^9*b^2*c^8))*(-(9*b^13 - 9*b^4*(-(4*a*c - b
^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 302
40*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) -
213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^
11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*
c^4 - 6144*a^10*b^2*c^5)))^(1/2)*2i - atan((((-(9*b^13 + 9*b^4*(-(4*a*c - b
^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 302
40*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) -
213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^
11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*
c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(851968*a^14*b*c^8 + 192*a^8*b^13*c^2 - 46
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72*a^9*b^11*c^3 + 47360*a^10*b^9*c^4 - 256000*a^11*b^7*c^5 + 778240*a^12*b^
5*c^6 - 1261568*a^13*b^3*c^7 + x*(-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2)
 + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c
^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c
 - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*
a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a
^10*b^2*c^5)))^(1/2)*(1048576*a^16*b*c^8 + 256*a^10*b^13*c^2 - 6144*a^11*b^
11*c^3 + 61440*a^12*b^9*c^4 - 327680*a^13*b^7*c^5 + 983040*a^14*b^5*c^6 - 1
572864*a^15*b^3*c^7)) + x*(204800*a^12*c^9 + 144*a^6*b^12*c^3 - 3264*a^7*b^
10*c^4 + 30112*a^8*b^8*c^5 - 143360*a^9*b^6*c^6 + 365568*a^10*b^4*c^7 - 458
752*a^11*b^2*c^8))*(-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b
*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5
*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*
(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 2
40*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))
^(1/2)*1i - ((-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 +
 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c
^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a
*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7
*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)
*(851968*a^14*b*c^8 + 192*a^8*b^13*c^2 - 4672*a^9*b^11*c^3 + 47360*a^10*b^9
*c^4 - 256000*a^11*b^7*c^5 + 778240*a^12*b^5*c^6 - 1261568*a^13*b^3*c^7 - x
*(-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^
9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*
c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)
^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1
280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(1048576*a^
16*b*c^8 + 256*a^10*b^13*c^2 - 6144*a^11*b^11*c^3 + 61440*a^12*b^9*c^4 - 32
7680*a^13*b^7*c^5 + 983040*a^14*b^5*c^6 - 1572864*a^15*b^3*c^7)) - x*(20480
0*a^12*c^9 + 144*a^6*b^12*c^3 - 3264*a^7*b^10*c^4 + 30112*a^8*b^8*c^5 - 143
360*a^9*b^6*c^6 + 365568*a^10*b^4*c^7 - 458752*a^11*b^2*c^8))*(-(9*b^13 + 9
*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*
a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c -
 b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a
^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^
3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*1i)/(((-(9*b^13 + 9*b^4*(
-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^
7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^
9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^1
2 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 38
40*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(851968*a^14*b*c^8 + 192*a^8*b^
13*c^2 - 4672*a^9*b^11*c^3 + 47360*a^10*b^9*c^4 - 256000*a^11*b^7*c^5 + 778
240*a^12*b^5*c^6 - 1261568*a^13*b^3*c^7 + x*(-(9*b^13 + 9*b^4*(-(4*a*c - b^
2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 3024
0*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 2
13*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^1
1*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c
^4 - 6144*a^10*b^2*c^5)))^(1/2)*(1048576*a^16*b*c^8 + 256*a^10*b^13*c^2 - 6
144*a^11*b^11*c^3 + 61440*a^12*b^9*c^4 - 327680*a^13*b^7*c^5 + 983040*a^14*
b^5*c^6 - 1572864*a^15*b^3*c^7)) + x*(204800*a^12*c^9 + 144*a^6*b^12*c^3 -
3264*a^7*b^10*c^4 + 30112*a^8*b^8*c^5 - 143360*a^9*b^6*c^6 + 365568*a^10*b^
4*c^7 - 458752*a^11*b^2*c^8))*(-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) +
26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4
- 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c -
51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6
*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10
*b^2*c^5)))^(1/2) + ((-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6
*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a
^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*
```

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c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c +
 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)
))^(1/2)*(851968*a^14*b*c^8 + 192*a^8*b^13*c^2 - 4672*a^9*b^11*c^3 + 47360*
a^10*b^9*c^4 - 256000*a^11*b^7*c^5 + 778240*a^12*b^5*c^6 - 1261568*a^13*b^3
*c^7 - x*(-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 207
7*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 +
 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c -
 b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8
*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(10
48576*a^16*b*c^8 + 256*a^10*b^13*c^2 - 6144*a^11*b^11*c^3 + 61440*a^12*b^9*
c^4 - 327680*a^13*b^7*c^5 + 983040*a^14*b^5*c^6 - 1572864*a^15*b^3*c^7)) -
x*(204800*a^12*c^9 + 144*a^6*b^12*c^3 - 3264*a^7*b^10*c^4 + 30112*a^8*b^8*c
^5 - 143360*a^9*b^6*c^6 + 365568*a^10*b^4*c^7 - 458752*a^11*b^2*c^8))*(-(9*
b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2
- 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-
(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2)
)/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^
8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2) + 128000*a^10*c^9
 + 504*a^6*b^8*c^5 - 8112*a^7*b^6*c^6 + 48704*a^8*b^4*c^7 - 129280*a^9*b^2*
c^8))*(-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a
^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25
*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^
2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^
2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*2i - (
1/a + (b*x^2*(11*a*c - 3*b^2))/(2*a^2*(4*a*c - b^2)) + (c*x^4*(10*a*c - 3*b
^2))/(2*a^2*(4*a*c - b^2)))/(a*x + b*x^3 + c*x^5)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] Timed out
```

3.101 $\int \frac{1}{x\left(ax+bx^3+cx^5\right)^2}\,dx$

Optimal. Leaf size=162

$$\frac{b\log\left(a+bx^2+cx^4\right)}{2a^3}-\frac{2b\log(x)}{a^3}-\frac{b^2-3ac}{a^2x^2\left(b^2-4ac\right)}-\frac{\left(6a^2c^2-6ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{a^3\left(b^2-4ac\right)^{3/2}}+\frac{-2ac+b^2+b}{2ax^2\left(b^2-4ac\right)\left(a+\right.}$$

```
[Out] (3*a*c-b^2)/a^2/(-4*a*c+b^2)/x^2+1/2*(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/x^2
/(c*x^4+b*x^2+a)-(6*a^2*c^2-6*a*b^2*c+b^4)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)
^(1/2))/a^3/(-4*a*c+b^2)^(3/2)-2*b*ln(x)/a^3+1/2*b*ln(c*x^4+b*x^2+a)/a^3
```

Rubi [A] time = 0.25, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {1585, 1114, 740, 800, 634, 618, 206, 628}

$$-\frac{\left(6a^2c^2-6ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{a^3\left(b^2-4ac\right)^{3/2}}-\frac{b^2-3ac}{a^2x^2\left(b^2-4ac\right)}+\frac{b\log\left(a+bx^2+cx^4\right)}{2a^3}-\frac{2b\log(x)}{a^3}+\frac{-2ac+b^2+}{2ax^2\left(b^2-4ac\right)\left(a-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a*x + b*x^3 + c*x^5)^2),x]
```

```
[Out] -((b^2 - 3*a*c)/(a^2*(b^2 - 4*a*c)*x^2)) + (b^2 - 2*a*c + b*c*x^2)/(2*a*(b^
2 - 4*a*c)*x^2*(a + b*x^2 + c*x^4)) - ((b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTan
h[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(a^3*(b^2 - 4*a*c)^(3/2)) - (2*b*Log[x]
)/a^3 + (b*Log[a + b*x^2 + c*x^4])/(2*a^3)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
```

```
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\int \frac{1}{x\left(ax+bx^3+cx^5\right)^2}\,dx = \int \frac{1}{x^3\left(a+bx^2+cx^4\right)^2}\,dx$$

$$= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^2\left(a+bx+cx^2\right)^2}\,dx,x,x^2\right)$$

$$= \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int \frac{-2\left(b^2-3ac\right)-2bcx}{x^2\left(a+bx+cx^2\right)}\,dx,x,x^2\right)}{2a\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int \left(\frac{2\left(-b^2+3ac\right)}{ax^2} - \frac{2b\left(-b^2+4ac\right)}{a^2x} + \frac{2\left(-b^4+5ab^2c\cdots\right.}{a^2\left(\cdots\right.}\right.\right.}{2a\left(b^2-4ac\right)}$$

$$= -\frac{b^2-3ac}{a^2\left(b^2-4ac\right)x^2} + \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)} - \frac{2b\log(x)}{a^3} - \frac{\text{Subst}\left(\int \frac{-b^4+\cdots}{\cdots}\right.}{\cdots}$$

$$= -\frac{b^2-3ac}{a^2\left(b^2-4ac\right)x^2} + \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)} - \frac{2b\log(x)}{a^3} + \frac{b\,\text{Subst}\left(\int \frac{b\cdots}{a+\cdots}\right.}{2\cdots}$$

$$= -\frac{b^2-3ac}{a^2\left(b^2-4ac\right)x^2} + \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)} - \frac{2b\log(x)}{a^3} + \frac{b\log\left(a+bx^2\cdots\right.}{2a^3}$$

$$= -\frac{b^2-3ac}{a^2\left(b^2-4ac\right)x^2} + \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)} - \frac{\left(b^4-6ab^2c+6a^2c^2\right)\tanh\cdots}{a^3\left(b^2-4ac\right)^{3/\cdots}}$$

Mathematica [A] time = 0.26, size = 248, normalized size = 1.53

$$\frac{\frac{\left(6a^2c^2-6ab^2c-4abc\sqrt{b^2-4ac}+b^3\sqrt{b^2-4ac}+b^4\right)\log\left(-\sqrt{b^2-4ac}+b+2cx^2\right)}{\left(b^2-4ac\right)^{3/2}}+\frac{\left(-6a^2c^2+6ab^2c-4abc\sqrt{b^2-4ac}+b^3\sqrt{b^2-4ac}-b^4\right)\log\left(\sqrt{b^2-4ac}+b+2cx^2\right)}{\left(b^2-4ac\right)^{3/2}}}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a*x + b*x^3 + c*x^5)^2),x]
```

```
[Out] (-(a/x^2) - (a*(b^3 - 3*a*b*c + b^2*c*x^2 - 2*a*c^2*x^2))/((b^2 - 4*a*c)*(a
 + b*x^2 + c*x^4)) - 4*b*Log[x] + ((b^4 - 6*a*b^2*c + 6*a^2*c^2 + b^3*Sqrt[
b^2 - 4*a*c] - 4*a*b*c*Sqrt[b^2 - 4*a*c])*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x
^2])/(b^2 - 4*a*c)^(3/2) + ((-b^4 + 6*a*b^2*c - 6*a^2*c^2 + b^3*Sqrt[b^2 -
4*a*c] - 4*a*b*c*Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(
b^2 - 4*a*c)^(3/2))/(2*a^3)
```

fricas [B] time = 1.23, size = 1007, normalized size = 6.22

$$\left[-\frac{a^2b^4 - 8\,a^3b^2c + 16\,a^4c^2 + 2\left(ab^4c - 7\,a^2b^2c^2 + 12\,a^3c^3\right)x^4 + \left(2\,ab^5 - 15\,a^2b^3c + 28\,a^3bc^2\right)x^2 + \left(\left(b^4c - 6\,ab^2c^2 \right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + 2*(a*b^4*c - 7*a^2*b^2*c^2 + 12
*a^3*c^3)*x^4 + (2*a*b^5 - 15*a^2*b^3*c + 28*a^3*b*c^2)*x^2 + ((b^4*c - 6*a
*b^2*c^2 + 6*a^2*c^3)*x^6 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*x^4 + (a*b^4 -
6*a^2*b^2*c + 6*a^3*c^2)*x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2
+ b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - ((b
^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)
*x^4 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*log(c*x^4 + b*x^2 + a) + 4
*((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*
c^2)*x^4 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*log(x))/((a^3*b^4*c -
8*a^4*b^2*c^2 + 16*a^5*c^3)*x^6 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^
4 + (a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*x^2), -1/2*(a^2*b^4 - 8*a^3*b^2*c
+ 16*a^4*c^2 + 2*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*x^4 + (2*a*b^5 - 15
*a^2*b^3*c + 28*a^3*b*c^2)*x^2 + 2*((b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*x^6 +
 (b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*x^4 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*x^
2)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c
)) - ((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + (b^6 - 8*a*b^4*c + 16*a^2*
b^2*c^2)*x^4 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*log(c*x^4 + b*x^2
+ a) + 4*((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + (b^6 - 8*a*b^4*c + 16*
a^2*b^2*c^2)*x^4 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*log(x))/((a^3*
b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*x^6 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b
*c^2)*x^4 + (a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*x^2)]
```

giac [A] time = 2.41, size = 182, normalized size = 1.12

$$\frac{\left(b^4 - 6\,ab^2c + 6\,a^2c^2\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(a^3b^2 - 4\,a^4c\right)\sqrt{-b^2 + 4\,ac}} - \frac{2\,b^2cx^4 - 6\,ac^2x^4 + 2\,b^3x^2 - 7\,abcx^2 + ab^2 - 4\,a^2c}{2\left(cx^6 + bx^4 + ax^2\right)\left(a^2b^2 - 4\,a^3c\right)} + \frac{b\log\left(cx^4 + bx^2 + \right.}{2\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] (b^4 - 6*a*b^2*c + 6*a^2*c^2)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((a^
3*b^2 - 4*a^4*c)*sqrt(-b^2 + 4*a*c)) - 1/2*(2*b^2*c*x^4 - 6*a*c^2*x^4 + 2*b
^3*x^2 - 7*a*b*c*x^2 + a*b^2 - 4*a^2*c)/((c*x^6 + b*x^4 + a*x^2)*(a^2*b^2 -
 4*a^3*c)) + 1/2*b*log(c*x^4 + b*x^2 + a)/a^3 - b*log(x^2)/a^3
```

maple [B] time = 0.02, size = 352, normalized size = 2.17

$$-\frac{c^2x^2}{\left(cx^4+bx^2+a\right)\left(4ac-b^2\right)a}+\frac{b^2cx^2}{2\left(cx^4+bx^2+a\right)\left(4ac-b^2\right)a^2}-\frac{6c^2\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}a}+\frac{6b^2c\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^5+b*x^3+a*x)^2,x)
```

```
[Out] -1/2/a^2/x^2-2/a^3*b*ln(x)-1/a/(c*x^4+b*x^2+a)*c^2/(4*a*c-b^2)*x^2+1/2/a^2/
(c*x^4+b*x^2+a)*c/(4*a*c-b^2)*x^2*b^2-3/2/a/(c*x^4+b*x^2+a)*b/(4*a*c-b^2)*c
+1/2/a^2/(c*x^4+b*x^2+a)*b^3/(4*a*c-b^2)+2/a^2/(4*a*c-b^2)*c*ln(c*x^4+b*x^2
+a)*b-1/2/a^3/(4*a*c-b^2)*ln(c*x^4+b*x^2+a)*b^3-6/a/(4*a*c-b^2)^(3/2)*arcta
n((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*c^2+6/a^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x^
2+b)/(4*a*c-b^2)^(1/2))*b^2*c-1/a^3/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4
*a*c-b^2)^(1/2))*b^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2\left(b^2c-3ac^2\right)x^4+ab^2-4a^2c+\left(2b^3-7abc\right)x^2}{2\left(\left(a^2b^2c-4a^3c^2\right)x^6+\left(a^2b^3-4a^3bc\right)x^4+\left(a^3b^2-4a^4c\right)x^2\right)}-\frac{-2\int\frac{\left(b^3c-4abc^2\right)x^3+\left(b^4-5ab^2c+3a^2c^2\right)x}{cx^4+bx^2+a}\,dx}{a^3b^2-4a^4c}-\frac{2b\log}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*(b^2*c - 3*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (2*b^3 - 7*a*b*c)*x^2)/((
a^2*b^2*c - 4*a^3*c^2)*x^6 + (a^2*b^3 - 4*a^3*b*c)*x^4 + (a^3*b^2 - 4*a^4*c
)*x^2) - 2*integrate(-((b^3*c - 4*a*b*c^2)*x^3 + (b^4 - 5*a*b^2*c + 3*a^2*c
^2)*x)/(c*x^4 + b*x^2 + a), x)/(a^3*b^2 - 4*a^4*c) - 2*b*log(x)/a^3
```

mupad [B] time = 6.77, size = 5491, normalized size = 33.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a*x + b*x^3 + c*x^5)^2),x)
```

```
[Out] (log(a + b*x^2 + c*x^4)*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c))
/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)) - (1/(2*a) - (x
^2*(2*b^3 - 7*a*b*c))/(2*a^2*(4*a*c - b^2)) + (c*x^4*(3*a*c - b^2))/(a^2*(4
*a*c - b^2)))/(a*x^2 + b*x^4 + c*x^6) - (2*b*log(x))/a^3 + (atan(((2*a^9*b^
6*(4*a*c - b^2)^(9/2) - 128*a^12*c^3*(4*a*c - b^2)^(9/2) - 24*a^10*b^4*c*(4
*a*c - b^2)^(9/2) + 96*a^11*b^2*c^2*(4*a*c - b^2)^(9/2))*(3*b^6 - 3*a^3*c^3
 + 36*a^2*b^2*c^2 - 21*a*b^4*c)*((4*(2*b^5*c^4 - 12*a*b^3*c^5 + 18*a^2*b*c^
6))/(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c) + (((4*(9*a^5*c^6 - 4*a^2*b^6*c^3
+ 29*a^3*b^4*c^4 - 54*a^4*b^2*c^5))/(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c) -
(((4*(24*a^7*b*c^5 - 2*a^4*b^7*c^2 + 18*a^5*b^5*c^3 - 46*a^6*b^3*c^4))/(a^6
*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c) - (2*(a^7*b^6*c^2 - 8*a^8*b^4*c^3 + 16*a^9
*b^2*c^4)*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c))/((a^6*b^4 + 1
6*a^8*c^2 - 8*a^7*b^2*c)*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*
c^2)))*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c))/(2*(a^3*b^6 - 64
*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)))*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^
```

```
3*c^2 - 12*a*b^5*c))/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c
^2)) + (((((4*(24*a^7*b*c^5 - 2*a^4*b^7*c^2 + 18*a^5*b^5*c^3 - 46*a^6*b^3*c
^4))/(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c) - (2*(a^7*b^6*c^2 - 8*a^8*b^4*c^3
 + 16*a^9*b^2*c^4)*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c))/((a^
6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48
*a^5*b^2*c^2)))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(2*a^3*(4*a*c - b^2)^(3/2))
- ((a^7*b^6*c^2 - 8*a^8*b^4*c^3 + 16*a^9*b^2*c^4)*(b^4 + 6*a^2*c^2 - 6*a*b^
2*c)*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c))/(a^3*(4*a*c - b^2)
^(3/2)*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*
b^4*c + 48*a^5*b^2*c^2)))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(2*a^3*(4*a*c - b^
2)^(3/2)) - ((a^7*b^6*c^2 - 8*a^8*b^4*c^3 + 16*a^9*b^2*c^4)*(b^4 + 6*a^2*c^
2 - 6*a*b^2*c)^2*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c))/(2*a^6
*(4*a*c - b^2)^3*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)*(a^3*b^6 - 64*a^6*c^3
 - 12*a^4*b^4*c + 48*a^5*b^2*c^2))))/(8*a^3*c^2*(4*a*c - b^2)^3*(9*a^4*c^4
- 6*b^8 - 288*a^2*b^4*c^2 + 382*a^3*b^2*c^3 + 72*a*b^6*c)*(36*a^4*c^6 + b^8
*c^2 - 12*a*b^6*c^3 + 48*a^2*b^4*c^4 - 72*a^3*b^2*c^5)) - (x^2*((((4*(54*a^
3*c^8 - 2*b^6*c^5 + 18*a*b^4*c^6 - 54*a^2*b^2*c^7))/(a^6*b^6 - 64*a^9*c^3 -
 12*a^7*b^4*c + 48*a^8*b^2*c^2) - (((4*(276*a^5*b*c^7 - 6*a^2*b^7*c^4 + 65*
a^3*b^5*c^5 - 233*a^4*b^3*c^6))/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a
^8*b^2*c^2) - (((4*(480*a^8*c^7 - a^4*b^8*c^3 + 6*a^5*b^6*c^4 + 30*a^6*b^4*
c^5 - 272*a^7*b^2*c^6))/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c
^2) - (2*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c)*(640*a^10*b*c^6
 + 3*a^6*b^9*c^2 - 46*a^7*b^7*c^3 + 264*a^8*b^5*c^4 - 672*a^9*b^3*c^5))/((a
^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(a^6*b^6 - 64*a^9*c^3
- 12*a^7*b^4*c + 48*a^8*b^2*c^2)))*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 1
2*a*b^5*c))/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)))*(b^
7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c))/(2*(a^3*b^6 - 64*a^6*c^3 -
 12*a^4*b^4*c + 48*a^5*b^2*c^2)) - (((((4*(480*a^8*c^7 - a^4*b^8*c^3 + 6*a^
5*b^6*c^4 + 30*a^6*b^4*c^5 - 272*a^7*b^2*c^6))/(a^6*b^6 - 64*a^9*c^3 - 12*a
^7*b^4*c + 48*a^8*b^2*c^2) - (2*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a
*b^5*c)*(640*a^10*b*c^6 + 3*a^6*b^9*c^2 - 46*a^7*b^7*c^3 + 264*a^8*b^5*c^4
- 672*a^9*b^3*c^5))/((a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)
*(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)))*(b^4 + 6*a^2*c^2
- 6*a*b^2*c))/(2*a^3*(4*a*c - b^2)^(3/2)) - ((b^4 + 6*a^2*c^2 - 6*a*b^2*c)*
(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c)*(640*a^10*b*c^6 + 3*a^6*
b^9*c^2 - 46*a^7*b^7*c^3 + 264*a^8*b^5*c^4 - 672*a^9*b^3*c^5))/(a^3*(4*a*c
- b^2)^(3/2)*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(a^6*b^
6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)))*(b^4 + 6*a^2*c^2 - 6*a*b^
2*c))/(2*a^3*(4*a*c - b^2)^(3/2)) + ((b^4 + 6*a^2*c^2 - 6*a*b^2*c)^2*(b^7 -
 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c)*(640*a^10*b*c^6 + 3*a^6*b^9*c^
2 - 46*a^7*b^7*c^3 + 264*a^8*b^5*c^4 - 672*a^9*b^3*c^5))/(2*a^6*(4*a*c - b^
2)^3*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(a^6*b^6 - 64*a
^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)))*(3*b^6 - 3*a^3*c^3 + 36*a^2*b^2*c
^2 - 21*a*b^4*c))/(8*a^3*c^2*(4*a*c - b^2)^3*(9*a^4*c^4 - 6*b^8 - 288*a^2*b
^4*c^2 + 382*a^3*b^2*c^3 + 72*a*b^6*c)) - (b*((((((4*(480*a^8*c^7 - a^4*b^8
*c^3 + 6*a^5*b^6*c^4 + 30*a^6*b^4*c^5 - 272*a^7*b^2*c^6))/(a^6*b^6 - 64*a^9
*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) - (2*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3
*c^2 - 12*a*b^5*c)*(640*a^10*b*c^6 + 3*a^6*b^9*c^2 - 46*a^7*b^7*c^3 + 264*a
^8*b^5*c^4 - 672*a^9*b^3*c^5))/((a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a
^5*b^2*c^2)*(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)))*(b^4 +
 6*a^2*c^2 - 6*a*b^2*c))/(2*a^3*(4*a*c - b^2)^(3/2)) - ((b^4 + 6*a^2*c^2 -
6*a*b^2*c)*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c)*(640*a^10*b*c
^6 + 3*a^6*b^9*c^2 - 46*a^7*b^7*c^3 + 264*a^8*b^5*c^4 - 672*a^9*b^3*c^5))/(
a^3*(4*a*c - b^2)^(3/2)*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c
^2)*(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)))*(b^7 - 64*a^3*
b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c))/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4
*c + 48*a^5*b^2*c^2)) - (((4*(276*a^5*b*c^7 - 6*a^2*b^7*c^4 + 65*a^3*b^5*c^
5 - 233*a^4*b^3*c^6))/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2
) - (((4*(480*a^8*c^7 - a^4*b^8*c^3 + 6*a^5*b^6*c^4 + 30*a^6*b^4*c^5 - 272*
```

```
a^7*b^2*c^6))/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) - (2*(
b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c)*(640*a^10*b*c^6 + 3*a^6*b
^9*c^2 - 46*a^7*b^7*c^3 + 264*a^8*b^5*c^4 - 672*a^9*b^3*c^5))/((a^3*b^6 - 6
4*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b
^4*c + 48*a^8*b^2*c^2)))*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c)
)/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)))*(b^4 + 6*a^2*
c^2 - 6*a*b^2*c))/(2*a^3*(4*a*c - b^2)^(3/2)) + ((b^4 + 6*a^2*c^2 - 6*a*b^2
*c)^3*(640*a^10*b*c^6 + 3*a^6*b^9*c^2 - 46*a^7*b^7*c^3 + 264*a^8*b^5*c^4 -
672*a^9*b^3*c^5))/(2*a^9*(4*a*c - b^2)^(9/2)*(a^6*b^6 - 64*a^9*c^3 - 12*a^7
*b^4*c + 48*a^8*b^2*c^2)))*(3*b^6 - 49*a^3*c^3 + 72*a^2*b^2*c^2 - 27*a*b^4*
c))/(8*a^3*c^2*(4*a*c - b^2)^(7/2)*(9*a^4*c^4 - 6*b^8 - 288*a^2*b^4*c^2 + 3
82*a^3*b^2*c^3 + 72*a*b^6*c)))*(2*a^9*b^6*(4*a*c - b^2)^(9/2) - 128*a^12*c^
3*(4*a*c - b^2)^(9/2) - 24*a^10*b^4*c*(4*a*c - b^2)^(9/2) + 96*a^11*b^2*c^2
*(4*a*c - b^2)^(9/2)))/(36*a^4*c^6 + b^8*c^2 - 12*a*b^6*c^3 + 48*a^2*b^4*c^
4 - 72*a^3*b^2*c^5) + (b*((((((4*(24*a^7*b*c^5 - 2*a^4*b^7*c^2 + 18*a^5*b^5
*c^3 - 46*a^6*b^3*c^4))/(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c) - (2*(a^7*b^6*
c^2 - 8*a^8*b^4*c^3 + 16*a^9*b^2*c^4)*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2
- 12*a*b^5*c))/((a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)*(a^3*b^6 - 64*a^6*c^3
- 12*a^4*b^4*c + 48*a^5*b^2*c^2)))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(2*a^3*(4
*a*c - b^2)^(3/2)) - ((a^7*b^6*c^2 - 8*a^8*b^4*c^3 + 16*a^9*b^2*c^4)*(b^4 +
 6*a^2*c^2 - 6*a*b^2*c)*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c))
/(a^3*(4*a*c - b^2)^(3/2)*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)*(a^3*b^6 - 6
4*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)))*(b^7 - 64*a^3*b*c^3 + 48*a^2*b
^3*c^2 - 12*a*b^5*c))/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*
c^2)) - (((4*(9*a^5*c^6 - 4*a^2*b^6*c^3 + 29*a^3*b^4*c^4 - 54*a^4*b^2*c^5))
/(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c) - (((4*(24*a^7*b*c^5 - 2*a^4*b^7*c^2
+ 18*a^5*b^5*c^3 - 46*a^6*b^3*c^4))/(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c) -
(2*(a^7*b^6*c^2 - 8*a^8*b^4*c^3 + 16*a^9*b^2*c^4)*(b^7 - 64*a^3*b*c^3 + 48*
a^2*b^3*c^2 - 12*a*b^5*c))/((a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)*(a^3*b^6 -
 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)))*(b^7 - 64*a^3*b*c^3 + 48*a^2
*b^3*c^2 - 12*a*b^5*c))/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^
2*c^2)))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(2*a^3*(4*a*c - b^2)^(3/2)) + ((a^7
*b^6*c^2 - 8*a^8*b^4*c^3 + 16*a^9*b^2*c^4)*(b^4 + 6*a^2*c^2 - 6*a*b^2*c)^3)
/(2*a^9*(4*a*c - b^2)^(9/2)*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))*(2*a^9*b
^6*(4*a*c - b^2)^(9/2) - 128*a^12*c^3*(4*a*c - b^2)^(9/2) - 24*a^10*b^4*c*(
4*a*c - b^2)^(9/2) + 96*a^11*b^2*c^2*(4*a*c - b^2)^(9/2))*(3*b^6 - 49*a^3*c
^3 + 72*a^2*b^2*c^2 - 27*a*b^4*c))/(8*a^3*c^2*(4*a*c - b^2)^(7/2)*(9*a^4*c^
4 - 6*b^8 - 288*a^2*b^4*c^2 + 382*a^3*b^2*c^3 + 72*a*b^6*c)*(36*a^4*c^6 + b
^8*c^2 - 12*a*b^6*c^3 + 48*a^2*b^4*c^4 - 72*a^3*b^2*c^5)))*(b^4 + 6*a^2*c^2
 - 6*a*b^2*c))/(a^3*(4*a*c - b^2)^(3/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] Timed out
```

3.102 $\int \frac{1}{x^2\left(ax+bx^3+cx^5\right)^2}\,dx$

Optimal. Leaf size=361

$$\frac{b\left(5b^2-19ac\right)}{2a^3x\left(b^2-4ac\right)}-\frac{5b^2-14ac}{6a^2x^3\left(b^2-4ac\right)}+\frac{\sqrt{c}\left(28a^2c^2-29ab^2c+b\left(5b^2-19ac\right)\sqrt{b^2-4ac}+5b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^3\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}$$

```
[Out] 1/6*(14*a*c-5*b^2)/a^2/(-4*a*c+b^2)/x^3+1/2*b*(-19*a*c+5*b^2)/a^3/(-4*a*c+b
^2)/x+1/2*(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/x^3/(c*x^4+b*x^2+a)+1/4*arctan
(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(5*b^4-29*a*b^2*c+
28*a^2*c^2+b*(-19*a*c+5*b^2)*(-4*a*c+b^2)^(1/2))/a^3/(-4*a*c+b^2)^(3/2)*2^(
1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/4*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b
^2)^(1/2))^(1/2))*c^(1/2)*(5*b^4-29*a*b^2*c+28*a^2*c^2-b*(-19*a*c+5*b^2)*(-
4*a*c+b^2)^(1/2))/a^3/(-4*a*c+b^2)^(3/2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/
2)
```

Rubi [A] time = 3.08, antiderivative size = 361, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1585, 1121, 1281, 1166, 205}

$$\frac{\sqrt{c}\left(28a^2c^2-29ab^2c+b\left(5b^2-19ac\right)\sqrt{b^2-4ac}+5b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^3\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(28a^2c^2-29ab^2c-b\left(5b^2-19\right.\right.}{2\sqrt{2}a^3\left(b^2-4\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a*x + b*x^3 + c*x^5)^2),x]
```

```
[Out] -(5*b^2 - 14*a*c)/(6*a^2*(b^2 - 4*a*c)*x^3) + (b*(5*b^2 - 19*a*c))/(2*a^3*(
b^2 - 4*a*c)*x) + (b^2 - 2*a*c + b*c*x^2)/(2*a*(b^2 - 4*a*c)*x^3*(a + b*x^2
 + c*x^4)) + (Sqrt[c]*(5*b^4 - 29*a*b^2*c + 28*a^2*c^2 + b*(5*b^2 - 19*a*c)
*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]
)/(2*Sqrt[2]*a^3*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c
]*(5*b^4 - 29*a*b^2*c + 28*a^2*c^2 - b*(5*b^2 - 19*a*c)*Sqrt[b^2 - 4*a*c])*
ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^3*(b^
2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1121

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> -Simp[((d*x)^(m + 1)*(b^2 - 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1
))/(2*a*d*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)),
Int[(d*x)^m*(a + b*x^2 + c*x^4)^(p + 1)*Simp[b^2*(m + 2*p + 3) - 2*a*c*(m +
 4*p + 5) + b*c*(m + 4*p + 7)*x^2, x], x], x] /; FreeQ[{a, b, c, d, m}, x]
&& NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] || In
tegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
```

```
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\int \frac{1}{x^2\left(ax+bx^3+cx^5\right)^2}\,dx = \int \frac{1}{x^4\left(a+bx^2+cx^4\right)^2}\,dx$$

$$= \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x^3\left(a+bx^2+cx^4\right)} - \frac{\int \frac{-5b^2+14ac-5bcx^2}{x^4\left(a+bx^2+cx^4\right)}\,dx}{2a\left(b^2-4ac\right)}$$

$$= -\frac{5b^2-14ac}{6a^2\left(b^2-4ac\right)x^3} + \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x^3\left(a+bx^2+cx^4\right)} + \frac{\int \frac{-3b\left(5b^2-19ac\right)-3c\left(5b^2-14a\right.}{x^2\left(a+bx^2+cx^4\right)}}{6a^2\left(b^2-4ac\right)}$$

$$= -\frac{5b^2-14ac}{6a^2\left(b^2-4ac\right)x^3} + \frac{b\left(5b^2-19ac\right)}{2a^3\left(b^2-4ac\right)x} + \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x^3\left(a+bx^2+cx^4\right)} - \frac{\int \frac{-3}{}}{}$$

$$= -\frac{5b^2-14ac}{6a^2\left(b^2-4ac\right)x^3} + \frac{b\left(5b^2-19ac\right)}{2a^3\left(b^2-4ac\right)x} + \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x^3\left(a+bx^2+cx^4\right)} - \frac{\left(c\left(5\right.\right.}{}$$

$$= -\frac{5b^2-14ac}{6a^2\left(b^2-4ac\right)x^3} + \frac{b\left(5b^2-19ac\right)}{2a^3\left(b^2-4ac\right)x} + \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x^3\left(a+bx^2+cx^4\right)} + \frac{\sqrt{c}\,\left(\right.}{}$$

Mathematica [A] time = 0.71, size = 344, normalized size = 0.95

$$\frac{\frac{6x\left(2a^2c^2-4ab^2c-3abc^2x^2+b^4+b^3cx^2\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{3\sqrt{2}\sqrt{c}\left(28a^2c^2-29ab^2c-19abc\sqrt{b^2-4ac}+5b^3\sqrt{b^2-4ac}+5b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{3\sqrt{2}\sqrt{c}\left(-28a^2c^2\right.}{}}{12a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a*x + b*x^3 + c*x^5)^2),x]
```

```
[Out] ((-4*a)/x^3 + (24*b)/x + (6*x*(b^4 - 4*a*b^2*c + 2*a^2*c^2 + b^3*c*x^2 - 3*
a*b*c^2*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (3*Sqrt[2]*Sqrt[c]*(5*b
^4 - 29*a*b^2*c + 28*a^2*c^2 + 5*b^3*Sqrt[b^2 - 4*a*c] - 19*a*b*c*Sqrt[b^2
- 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 -
4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2]*Sqrt[c]*(-5*b^4 + 29
*a*b^2*c - 28*a^2*c^2 + 5*b^3*Sqrt[b^2 - 4*a*c] - 19*a*b*c*Sqrt[b^2 - 4*a*c
])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^
(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(12*a^3)
```

fricas [B] time = 1.50, size = 3435, normalized size = 9.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(6*(5*b^3*c - 19*a*b*c^2)*x^6 + 2*(15*b^4 - 62*a*b^2*c + 14*a^2*c^2)*x
^4 - 4*a^2*b^2 + 16*a^3*c + 20*(a*b^3 - 4*a^2*b*c)*x^2 + 3*sqrt(1/2)*((a^3*
b^2*c - 4*a^4*c^2)*x^7 + (a^3*b^3 - 4*a^4*b*c)*x^5 + (a^4*b^2 - 4*a^5*c)*x^
3)*sqrt(-(25*b^9 - 315*a*b^7*c + 1386*a^2*b^5*c^2 - 2415*a^3*b^3*c^3 + 1260
*a^4*b*c^4 + (a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*sqrt((
625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^
4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 4
8*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 -
64*a^10*c^3))*log((1125*b^8*c^4 - 12325*a*b^6*c^5 + 43410*a^2*b^4*c^6 - 504
21*a^3*b^2*c^7 + 9604*a^4*c^8)*x + 1/2*sqrt(1/2)*(125*b^14 - 2425*a*b^12*c
+ 18940*a^2*b^10*c^2 - 75579*a^3*b^8*c^3 + 160932*a^4*b^6*c^4 - 172990*a^5*
b^4*c^5 + 79408*a^6*b^2*c^6 - 10976*a^7*c^7 - (5*a^7*b^11 - 94*a^8*b^9*c +
700*a^9*b^7*c^2 - 2576*a^10*b^5*c^3 + 4672*a^11*b^3*c^4 - 3328*a^12*b*c^5)*
sqrt((625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76
686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/(a^14*b^6 - 12*a^15*b^4
*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))*sqrt(-(25*b^9 - 315*a*b^7*c + 1386*a^
2*b^5*c^2 - 2415*a^3*b^3*c^3 + 1260*a^4*b*c^4 + (a^7*b^6 - 12*a^8*b^4*c + 4
8*a^9*b^2*c^2 - 64*a^10*c^3)*sqrt((625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8
*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6
*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^6
 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3))) - 3*sqrt(1/2)*((a^3*b^2*c
 - 4*a^4*c^2)*x^7 + (a^3*b^3 - 4*a^4*b*c)*x^5 + (a^4*b^2 - 4*a^5*c)*x^3)*sq
rt(-(25*b^9 - 315*a*b^7*c + 1386*a^2*b^5*c^2 - 2415*a^3*b^3*c^3 + 1260*a^4*
b*c^4 + (a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*sqrt((625*b
^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4
*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^1
6*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^
10*c^3))*log((1125*b^8*c^4 - 12325*a*b^6*c^5 + 43410*a^2*b^4*c^6 - 50421*a^
3*b^2*c^7 + 9604*a^4*c^8)*x - 1/2*sqrt(1/2)*(125*b^14 - 2425*a*b^12*c + 189
40*a^2*b^10*c^2 - 75579*a^3*b^8*c^3 + 160932*a^4*b^6*c^4 - 172990*a^5*b^4*c
^5 + 79408*a^6*b^2*c^6 - 10976*a^7*c^7 - (5*a^7*b^11 - 94*a^8*b^9*c + 700*a
^9*b^7*c^2 - 2576*a^10*b^5*c^3 + 4672*a^11*b^3*c^4 - 3328*a^12*b*c^5)*sqrt(
(625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a
^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c +
48*a^16*b^2*c^2 - 64*a^17*c^3)))*sqrt(-(25*b^9 - 315*a*b^7*c + 1386*a^2*b^5
*c^2 - 2415*a^3*b^3*c^3 + 1260*a^4*b*c^4 + (a^7*b^6 - 12*a^8*b^4*c + 48*a^9
*b^2*c^2 - 64*a^10*c^3)*sqrt((625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2
- 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)
/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^6 - 12
*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3))) + 3*sqrt(1/2)*((a^3*b^2*c - 4*
a^4*c^2)*x^7 + (a^3*b^3 - 4*a^4*b*c)*x^5 + (a^4*b^2 - 4*a^5*c)*x^3)*sqrt(-(
25*b^9 - 315*a*b^7*c + 1386*a^2*b^5*c^2 - 2415*a^3*b^3*c^3 + 1260*a^4*b*c^4
 - (a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*sqrt((625*b^12 -
 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4
```

```
- 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2
*c^2 - 64*a^17*c^3)))/(a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^
3))*log((1125*b^8*c^4 - 12325*a*b^6*c^5 + 43410*a^2*b^4*c^6 - 50421*a^3*b^2
*c^7 + 9604*a^4*c^8)*x + 1/2*sqrt(1/2)*(125*b^14 - 2425*a*b^12*c + 18940*a^
2*b^10*c^2 - 75579*a^3*b^8*c^3 + 160932*a^4*b^6*c^4 - 172990*a^5*b^4*c^5 +
79408*a^6*b^2*c^6 - 10976*a^7*c^7 + (5*a^7*b^11 - 94*a^8*b^9*c + 700*a^9*b^
7*c^2 - 2576*a^10*b^5*c^3 + 4672*a^11*b^3*c^4 - 3328*a^12*b*c^5)*sqrt((625*
b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^
4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^
16*b^2*c^2 - 64*a^17*c^3)))*sqrt(-(25*b^9 - 315*a*b^7*c + 1386*a^2*b^5*c^2
- 2415*a^3*b^3*c^3 + 1260*a^4*b*c^4 - (a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*
c^2 - 64*a^10*c^3)*sqrt((625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 836
30*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/(a^1
4*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^6 - 12*a^8*
b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3))) - 3*sqrt(1/2)*((a^3*b^2*c - 4*a^4*c
^2)*x^7 + (a^3*b^3 - 4*a^4*b*c)*x^5 + (a^4*b^2 - 4*a^5*c)*x^3)*sqrt(-(25*b^
9 - 315*a*b^7*c + 1386*a^2*b^5*c^2 - 2415*a^3*b^3*c^3 + 1260*a^4*b*c^4 - (a
^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*sqrt((625*b^12 - 8250
*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 241
08*a^5*b^2*c^5 + 2401*a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2
- 64*a^17*c^3)))/(a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3))*l
og((1125*b^8*c^4 - 12325*a*b^6*c^5 + 43410*a^2*b^4*c^6 - 50421*a^3*b^2*c^7
+ 9604*a^4*c^8)*x - 1/2*sqrt(1/2)*(125*b^14 - 2425*a*b^12*c + 18940*a^2*b^1
0*c^2 - 75579*a^3*b^8*c^3 + 160932*a^4*b^6*c^4 - 172990*a^5*b^4*c^5 + 79408
*a^6*b^2*c^6 - 10976*a^7*c^7 + (5*a^7*b^11 - 94*a^8*b^9*c + 700*a^9*b^7*c^2
 - 2576*a^10*b^5*c^3 + 4672*a^11*b^3*c^4 - 3328*a^12*b*c^5)*sqrt((625*b^12
- 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4
 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^
2*c^2 - 64*a^17*c^3)))*sqrt(-(25*b^9 - 315*a*b^7*c + 1386*a^2*b^5*c^2 - 241
5*a^3*b^3*c^3 + 1260*a^4*b*c^4 - (a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 -
 64*a^10*c^3)*sqrt((625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^
3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/(a^14*b^6
 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^6 - 12*a^8*b^4*c
 + 48*a^9*b^2*c^2 - 64*a^10*c^3))))/((a^3*b^2*c - 4*a^4*c^2)*x^7 + (a^3*b^3
 - 4*a^4*b*c)*x^5 + (a^4*b^2 - 4*a^5*c)*x^3)
```

giac [B] time = 3.92, size = 3651, normalized size = 10.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^3*c*x^3 - 3*a*b*c^2*x^3 + b^4*x - 4*a*b^2*c*x + 2*a^2*c^2*x)/((a^3*b
^2 - 4*a^4*c)*(c*x^4 + b*x^2 + a)) + 1/16*(10*a^6*b^9*c^2 - 138*a^7*b^7*c^3
 + 680*a^8*b^5*c^4 - 1376*a^9*b^3*c^5 + 896*a^10*b*c^6 - 5*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^9 + 69*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^7*c + 10*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^8*c - 340*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b^5*c^2 - 98*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^6*c^2 - 5*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^7*c^2 + 688*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^9*b^3*c^3 + 288*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b^4*c^3 + 49*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^5*c^3 - 448*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^10*b*c^4 - 224*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^9*b^2*c^4 - 144*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b^3*c^4 + 112*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^9*b*c^5 - 10*(b^2 - 4*a*c)*a^6*b^7
*c^2 + 98*(b^2 - 4*a*c)*a^7*b^5*c^3 - 288*(b^2 - 4*a*c)*a^8*b^3*c^4 + 224*(
```

```
b^2 - 4*a*c)*a^9*b*c^5 + (10*b^5*c^2 - 78*a*b^3*c^3 + 152*a^2*b*c^4 - 5*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 + 39*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 10*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 76*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 38*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 19*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 10*(b^2 - 4*a*c)*b^3*c^2 + 38*(b^2 -
4*a*c)*a*b*c^3)*(a^3*b^2 - 4*a^4*c)^2 + 2*(5*sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^3*b^8 - 64*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c -
10*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^7*c - 10*a^3*b^8*c + 286*s
qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^2 + 88*sqrt(2)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c^2 + 5*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^3*b^6*c^2 + 128*a^4*b^6*c^2 - 496*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^6*b^2*c^3 - 220*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^3 -
44*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^3 - 572*a^5*b^4*c^3 +
224*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*c^4 + 112*sqrt(2)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^6*b*c^4 + 110*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^5*b^2*c^4 + 992*a^6*b^2*c^4 - 56*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^6*c^5 - 448*a^7*c^5 + 10*(b^2 - 4*a*c)*a^3*b^6*c - 88*(b^2 - 4*a*c)*a
^4*b^4*c^2 + 220*(b^2 - 4*a*c)*a^5*b^2*c^3 - 112*(b^2 - 4*a*c)*a^6*c^4)*abs
(a^3*b^2 - 4*a^4*c))*arctan(2*sqrt(1/2)*x/sqrt((a^3*b^3 - 4*a^4*b*c + sqrt(
(a^3*b^3 - 4*a^4*b*c)^2 - 4*(a^4*b^2 - 4*a^5*c)*(a^3*b^2*c - 4*a^4*c^2)))/(
a^3*b^2*c - 4*a^4*c^2)))/((a^7*b^6 - 12*a^8*b^4*c - 2*a^7*b^5*c + 48*a^9*b^
2*c^2 + 16*a^8*b^3*c^2 + a^7*b^4*c^2 - 64*a^10*c^3 - 32*a^9*b*c^3 - 8*a^8*b
^2*c^3 + 16*a^9*c^4)*abs(a^3*b^2 - 4*a^4*c)*abs(c)) - 1/16*(10*a^6*b^9*c^2
- 138*a^7*b^7*c^3 + 680*a^8*b^5*c^4 - 1376*a^9*b^3*c^5 + 896*a^10*b*c^6 - 5
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^9 + 69*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^7*c + 10*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^8*c - 340*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^5*c^2 - 98*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^6*c^2 - 5*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^7*c^2 + 688*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b^3*c^3 + 288*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^4*c^3 + 49*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^5*c^3 - 448*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^10*b*c^4 - 224*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b^2*c^4 - 144*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^3*c^4 + 112*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b*c^5 - 10*(b^2
 - 4*a*c)*a^6*b^7*c^2 + 98*(b^2 - 4*a*c)*a^7*b^5*c^3 - 288*(b^2 - 4*a*c)*a^
8*b^3*c^4 + 224*(b^2 - 4*a*c)*a^9*b*c^5 + (10*b^5*c^2 - 78*a*b^3*c^3 + 152*
a^2*b*c^4 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5
 + 39*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 1
0*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c - 76*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 38*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 5*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 19*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 10*(b^2 - 4*a*c)*b^3
*c^2 + 38*(b^2 - 4*a*c)*a*b*c^3)*(a^3*b^2 - 4*a^4*c)^2 - 2*(5*sqrt(2)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^8 - 64*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^4*b^6*c - 10*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^7*c + 10*
a^3*b^8*c + 286*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^2 + 88*sq
rt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c^2 + 5*sqrt(2)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^3*b^6*c^2 - 128*a^4*b^6*c^2 - 496*sqrt(2)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^6*b^2*c^3 - 220*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a^5*b^3*c^3 - 44*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^3 + 5
72*a^5*b^4*c^3 + 224*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*c^4 + 112*
sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b*c^4 + 110*sqrt(2)*sqrt(b*c -
```

```
sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^4 - 992*a^6*b^2*c^4 - 56*sqrt(2)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^6*c^5 + 448*a^7*c^5 - 10*(b^2 - 4*a*c)*a^3*b^6*c + 8
8*(b^2 - 4*a*c)*a^4*b^4*c^2 - 220*(b^2 - 4*a*c)*a^5*b^2*c^3 + 112*(b^2 - 4*
a*c)*a^6*c^4)*abs(a^3*b^2 - 4*a^4*c))*arctan(2*sqrt(1/2)*x/sqrt((a^3*b^3 -
4*a^4*b*c - sqrt((a^3*b^3 - 4*a^4*b*c)^2 - 4*(a^4*b^2 - 4*a^5*c)*(a^3*b^2*c
 - 4*a^4*c^2)))/(a^3*b^2*c - 4*a^4*c^2)))/((a^7*b^6 - 12*a^8*b^4*c - 2*a^7*
b^5*c + 48*a^9*b^2*c^2 + 16*a^8*b^3*c^2 + a^7*b^4*c^2 - 64*a^10*c^3 - 32*a^
9*b*c^3 - 8*a^8*b^2*c^3 + 16*a^9*c^4)*abs(a^3*b^2 - 4*a^4*c)*abs(c)) + 1/3*
(6*b*x^2 - a)/(a^3*x^3)
```

maple [B] time = 0.04, size = 913, normalized size = 2.53

$$\frac{3b\,c^2x^3}{2\left(c\,x^4+b\,x^2+a\right)\left(4ac-b^2\right)a^2}-\frac{b^3c\,x^3}{2\left(c\,x^4+b\,x^2+a\right)\left(4ac-b^2\right)a^3}+\frac{7\sqrt{2}\,c^3\operatorname{arctanh}\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{\left(4ac-b^2\right)\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b\right.}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^5+b*x^3+a*x)^2,x)
```

```
[Out] -1/3/a^2/x^3+2/a^3*b/x+3/2/a^2/(c*x^4+b*x^2+a)*b*c^2/(4*a*c-b^2)*x^3-1/2/a^
3/(c*x^4+b*x^2+a)*b^3*c/(4*a*c-b^2)*x^3-1/a/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x*c
^2+2/a^2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x*b^2*c-1/2/a^3/(c*x^4+b*x^2+a)/(4*a*c
-b^2)*x*b^4-19/4/a^2*c^2/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b+5/4/a^3*c/(4*a
*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*
a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^3+7/a*c^3/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^
(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(
1/2))*c)^(1/2)*c*x)-29/4/a^2*c^2/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-
b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^
(1/2)*c*x)*b^2+5/4/a^3*c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*
c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*
x)*b^4+19/4/a^2*c^2/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*ar
ctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b-5/4/a^3*c/(4*a*c-b^2)*
2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1
/2))*c)^(1/2)*c*x)*b^3+7/a*c^3/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(
-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*c*x)-29/4/a^2*c^2/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^
(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2+5/
4/a^3*c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(
1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{3\left(5\,b^3c-19\,abc^2\right)x^6+\left(15\,b^4-62\,ab^2c+14\,a^2c^2\right)x^4-2\,a^2b^2+8\,a^3c+10\left(ab^3-4\,a^2bc\right)x^2}{6\left(\left(a^3b^2c-4\,a^4c^2\right)x^7+\left(a^3b^3-4\,a^4bc\right)x^5+\left(a^4b^2-4\,a^5c\right)x^3\right)}-\frac{-\int\frac{5\,b^4-24\,ab^2c+14}{cx}}{2\left(a^3b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/6*(3*(5*b^3*c - 19*a*b*c^2)*x^6 + (15*b^4 - 62*a*b^2*c + 14*a^2*c^2)*x^4
- 2*a^2*b^2 + 8*a^3*c + 10*(a*b^3 - 4*a^2*b*c)*x^2)/((a^3*b^2*c - 4*a^4*c^2
)*x^7 + (a^3*b^3 - 4*a^4*b*c)*x^5 + (a^4*b^2 - 4*a^5*c)*x^3) - 1/2*integrat
e(-(5*b^4 - 24*a*b^2*c + 14*a^2*c^2 + (5*b^3*c - 19*a*b*c^2)*x^2)/(c*x^4 +
b*x^2 + a), x)/(a^3*b^2 - 4*a^4*c)
```

mupad [B] time = 4.91, size = 8739, normalized size = 24.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a*x + b*x^3 + c*x^5)^2),x)
```

```
[Out] atan((((-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 636
6*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^
5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c
 - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*(-(4*a*c - b^2)^9
)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 -
1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*(320*a^1
2*b^14*c^2 - 917504*a^19*c^9 - 7936*a^13*b^12*c^3 + 82816*a^14*b^10*c^4 - 4
68480*a^15*b^8*c^5 + 1536000*a^16*b^6*c^6 - 2867200*a^17*b^4*c^7 + 2719744*
a^18*b^2*c^8 + x*(-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b
*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*
a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 61
5*a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*(-(4*a*
c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*
b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2
)*(1048576*a^21*b*c^8 + 256*a^15*b^13*c^2 - 6144*a^16*b^11*c^3 + 61440*a^17
*b^9*c^4 - 327680*a^18*b^7*c^5 + 983040*a^19*b^5*c^6 - 1572864*a^20*b^3*c^7
)) - x*(401408*a^16*c^10 - 400*a^9*b^14*c^3 + 9440*a^10*b^12*c^4 - 92816*a^
11*b^10*c^5 + 488096*a^12*b^8*c^6 - 1458688*a^13*b^6*c^7 + 2401280*a^14*b^4
*c^8 - 1871872*a^15*b^2*c^9))*(-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2)
- 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*
c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^
9)^(1/2) - 615*a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*
b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10
*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^
2*c^5)))^(1/2)*1i + ((-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a
^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219
744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2)
- 615*a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*(-(
4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*
a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^
(1/2)*(917504*a^19*c^9 - 320*a^12*b^14*c^2 + 7936*a^13*b^12*c^3 - 82816*a^1
4*b^10*c^4 + 468480*a^15*b^8*c^5 - 1536000*a^16*b^6*c^6 + 2867200*a^17*b^4*
c^7 - 2719744*a^18*b^2*c^8 + x*(-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2)
 - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7
*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)
^9)^(1/2) - 615*a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a
*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^1
0*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b
^2*c^5)))^(1/2)*(1048576*a^21*b*c^8 + 256*a^15*b^13*c^2 - 6144*a^16*b^11*c^
3 + 61440*a^17*b^9*c^4 - 327680*a^18*b^7*c^5 + 983040*a^19*b^5*c^6 - 157286
4*a^20*b^3*c^7)) - x*(401408*a^16*c^10 - 400*a^9*b^14*c^3 + 9440*a^10*b^12*
c^4 - 92816*a^11*b^10*c^5 + 488096*a^12*b^8*c^6 - 1458688*a^13*b^6*c^7 + 24
01280*a^14*b^4*c^8 - 1871872*a^15*b^2*c^9))*(-(25*b^15 - 25*b^6*(-(4*a*c -
b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 1
16928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-
(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^
(1/2) + 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6
 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4
- 6144*a^12*b^2*c^5)))^(1/2)*1i)/(((-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(
1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4
*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c -
b^2)^9)^(1/2) - 615*a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 1
65*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8
*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^
12*b^2*c^5)))^(1/2)*(320*a^12*b^14*c^2 - 917504*a^19*c^9 - 7936*a^13*b^12*c
^3 + 82816*a^14*b^10*c^4 - 468480*a^15*b^8*c^5 + 1536000*a^16*b^6*c^6 - 286
```

7200*a^17*b^4*c^7 + 2719744*a^18*b^2*c^8 + x*(-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*(1048576*a^21*b*c^8 + 256*a^15*b^13*c^2 - 6144*a^16*b^11*c^3 + 61440*a^17*b^9*c^4 - 327680*a^18*b^7*c^5 + 983040*a^19*b^5*c^6 - 1572864*a^20*b^3*c^7)) - x*(401408*a^16*c^10 - 400*a^9*b^14*c^3 + 9440*a^10*b^12*c^4 - 92816*a^11*b^10*c^5 + 488096*a^12*b^8*c^6 - 1458688*a^13*b^6*c^7 + 2401280*a^14*b^4*c^8 - 1871872*a^15*b^2*c^9))*(-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2) - ((-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*(917504*a^19*c^9 - 320*a^12*b^14*c^2 + 7936*a^13*b^12*c^3 - 82816*a^14*b^10*c^4 + 468480*a^15*b^8*c^5 - 1536000*a^16*b^6*c^6 + 2867200*a^17*b^4*c^7 - 2719744*a^18*b^2*c^8 + x*(-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*(1048576*a^21*b*c^8 + 256*a^15*b^13*c^2 - 6144*a^16*b^11*c^3 + 61440*a^17*b^9*c^4 - 327680*a^18*b^7*c^5 + 983040*a^19*b^5*c^6 - 1572864*a^20*b^3*c^7)) - x*(401408*a^16*c^10 - 400*a^9*b^14*c^3 + 9440*a^10*b^12*c^4 - 92816*a^11*b^10*c^5 + 488096*a^12*b^8*c^6 - 1458688*a^13*b^6*c^7 + 2401280*a^14*b^4*c^8 - 1871872*a^15*b^2*c^9))*(-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2) + 476672*a^13*b*c^10 + 1800*a^9*b^9*c^6 - 29080*a^10*b^7*c^7 + 176032*a^11*b^5*c^8 - 473216*a^12*b^3*c^9))*(-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*2i - (1/(3*a) - (5*b*x^2)/(3*a^2) + (x^4*(15*b^4 + 14*a^2*c^2 - 62*a*b^2*c))/(6*a^3*(4*a*c - b^2)) + (c*x^6*(5*b^3 - 19*a*b*c))/(2*a^3*(4*a*c - b^2)))/(a*x^3 + b*x^5 + c*x^7) + atan((((-(25*b^15 + 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 - 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c + 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*(320*a^12*b^14*c^2 - 917504*a^19*c^9 - 7936*a^13*b^12*c^3 + 82816*a^14*b^10*c^4 - 468480*a^15*b^8*c^5 + 1536000*a^16*b^6*c^6 - 2867200*a^17*b^4*c^7 + 2719744*a^18*b^2*c^8 + x*(-(25*b^15 + 25*b^6*(-(4*a*c - b^2)

^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 11692
8*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 - 49*a^3*c^3*(-(4*a
*c - b^2)^9)^(1/2) - 615*a*b^13*c + 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2
) - 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 2
4*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 61
44*a^12*b^2*c^5)))^(1/2)*(1048576*a^21*b*c^8 + 256*a^15*b^13*c^2 - 6144*a^1
6*b^11*c^3 + 61440*a^17*b^9*c^4 - 327680*a^18*b^7*c^5 + 983040*a^19*b^5*c^6
 - 1572864*a^20*b^3*c^7)) - x*(401408*a^16*c^10 - 400*a^9*b^14*c^3 + 9440*a
^10*b^12*c^4 - 92816*a^11*b^10*c^5 + 488096*a^12*b^8*c^6 - 1458688*a^13*b^6
*c^7 + 2401280*a^14*b^4*c^8 - 1871872*a^15*b^2*c^9))*(-(25*b^15 + 25*b^6*(-
(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^
9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 - 49*a
^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c + 246*a^2*b^2*c^2*(-(4*a*c -
 b^2)^9)^(1/2) - 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096
*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11
*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*1i + ((-(25*b^15 + 25*b^6*(-(4*a*c -
b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 1
16928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 - 49*a^3*c^3*(-
(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c + 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^
(1/2) - 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6
 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4
- 6144*a^12*b^2*c^5)))^(1/2)*(917504*a^19*c^9 - 320*a^12*b^14*c^2 + 7936*a^
13*b^12*c^3 - 82816*a^14*b^10*c^4 + 468480*a^15*b^8*c^5 - 1536000*a^16*b^6*
c^6 + 2867200*a^17*b^4*c^7 - 2719744*a^18*b^2*c^8 + x*(-(25*b^15 + 25*b^6*(
-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b
^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 - 49*
a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c + 246*a^2*b^2*c^2*(-(4*a*c
- b^2)^9)^(1/2) - 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 409
6*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^1
1*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*(1048576*a^21*b*c^8 + 256*a^15*b^13*
c^2 - 6144*a^16*b^11*c^3 + 61440*a^17*b^9*c^4 - 327680*a^18*b^7*c^5 + 98304
0*a^19*b^5*c^6 - 1572864*a^20*b^3*c^7)) - x*(401408*a^16*c^10 - 400*a^9*b^1
4*c^3 + 9440*a^10*b^12*c^4 - 92816*a^11*b^10*c^5 + 488096*a^12*b^8*c^6 - 14
58688*a^13*b^6*c^7 + 2401280*a^14*b^4*c^8 - 1871872*a^15*b^2*c^9))*(-(25*b^
15 + 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2
- 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*
b^3*c^6 - 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c + 246*a^2*b^2*
c^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a
^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c
^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*1i)/(((-(25*b^15 + 25*b
^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a
^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 -
 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c + 246*a^2*b^2*c^2*(-(4*
a*c - b^2)^9)^(1/2) - 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 +
 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840
*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*(320*a^12*b^14*c^2 - 917504*a^19
*c^9 - 7936*a^13*b^12*c^3 + 82816*a^14*b^10*c^4 - 468480*a^15*b^8*c^5 + 153
6000*a^16*b^6*c^6 - 2867200*a^17*b^4*c^7 + 2719744*a^18*b^2*c^8 + x*(-(25*b
^15 + 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2
 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6
*b^3*c^6 - 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c + 246*a^2*b^2
c^2(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(
a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*
c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*(1048576*a^21*b*c^8 +
256*a^15*b^13*c^2 - 6144*a^16*b^11*c^3 + 61440*a^17*b^9*c^4 - 327680*a^18*b
^7*c^5 + 983040*a^19*b^5*c^6 - 1572864*a^20*b^3*c^7)) - x*(401408*a^16*c^10
 - 400*a^9*b^14*c^3 + 9440*a^10*b^12*c^4 - 92816*a^11*b^10*c^5 + 488096*a^1
2*b^8*c^6 - 1458688*a^13*b^6*c^7 + 2401280*a^14*b^4*c^8 - 1871872*a^15*b^2*
c^9))*(-(25*b^15 + 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366

```
*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5
 + 215040*a^6*b^3*c^6 - 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c
+ 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*(-(4*a*c - b^2)^9)
^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1
280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2) - ((-(25*
b^15 + 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^
2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^
6*b^3*c^6 - 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c + 246*a^2*b^
2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*
(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6
*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*(917504*a^19*c^9 - 32
0*a^12*b^14*c^2 + 7936*a^13*b^12*c^3 - 82816*a^14*b^10*c^4 + 468480*a^15*b^
8*c^5 - 1536000*a^16*b^6*c^6 + 2867200*a^17*b^4*c^7 - 2719744*a^18*b^2*c^8
+ x*(-(25*b^15 + 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a
^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 +
 215040*a^6*b^3*c^6 - 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c +
246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*(-(4*a*c - b^2)^9)^(
1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 128
0*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*(1048576*a^
21*b*c^8 + 256*a^15*b^13*c^2 - 6144*a^16*b^11*c^3 + 61440*a^17*b^9*c^4 - 32
7680*a^18*b^7*c^5 + 983040*a^19*b^5*c^6 - 1572864*a^20*b^3*c^7)) - x*(40140
8*a^16*c^10 - 400*a^9*b^14*c^3 + 9440*a^10*b^12*c^4 - 92816*a^11*b^10*c^5 +
 488096*a^12*b^8*c^6 - 1458688*a^13*b^6*c^7 + 2401280*a^14*b^4*c^8 - 187187
2*a^15*b^2*c^9))*(-(25*b^15 + 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b
*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*
a^5*b^5*c^5 + 215040*a^6*b^3*c^6 - 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 61
5*a*b^13*c + 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*(-(4*a*
c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*
b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2
) + 476672*a^13*b*c^10 + 1800*a^9*b^9*c^6 - 29080*a^10*b^7*c^7 + 176032*a^1
1*b^5*c^8 - 473216*a^12*b^3*c^9))*(-(25*b^15 + 25*b^6*(-(4*a*c - b^2)^9)^(1
/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*
b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 - 49*a^3*c^3*(-(4*a*c - b
^2)^9)^(1/2) - 615*a*b^13*c + 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 16
5*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*
b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^1
2*b^2*c^5)))^(1/2)*2i
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**5+b*x**3+a*x)**2,x)
[Out] Timed out
```

3.103 $\int \frac{1}{x^3\left(ax+bx^3+cx^5\right)^2}\,dx$

Optimal. Leaf size=219

$$-\frac{\left(3b^2-2ac\right)\log\left(a+bx^2+cx^4\right)}{4a^4}+\frac{\log(x)\left(3b^2-2ac\right)}{a^4}+\frac{b\left(3b^2-11ac\right)}{2a^3x^2\left(b^2-4ac\right)}-\frac{3b^2-8ac}{4a^2x^4\left(b^2-4ac\right)}+\frac{b\left(30a^2c^2-20ab^2c+\right.}{2a^4\left(b^2\right.}$$

```
[Out] 1/4*(8*a*c-3*b^2)/a^2/(-4*a*c+b^2)/x^4+1/2*b*(-11*a*c+3*b^2)/a^3/(-4*a*c+b^
2)/x^2+1/2*(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/x^4/(c*x^4+b*x^2+a)+1/2*b*(30
*a^2*c^2-20*a*b^2*c+3*b^4)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/a^4/(-4*
a*c+b^2)^(3/2)+(-2*a*c+3*b^2)*ln(x)/a^4-1/4*(-2*a*c+3*b^2)*ln(c*x^4+b*x^2+a
)/a^4
```

Rubi [A] time = 0.31, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {1585, 1114, 740, 800, 634, 618, 206, 628}

$$\frac{b\left(30a^2c^2-20ab^2c+3b^4\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^4\left(b^2-4ac\right)^{3/2}}+\frac{b\left(3b^2-11ac\right)}{2a^3x^2\left(b^2-4ac\right)}-\frac{3b^2-8ac}{4a^2x^4\left(b^2-4ac\right)}-\frac{\left(3b^2-2ac\right)\log\left(a+bx^2+cx^4\right)}{4a^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a*x + b*x^3 + c*x^5)^2),x]
```

```
[Out] -(3*b^2 - 8*a*c)/(4*a^2*(b^2 - 4*a*c)*x^4) + (b*(3*b^2 - 11*a*c))/(2*a^3*(b
^2 - 4*a*c)*x^2) + (b^2 - 2*a*c + b*c*x^2)/(2*a*(b^2 - 4*a*c)*x^4*(a + b*x^
2 + c*x^4)) + (b*(3*b^4 - 20*a*b^2*c + 30*a^2*c^2)*ArcTanh[(b + 2*c*x^2)/Sq
rt[b^2 - 4*a*c]])/(2*a^4*(b^2 - 4*a*c)^(3/2)) + ((3*b^2 - 2*a*c)*Log[x])/a^
4 - ((3*b^2 - 2*a*c)*Log[a + b*x^2 + c*x^4])/(4*a^4)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\int \frac{1}{x^3\left(ax+bx^3+cx^5\right)^2}\,dx = \int \frac{1}{x^5\left(a+bx^2+cx^4\right)^2}\,dx$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^3\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right)$$
$$= \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x^4\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int \frac{-3b^2+8ac-3bcx}{x^3(a+bx+cx^2)}\,dx, x, x^2\right)}{2a\left(b^2-4ac\right)}$$
$$= \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x^4\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int\left(\frac{-3b^2+8ac}{ax^3} + \frac{3b^3-11abc}{a^2x^2} + \frac{(b^2-4ac)(-3b^2+2ac}{a^3x}\right.\right.}{2a\,(b^2}$$
$$= -\frac{3b^2-8ac}{4a^2\left(b^2-4ac\right)x^4} + \frac{b\left(3b^2-11ac\right)}{2a^3\left(b^2-4ac\right)x^2} + \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x^4\left(a+bx^2+cx^4\right)} + \frac{(3b^2-}{}$$
$$= -\frac{3b^2-8ac}{4a^2\left(b^2-4ac\right)x^4} + \frac{b\left(3b^2-11ac\right)}{2a^3\left(b^2-4ac\right)x^2} + \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x^4\left(a+bx^2+cx^4\right)} + \frac{(3b^2-}{}$$
$$= -\frac{3b^2-8ac}{4a^2\left(b^2-4ac\right)x^4} + \frac{b\left(3b^2-11ac\right)}{2a^3\left(b^2-4ac\right)x^2} + \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x^4\left(a+bx^2+cx^4\right)} + \frac{(3b^2-}{}$$
$$= -\frac{3b^2-8ac}{4a^2\left(b^2-4ac\right)x^4} + \frac{b\left(3b^2-11ac\right)}{2a^3\left(b^2-4ac\right)x^2} + \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x^4\left(a+bx^2+cx^4\right)} + \frac{b\,(3b^4}{}$$

Mathematica [**A**] time = 0.38, size = 328, normalized size = 1.50

$$\frac{\frac{2a\left(2a^2c^2-4ab^2c-3abc^2x^2+b^4+b^3cx^2\right)}{(b^2-4ac)(a+bx^2+cx^4)} - \frac{\left(8a^2c^2\sqrt{b^2-4ac}+30a^2bc^2-20ab^3c-14ab^2c\sqrt{b^2-4ac}+3b^4\sqrt{b^2-4ac}+3b^5\right)\log\left(-\sqrt{b^2-4ac}+b+2cx^2\right)}{(b^2-4ac)^{3/2}} + \frac{(-8a^2}{}}{4a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a*x + b*x^3 + c*x^5)^2),x]
```

```
[Out] (-(a^2/x^4) + (4*a*b)/x^2 + (2*a*(b^4 - 4*a*b^2*c + 2*a^2*c^2 + b^3*c*x^2 -
 3*a*b*c^2*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + 4*(3*b^2 - 2*a*c)*Lo
g[x] - ((3*b^5 - 20*a*b^3*c + 30*a^2*b*c^2 + 3*b^4*Sqrt[b^2 - 4*a*c] - 14*a
*b^2*c*Sqrt[b^2 - 4*a*c] + 8*a^2*c^2*Sqrt[b^2 - 4*a*c])*Log[b - Sqrt[b^2 -
4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2) + ((3*b^5 - 20*a*b^3*c + 30*a^2*b*c^
2 - 3*b^4*Sqrt[b^2 - 4*a*c] + 14*a*b^2*c*Sqrt[b^2 - 4*a*c] - 8*a^2*c^2*Sqrt
[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/(
4*a^4)
```

fricas [**B**] time = 1.41, size = 1242, normalized size = 5.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 - 2*(3*a*b^5*c - 23*a^2*b^3*c^2 +
 44*a^3*b*c^3)*x^6 - (6*a*b^6 - 49*a^2*b^4*c + 108*a^3*b^2*c^2 - 32*a^4*c^3
```

```
)*x^4 - 3*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2 + ((3*b^5*c - 20*a*b^3
*c^2 + 30*a^2*b*c^3)*x^8 + (3*b^6 - 20*a*b^4*c + 30*a^2*b^2*c^2)*x^6 + (3*a
*b^5 - 20*a^2*b^3*c + 30*a^3*b*c^2)*x^4)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 +
 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2
+ a)) + ((3*b^6*c - 26*a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*x^8 + (3*b^
7 - 26*a*b^5*c + 64*a^2*b^3*c^2 - 32*a^3*b*c^3)*x^6 + (3*a*b^6 - 26*a^2*b^4
*c + 64*a^3*b^2*c^2 - 32*a^4*c^3)*x^4)*log(c*x^4 + b*x^2 + a) - 4*((3*b^6*c
 - 26*a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*x^8 + (3*b^7 - 26*a*b^5*c +
64*a^2*b^3*c^2 - 32*a^3*b*c^3)*x^6 + (3*a*b^6 - 26*a^2*b^4*c + 64*a^3*b^2*c
^2 - 32*a^4*c^3)*x^4)*log(x))/((a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*x^8
 + (a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*x^6 + (a^5*b^4 - 8*a^6*b^2*c + 16
*a^7*c^2)*x^4), -1/4*(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 - 2*(3*a*b^5*c - 2
3*a^2*b^3*c^2 + 44*a^3*b*c^3)*x^6 - (6*a*b^6 - 49*a^2*b^4*c + 108*a^3*b^2*c
^2 - 32*a^4*c^3)*x^4 - 3*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2 - 2*((3
*b^5*c - 20*a*b^3*c^2 + 30*a^2*b*c^3)*x^8 + (3*b^6 - 20*a*b^4*c + 30*a^2*b^
2*c^2)*x^6 + (3*a*b^5 - 20*a^2*b^3*c + 30*a^3*b*c^2)*x^4)*sqrt(-b^2 + 4*a*c
)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + ((3*b^6*c - 26*
a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*x^8 + (3*b^7 - 26*a*b^5*c + 64*a^2
*b^3*c^2 - 32*a^3*b*c^3)*x^6 + (3*a*b^6 - 26*a^2*b^4*c + 64*a^3*b^2*c^2 - 3
2*a^4*c^3)*x^4)*log(c*x^4 + b*x^2 + a) - 4*((3*b^6*c - 26*a*b^4*c^2 + 64*a^
2*b^2*c^3 - 32*a^3*c^4)*x^8 + (3*b^7 - 26*a*b^5*c + 64*a^2*b^3*c^2 - 32*a^3
*b*c^3)*x^6 + (3*a*b^6 - 26*a^2*b^4*c + 64*a^3*b^2*c^2 - 32*a^4*c^3)*x^4)*l
og(x))/((a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*x^8 + (a^4*b^5 - 8*a^5*b^3
*c + 16*a^6*b*c^2)*x^6 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*x^4)]
```

giac [A] time = 2.04, size = 274, normalized size = 1.25

$$-\frac{\left(3\,b^5-20\,ab^3c+30\,a^2bc^2\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\left(a^4b^2-4\,a^5c\right)\sqrt{-b^2+4\,ac}}+\frac{3\,b^4cx^4-14\,ab^2c^2x^4+8\,a^2c^3x^4+3\,b^5x^2-12\,ab^3cx^2+2\,a^2bc}{4\left(a^4b^2-4\,a^5c\right)\left(cx^4+bx^2+a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] -1/2*(3*b^5 - 20*a*b^3*c + 30*a^2*b*c^2)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4
*a*c))/((a^4*b^2 - 4*a^5*c)*sqrt(-b^2 + 4*a*c)) + 1/4*(3*b^4*c*x^4 - 14*a*b
^2*c^2*x^4 + 8*a^2*c^3*x^4 + 3*b^5*x^2 - 12*a*b^3*c*x^2 + 2*a^2*b*c^2*x^2 +
 5*a*b^4 - 22*a^2*b^2*c + 12*a^3*c^2)/((a^4*b^2 - 4*a^5*c)*(c*x^4 + b*x^2 +
 a)) - 1/4*(3*b^2 - 2*a*c)*log(c*x^4 + b*x^2 + a)/a^4 + 1/2*(3*b^2 - 2*a*c)
*log(x^2)/a^4 - 1/4*(9*b^2*x^4 - 6*a*c*x^4 - 4*a*b*x^2 + a^2)/(a^4*x^4)
```

maple [B] time = 0.02, size = 443, normalized size = 2.02

$$\frac{3b\,c^2x^2}{2\left(c\,x^4+b\,x^2+a\right)\left(4ac-b^2\right)a^2}-\frac{b^3c\,x^2}{2\left(c\,x^4+b\,x^2+a\right)\left(4ac-b^2\right)a^3}+\frac{15b\,c^2\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}a^2}-\frac{10b^3c\arctan\left(\frac{2c\,x}{\sqrt{4a}}\right.}{\left(4ac-b^2\right)^{\frac{3}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(c*x^5+b*x^3+a*x)^2,x)
```

```
[Out] -1/4/a^2/x^4-2/a^3*c*ln(x)+3/a^4*b^2*ln(x)+1/a^3*b/x^2+3/2/a^2/(c*x^4+b*x^2
+a)*b*c^2/(4*a*c-b^2)*x^2-1/2/a^3/(c*x^4+b*x^2+a)*b^3*c/(4*a*c-b^2)*x^2-1/a
/(c*x^4+b*x^2+a)/(4*a*c-b^2)*c^2+2/a^2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*b^2*c-1/
2/a^3/(c*x^4+b*x^2+a)/(4*a*c-b^2)*b^4+2/a^2/(4*a*c-b^2)*c^2*ln(c*x^4+b*x^2+
a)-7/2/a^3/(4*a*c-b^2)*c*ln(c*x^4+b*x^2+a)*b^2+3/4/a^4/(4*a*c-b^2)*ln(c*x^4
+b*x^2+a)*b^4+15/a^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2)
)*b*c^2-10/a^3/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^3*
c+3/2/a^4/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2\left(3\,b^3c-11\,abc^2\right)x^6+\left(6\,b^4-25\,ab^2c+8\,a^2c^2\right)x^4-a^2b^2+4\,a^3c+3\left(ab^3-4\,a^2bc\right)x^2}{4\left(\left(a^3b^2c-4\,a^4c^2\right)x^8+\left(a^3b^3-4\,a^4bc\right)x^6+\left(a^4b^2-4\,a^5c\right)x^4\right)}-\frac{\frac{1}{4}\left(3\,b^4-14\,ab^2c+8\,a^2c^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(2*(3*b^3*c - 11*a*b*c^2)*x^6 + (6*b^4 - 25*a*b^2*c + 8*a^2*c^2)*x^4 -
a^2*b^2 + 4*a^3*c + 3*(a*b^3 - 4*a^2*b*c)*x^2)/((a^3*b^2*c - 4*a^4*c^2)*x^8
 + (a^3*b^3 - 4*a^4*b*c)*x^6 + (a^4*b^2 - 4*a^5*c)*x^4) - integrate(((3*b^4
*c - 14*a*b^2*c^2 + 8*a^2*c^3)*x^3 + (3*b^5 - 17*a*b^3*c + 19*a^2*b*c^2)*x)
/(c*x^4 + b*x^2 + a), x)/(a^4*b^2 - 4*a^5*c) + (3*b^2 - 2*a*c)*log(x)/a^4
```

mupad [B] time = 7.47, size = 5999, normalized size = 27.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a*x + b*x^3 + c*x^5)^2),x)
```

```
[Out] (b*atan((x^2*((((((b*((2240*a^10*b*c^7 - 6*a^6*b^9*c^3 + 40*a^7*b^7*c^4 + 1
08*a^8*b^5*c^5 - 1248*a^9*b^3*c^6)/(a^9*b^6 - 64*a^12*c^3 - 12*a^10*b^4*c +
 48*a^11*b^2*c^2) - ((2560*a^13*b*c^6 + 12*a^9*b^9*c^2 - 184*a^10*b^7*c^3 +
 1056*a^11*b^5*c^4 - 2688*a^12*b^3*c^5)*(6*b^8 + 256*a^4*c^4 + 336*a^2*b^4*
c^2 - 576*a^3*b^2*c^3 - 76*a*b^6*c))/(2*(a^9*b^6 - 64*a^12*c^3 - 12*a^10*b^
4*c + 48*a^11*b^2*c^2)*(4*a^4*b^6 - 256*a^7*c^3 - 48*a^5*b^4*c + 192*a^6*b^
2*c^2)))*(3*b^4 + 30*a^2*c^2 - 20*a*b^2*c))/(4*a^4*(4*a*c - b^2)^(3/2)) - (
b*(3*b^4 + 30*a^2*c^2 - 20*a*b^2*c)*(2560*a^13*b*c^6 + 12*a^9*b^9*c^2 - 184
*a^10*b^7*c^3 + 1056*a^11*b^5*c^4 - 2688*a^12*b^3*c^5)*(6*b^8 + 256*a^4*c^4
 + 336*a^2*b^4*c^2 - 576*a^3*b^2*c^3 - 76*a*b^6*c))/(8*a^4*(4*a*c - b^2)^(3
/2)*(a^9*b^6 - 64*a^12*c^3 - 12*a^10*b^4*c + 48*a^11*b^2*c^2)*(4*a^4*b^6 -
256*a^7*c^3 - 48*a^5*b^4*c + 192*a^6*b^2*c^2)))*(6*b^8 + 256*a^4*c^4 + 336*
a^2*b^4*c^2 - 576*a^3*b^2*c^3 - 76*a*b^6*c))/(2*(4*a^4*b^6 - 256*a^7*c^3 -
48*a^5*b^4*c + 192*a^6*b^2*c^2)) + (b*((1760*a^7*b*c^8 + 54*a^3*b^9*c^4 - 6
57*a^4*b^7*c^5 + 2775*a^5*b^5*c^6 - 4484*a^6*b^3*c^7)/(a^9*b^6 - 64*a^12*c^
3 - 12*a^10*b^4*c + 48*a^11*b^2*c^2) + (((2240*a^10*b*c^7 - 6*a^6*b^9*c^3 +
 40*a^7*b^7*c^4 + 108*a^8*b^5*c^5 - 1248*a^9*b^3*c^6)/(a^9*b^6 - 64*a^12*c^
3 - 12*a^10*b^4*c + 48*a^11*b^2*c^2) - ((2560*a^13*b*c^6 + 12*a^9*b^9*c^2 -
 184*a^10*b^7*c^3 + 1056*a^11*b^5*c^4 - 2688*a^12*b^3*c^5)*(6*b^8 + 256*a^4
*c^4 + 336*a^2*b^4*c^2 - 576*a^3*b^2*c^3 - 76*a*b^6*c))/(2*(a^9*b^6 - 64*a^
12*c^3 - 12*a^10*b^4*c + 48*a^11*b^2*c^2)*(4*a^4*b^6 - 256*a^7*c^3 - 48*a^5
*b^4*c + 192*a^6*b^2*c^2)))*(6*b^8 + 256*a^4*c^4 + 336*a^2*b^4*c^2 - 576*a^
3*b^2*c^3 - 76*a*b^6*c))/(2*(4*a^4*b^6 - 256*a^7*c^3 - 48*a^5*b^4*c + 192*a
^6*b^2*c^2)))*(3*b^4 + 30*a^2*c^2 - 20*a*b^2*c))/(4*a^4*(4*a*c - b^2)^(3/2)
) + (b^3*(3*b^4 + 30*a^2*c^2 - 20*a*b^2*c)^3*(2560*a^13*b*c^6 + 12*a^9*b^9*
c^2 - 184*a^10*b^7*c^3 + 1056*a^11*b^5*c^4 - 2688*a^12*b^3*c^5))/(64*a^12*(
4*a*c - b^2)^(9/2)*(a^9*b^6 - 64*a^12*c^3 - 12*a^10*b^4*c + 48*a^11*b^2*c^2
)))*(9*b^8 + 80*a^4*c^4 + 270*a^2*b^4*c^2 - 285*a^3*b^2*c^3 - 87*a*b^6*c))/
(8*a^3*c^2*(4*a*c - b^2)^(7/2)*(54*b^10 - 1600*a^5*c^5 + 3480*a^2*b^6*c^2 -
 7200*a^3*b^4*c^3 + 5775*a^4*b^2*c^4 - 720*a*b^8*c)) + (3*b*((27*b^9*c^5 -
297*a*b^7*c^6 + 1089*a^2*b^5*c^7 - 1331*a^3*b^3*c^8)/(a^9*b^6 - 64*a^12*c^3
 - 12*a^10*b^4*c + 48*a^11*b^2*c^2) - (((1760*a^7*b*c^8 + 54*a^3*b^9*c^4 -
657*a^4*b^7*c^5 + 2775*a^5*b^5*c^6 - 4484*a^6*b^3*c^7)/(a^9*b^6 - 64*a^12*c
^3 - 12*a^10*b^4*c + 48*a^11*b^2*c^2) + (((2240*a^10*b*c^7 - 6*a^6*b^9*c^3
+ 40*a^7*b^7*c^4 + 108*a^8*b^5*c^5 - 1248*a^9*b^3*c^6)/(a^9*b^6 - 64*a^12*c
^3 - 12*a^10*b^4*c + 48*a^11*b^2*c^2) - ((2560*a^13*b*c^6 + 12*a^9*b^9*c^2
```

- 184*a^10*b^7*c^3 + 1056*a^11*b^5*c^4 - 2688*a^12*b^3*c^5)*(6*b^8 + 256*a^
4*c^4 + 336*a^2*b^4*c^2 - 576*a^3*b^2*c^3 - 76*a*b^6*c))/(2*(a^9*b^6 - 64*a
^12*c^3 - 12*a^10*b^4*c + 48*a^11*b^2*c^2)*(4*a^4*b^6 - 256*a^7*c^3 - 48*a^
5*b^4*c + 192*a^6*b^2*c^2)))*(6*b^8 + 256*a^4*c^4 + 336*a^2*b^4*c^2 - 576*a
^3*b^2*c^3 - 76*a*b^6*c))/(2*(4*a^4*b^6 - 256*a^7*c^3 - 48*a^5*b^4*c + 192*
a^6*b^2*c^2)))*(6*b^8 + 256*a^4*c^4 + 336*a^2*b^4*c^2 - 576*a^3*b^2*c^3 - 7
6*a*b^6*c))/(2*(4*a^4*b^6 - 256*a^7*c^3 - 48*a^5*b^4*c + 192*a^6*b^2*c^2))
+ (b*((b*((2240*a^10*b*c^7 - 6*a^6*b^9*c^3 + 40*a^7*b^7*c^4 + 108*a^8*b^5*c
^5 - 1248*a^9*b^3*c^6)/(a^9*b^6 - 64*a^12*c^3 - 12*a^10*b^4*c + 48*a^11*b^2
*c^2) - ((2560*a^13*b*c^6 + 12*a^9*b^9*c^2 - 184*a^10*b^7*c^3 + 1056*a^11*b
^5*c^4 - 2688*a^12*b^3*c^5)*(6*b^8 + 256*a^4*c^4 + 336*a^2*b^4*c^2 - 576*a^
3*b^2*c^3 - 76*a*b^6*c))/(2*(a^9*b^6 - 64*a^12*c^3 - 12*a^10*b^4*c + 48*a^1
1*b^2*c^2)*(4*a^4*b^6 - 256*a^7*c^3 - 48*a^5*b^4*c + 192*a^6*b^2*c^2)))*(3*
b^4 + 30*a^2*c^2 - 20*a*b^2*c))/(4*a^4*(4*a*c - b^2)^(3/2)) - (b*(3*b^4 + 3
0*a^2*c^2 - 20*a*b^2*c)*(2560*a^13*b*c^6 + 12*a^9*b^9*c^2 - 184*a^10*b^7*c^
3 + 1056*a^11*b^5*c^4 - 2688*a^12*b^3*c^5)*(6*b^8 + 256*a^4*c^4 + 336*a^2*b
^4*c^2 - 576*a^3*b^2*c^3 - 76*a*b^6*c))/(8*a^4*(4*a*c - b^2)^(3/2)*(a^9*b^6
 - 64*a^12*c^3 - 12*a^10*b^4*c + 48*a^11*b^2*c^2)*(4*a^4*b^6 - 256*a^7*c^3
- 48*a^5*b^4*c + 192*a^6*b^2*c^2)))*(3*b^4 + 30*a^2*c^2 - 20*a*b^2*c))/(4*a
^4*(4*a*c - b^2)^(3/2)) - (b^2*(3*b^4 + 30*a^2*c^2 - 20*a*b^2*c)^2*(2560*a^
13*b*c^6 + 12*a^9*b^9*c^2 - 184*a^10*b^7*c^3 + 1056*a^11*b^5*c^4 - 2688*a^1
2*b^3*c^5)*(6*b^8 + 256*a^4*c^4 + 336*a^2*b^4*c^2 - 576*a^3*b^2*c^3 - 76*a*
b^6*c))/(32*a^8*(4*a*c - b^2)^3*(a^9*b^6 - 64*a^12*c^3 - 12*a^10*b^4*c + 48
*a^11*b^2*c^2)*(4*a^4*b^6 - 256*a^7*c^3 - 48*a^5*b^4*c + 192*a^6*b^2*c^2)))
*(3*b^6 - 25*a^3*c^3 + 50*a^2*b^2*c^2 - 23*a*b^4*c))/(8*a^3*c^2*(4*a*c - b^
2)^3*(54*b^10 - 1600*a^5*c^5 + 3480*a^2*b^6*c^2 - 7200*a^3*b^4*c^3 + 5775*a
^4*b^2*c^4 - 720*a*b^8*c)))*(16*a^12*b^6*(4*a*c - b^2)^(9/2) - 1024*a^15*c^
3*(4*a*c - b^2)^(9/2) - 192*a^13*b^4*c*(4*a*c - b^2)^(9/2) + 768*a^14*b^2*c
^2*(4*a*c - b^2)^(9/2)))/(9*b^10*c^2 - 120*a*b^8*c^3 + 580*a^2*b^6*c^4 - 12
00*a^3*b^4*c^5 + 900*a^4*b^2*c^6) + (((b*((36*a^3*b^8*c^3 - 309*a^4*b^6*c^4
 + 778*a^5*b^4*c^5 - 473*a^6*b^2*c^6)/(a^9*b^4 + 16*a^11*c^2 - 8*a^10*b^2*c
) - (((12*a^6*b^8*c^2 - 116*a^7*b^6*c^3 + 348*a^8*b^4*c^4 - 304*a^9*b^2*c^5
)/(a^9*b^4 + 16*a^11*c^2 - 8*a^10*b^2*c) + ((4*a^10*b^6*c^2 - 32*a^11*b^4*c
^3 + 64*a^12*b^2*c^4)*(6*b^8 + 256*a^4*c^4 + 336*a^2*b^4*c^2 - 576*a^3*b^2*
c^3 - 76*a*b^6*c))/(2*(a^9*b^4 + 16*a^11*c^2 - 8*a^10*b^2*c)*(4*a^4*b^6 - 2
56*a^7*c^3 - 48*a^5*b^4*c + 192*a^6*b^2*c^2)))*(6*b^8 + 256*a^4*c^4 + 336*a
^2*b^4*c^2 - 576*a^3*b^2*c^3 - 76*a*b^6*c))/(2*(4*a^4*b^6 - 256*a^7*c^3 - 4
8*a^5*b^4*c + 192*a^6*b^2*c^2)))*(3*b^4 + 30*a^2*c^2 - 20*a*b^2*c))/(4*a^4*
(4*a*c - b^2)^(3/2)) - (((b*((12*a^6*b^8*c^2 - 116*a^7*b^6*c^3 + 348*a^8*b^
4*c^4 - 304*a^9*b^2*c^5)/(a^9*b^4 + 16*a^11*c^2 - 8*a^10*b^2*c) + ((4*a^10*
b^6*c^2 - 32*a^11*b^4*c^3 + 64*a^12*b^2*c^4)*(6*b^8 + 256*a^4*c^4 + 336*a^2
*b^4*c^2 - 576*a^3*b^2*c^3 - 76*a*b^6*c))/(2*(a^9*b^4 + 16*a^11*c^2 - 8*a^1
0*b^2*c)*(4*a^4*b^6 - 256*a^7*c^3 - 48*a^5*b^4*c + 192*a^6*b^2*c^2)))*(3*b^
4 + 30*a^2*c^2 - 20*a*b^2*c))/(4*a^4*(4*a*c - b^2)^(3/2)) + (b*(3*b^4 + 30*
a^2*c^2 - 20*a*b^2*c)*(4*a^10*b^6*c^2 - 32*a^11*b^4*c^3 + 64*a^12*b^2*c^4)*
(6*b^8 + 256*a^4*c^4 + 336*a^2*b^4*c^2 - 576*a^3*b^2*c^3 - 76*a*b^6*c))/(8*
a^4*(4*a*c - b^2)^(3/2)*(a^9*b^4 + 16*a^11*c^2 - 8*a^10*b^2*c)*(4*a^4*b^6 -
 256*a^7*c^3 - 48*a^5*b^4*c + 192*a^6*b^2*c^2)))*(6*b^8 + 256*a^4*c^4 + 336
*a^2*b^4*c^2 - 576*a^3*b^2*c^3 - 76*a*b^6*c))/(2*(4*a^4*b^6 - 256*a^7*c^3 -
 48*a^5*b^4*c + 192*a^6*b^2*c^2)) + (b^3*(3*b^4 + 30*a^2*c^2 - 20*a*b^2*c)^
3*(4*a^10*b^6*c^2 - 32*a^11*b^4*c^3 + 64*a^12*b^2*c^4))/(64*a^12*(4*a*c - b
^2)^(9/2)*(a^9*b^4 + 16*a^11*c^2 - 8*a^10*b^2*c)))*(16*a^12*b^6*(4*a*c - b^
2)^(9/2) - 1024*a^15*c^3*(4*a*c - b^2)^(9/2) - 192*a^13*b^4*c*(4*a*c - b^2)
^(9/2) + 768*a^14*b^2*c^2*(4*a*c - b^2)^(9/2))*(9*b^8 + 80*a^4*c^4 + 270*a^
2*b^4*c^2 - 285*a^3*b^2*c^3 - 87*a*b^6*c))/(8*a^3*c^2*(4*a*c - b^2)^(7/2)*(
9*b^10*c^2 - 120*a*b^8*c^3 + 580*a^2*b^6*c^4 - 1200*a^3*b^4*c^5 + 900*a^4*b
^2*c^6)*(54*b^10 - 1600*a^5*c^5 + 3480*a^2*b^6*c^2 - 7200*a^3*b^4*c^3 + 577
5*a^4*b^2*c^4 - 720*a*b^8*c)) - (3*b*(16*a^12*b^6*(4*a*c - b^2)^(9/2) - 102
4*a^15*c^3*(4*a*c - b^2)^(9/2) - 192*a^13*b^4*c*(4*a*c - b^2)^(9/2) + 768*a

```
^14*b^2*c^2*(4*a*c - b^2)^(9/2))*((((36*a^3*b^8*c^3 - 309*a^4*b^6*c^4 + 778
*a^5*b^4*c^5 - 473*a^6*b^2*c^6)/(a^9*b^4 + 16*a^11*c^2 - 8*a^10*b^2*c) - ((
(12*a^6*b^8*c^2 - 116*a^7*b^6*c^3 + 348*a^8*b^4*c^4 - 304*a^9*b^2*c^5)/(a^9
*b^4 + 16*a^11*c^2 - 8*a^10*b^2*c) + ((4*a^10*b^6*c^2 - 32*a^11*b^4*c^3 + 6
4*a^12*b^2*c^4)*(6*b^8 + 256*a^4*c^4 + 336*a^2*b^4*c^2 - 576*a^3*b^2*c^3 -
76*a*b^6*c))/(2*(a^9*b^4 + 16*a^11*c^2 - 8*a^10*b^2*c)*(4*a^4*b^6 - 256*a^7
*c^3 - 48*a^5*b^4*c + 192*a^6*b^2*c^2)))*(6*b^8 + 256*a^4*c^4 + 336*a^2*b^4
*c^2 - 576*a^3*b^2*c^3 - 76*a*b^6*c))/(2*(4*a^4*b^6 - 256*a^7*c^3 - 48*a^5*
b^4*c + 192*a^6*b^2*c^2)))*(6*b^8 + 256*a^4*c^4 + 336*a^2*b^4*c^2 - 576*a^3
*b^2*c^3 - 76*a*b^6*c))/(2*(4*a^4*b^6 - 256*a^7*c^3 - 48*a^5*b^4*c + 192*a^
6*b^2*c^2)) - (27*b^8*c^4 - 216*a*b^6*c^5 + 495*a^2*b^4*c^6 - 242*a^3*b^2*c
^7)/(a^9*b^4 + 16*a^11*c^2 - 8*a^10*b^2*c) + (b*((b*((12*a^6*b^8*c^2 - 116*
a^7*b^6*c^3 + 348*a^8*b^4*c^4 - 304*a^9*b^2*c^5)/(a^9*b^4 + 16*a^11*c^2 - 8
*a^10*b^2*c) + ((4*a^10*b^6*c^2 - 32*a^11*b^4*c^3 + 64*a^12*b^2*c^4)*(6*b^8
 + 256*a^4*c^4 + 336*a^2*b^4*c^2 - 576*a^3*b^2*c^3 - 76*a*b^6*c))/(2*(a^9*b
^4 + 16*a^11*c^2 - 8*a^10*b^2*c)*(4*a^4*b^6 - 256*a^7*c^3 - 48*a^5*b^4*c +
192*a^6*b^2*c^2)))*(3*b^4 + 30*a^2*c^2 - 20*a*b^2*c))/(4*a^4*(4*a*c - b^2)^
(3/2)) + (b*(3*b^4 + 30*a^2*c^2 - 20*a*b^2*c)*(4*a^10*b^6*c^2 - 32*a^11*b^4
*c^3 + 64*a^12*b^2*c^4)*(6*b^8 + 256*a^4*c^4 + 336*a^2*b^4*c^2 - 576*a^3*b^
2*c^3 - 76*a*b^6*c))/(8*a^4*(4*a*c - b^2)^(3/2)*(a^9*b^4 + 16*a^11*c^2 - 8*
a^10*b^2*c)*(4*a^4*b^6 - 256*a^7*c^3 - 48*a^5*b^4*c + 192*a^6*b^2*c^2)))*(3
*b^4 + 30*a^2*c^2 - 20*a*b^2*c))/(4*a^4*(4*a*c - b^2)^(3/2)) + (b^2*(3*b^4
+ 30*a^2*c^2 - 20*a*b^2*c)^2*(4*a^10*b^6*c^2 - 32*a^11*b^4*c^3 + 64*a^12*b^
2*c^4)*(6*b^8 + 256*a^4*c^4 + 336*a^2*b^4*c^2 - 576*a^3*b^2*c^3 - 76*a*b^6*
c))/(32*a^8*(4*a*c - b^2)^3*(a^9*b^4 + 16*a^11*c^2 - 8*a^10*b^2*c)*(4*a^4*b
^6 - 256*a^7*c^3 - 48*a^5*b^4*c + 192*a^6*b^2*c^2)))*(3*b^6 - 25*a^3*c^3 +
50*a^2*b^2*c^2 - 23*a*b^4*c))/(8*a^3*c^2*(4*a*c - b^2)^3*(9*b^10*c^2 - 120*
a*b^8*c^3 + 580*a^2*b^6*c^4 - 1200*a^3*b^4*c^5 + 900*a^4*b^2*c^6)*(54*b^10
- 1600*a^5*c^5 + 3480*a^2*b^6*c^2 - 7200*a^3*b^4*c^3 + 5775*a^4*b^2*c^4 - 7
20*a*b^8*c)))*(3*b^4 + 30*a^2*c^2 - 20*a*b^2*c))/(2*a^4*(4*a*c - b^2)^(3/2)
) - (log(x)*(2*a*c - 3*b^2))/a^4 - (log(a + b*x^2 + c*x^4)*(6*b^8 + 256*a^4
*c^4 + 336*a^2*b^4*c^2 - 576*a^3*b^2*c^3 - 76*a*b^6*c))/(2*(4*a^4*b^6 - 256
*a^7*c^3 - 48*a^5*b^4*c + 192*a^6*b^2*c^2)) - (1/(4*a) - (3*b*x^2)/(4*a^2)
+ (x^4*(6*b^4 + 8*a^2*c^2 - 25*a*b^2*c))/(4*a^3*(4*a*c - b^2)) - (b*c*x^6*(
11*a*c - 3*b^2))/(2*a^3*(4*a*c - b^2)))/(a*x^4 + b*x^6 + c*x^8)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] Timed out
```

3.104 $\int \frac{x}{\sqrt{ax+bx^3+cx^5}}\,dx$

Optimal. Leaf size=142

$$\frac{2x^2\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3\sqrt{ax+bx^3+cx^5}}$$

```
[Out] 2/3*x^2*AppellF1(3/4,1/2,1/2,7/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^2/(
b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^2/
(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(c*x^5+b*x^3+a*x)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1930, 1141, 510}

$$\frac{2x^2\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3\sqrt{ax+bx^3+cx^5}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a*x + b*x^3 + c*x^5],x]
```

```
[Out] (2*x^2*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b +
Sqrt[b^2 - 4*a*c])]*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b - Sqrt[b^2 -
 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(3*Sqrt[a*x + b*x^3 + c*x^5]
)
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 1930

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Dist[(a*x^q + b*x^n + c*x^(2*n - q))^p/(x^(p*q)*(a + b*x^(n
 - q) + c*x^(2*(n - q)))^p), Int[x^(m + p*q)*(a + b*x^(n - q) + c*x^(2*(n -
 q)))^p, x], x] /; FreeQ[{a, b, c, m, n, p, q}, x] && EqQ[r, 2*n - q] &&  !
IntegerQ[p] && PosQ[n - q]
```

Rubi steps

$$\int \frac{x}{\sqrt{ax+bx^3+cx^5}}\,dx = \frac{\left(\sqrt{x}\sqrt{a+bx^2+cx^4}\right)\int \frac{\sqrt{x}}{\sqrt{a+bx^2+cx^4}}\,dx}{\sqrt{ax+bx^3+cx^5}}$$
$$= \frac{\left(\sqrt{x}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\right)\int \frac{\sqrt{x}}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx}{\sqrt{ax+bx^3+cx^5}}$$
$$= \frac{2x^2\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3\sqrt{ax+bx^3+cx^5}}$$

Mathematica [A] time = 0.09, size = 170, normalized size = 1.20

$$\frac{2x^2\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\,F_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)}{3\sqrt{x\left(a+bx^2+cx^4\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/Sqrt[a*x + b*x^3 + c*x^5],x]
```

```
[Out] (2*x^2*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt
[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/4, 1
/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2
- 4*a*c])])/(3*Sqrt[x*(a + b*x^2 + c*x^4)])
```

fricas [F] time = 0.88, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{cx^5+bx^3+ax}}{cx^4+bx^2+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^5 + b*x^3 + a*x)/(c*x^4 + b*x^2 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{cx^5+bx^3+ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/sqrt(c*x^5 + b*x^3 + a*x), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{c\,x^5+b\,x^3+ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^5+b*x^3+a*x)^(1/2),x)
```

```
[Out] int(x/(c*x^5+b*x^3+a*x)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{cx^5 + bx^3 + ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/sqrt(c*x^5 + b*x^3 + a*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\sqrt{c\,x^5 + b\,x^3 + a\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x + b*x^3 + c*x^5)^(1/2),x)
```

```
[Out] int(x/(a*x + b*x^3 + c*x^5)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{x\left(a + bx^2 + cx^4\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**5+b*x**3+a*x)**(1/2),x)
```

```
[Out] Integral(x/sqrt(x*(a + b*x**2 + c*x**4)), x)
```

3.105 $\int x^{3/2}\sqrt{ax+bx^3+cx^5}\,dx$

Optimal. Leaf size=380

$$-\frac{\sqrt[4]{a}\sqrt{x}\left(\sqrt{a}b\sqrt{c}-6ac+2b^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{30c^{7/4}\sqrt{ax+bx^3+cx^5}}+\frac{2\sqrt[4]{a}\sqrt{x}\left(b^2-3ac\right)\cdots}{\cdots}$$

```
[Out] -2/15*(-3*a*c+b^2)*x^(3/2)*(c*x^4+b*x^2+a)/c^(3/2)/(a^(1/2)+x^2*c^(1/2))/(c
*x^5+b*x^3+a*x)^(1/2)+1/15*(3*c*x^2+b)*x^(1/2)*(c*x^5+b*x^3+a*x)^(1/2)/c+2/
15*a^(1/4)*(-3*a*c+b^2)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*ar
ctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-
b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*x^(1/2)*((c*x^4+b*x^2+a)/(a
^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(7/4)/(c*x^5+b*x^3+a*x)^(1/2)-1/30*a^(1/4)*(
cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*
EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))
*(a^(1/2)+x^2*c^(1/2))*(2*b^2-6*a*c+b*a^(1/2)*c^(1/2))*x^(1/2)*((c*x^4+b*x^
2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(7/4)/(c*x^5+b*x^3+a*x)^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 380, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1919, 1953, 1197, 1103, 1195}

$$-\frac{2x^{3/2}\left(b^2-3ac\right)\left(a+bx^2+cx^4\right)}{15c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{ax+bx^3+cx^5}}-\frac{\sqrt[4]{a}\sqrt{x}\left(\sqrt{a}b\sqrt{c}-6ac+2b^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\cdots\right)\cdots\right)}{30c^{7/4}\sqrt{ax+bx^3+cx^5}}\cdots$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*Sqrt[a*x + b*x^3 + c*x^5],x]
```

```
[Out] (-2*(b^2 - 3*a*c)*x^(3/2)*(a + b*x^2 + c*x^4))/(15*c^(3/2)*(Sqrt[a] + Sqrt[
c]*x^2)*Sqrt[a*x + b*x^3 + c*x^5]) + (Sqrt[x]*(b + 3*c*x^2)*Sqrt[a*x + b*x^
3 + c*x^5])/(15*c) + (2*a^(1/4)*(b^2 - 3*a*c)*Sqrt[x]*(Sqrt[a] + Sqrt[c]*x^
2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(
c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(15*c^(7/4)*Sqrt[a*x + b
*x^3 + c*x^5]) - (a^(1/4)*(2*b^2 + Sqrt[a]*b*Sqrt[c] - 6*a*c)*Sqrt[x]*(Sqrt
[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Elli
pticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(30*c^(7
/4)*Sqrt[a*x + b*x^3 + c*x^5])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1919

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(x^(m - n + q + 1)*(b*(n - q)*p + c*(m + p*q + (n - q)
*(2*p - 1) + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(c*(m + p*(2*
n - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1)), x] + Dist[((n - q)*p)/(c*(m
 + p*(2*n - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1)), Int[x^(m - (n - 2*q
))*Simp[-(a*b*(m + p*q - n + q + 1)) + (2*a*c*(m + p*q + (n - q)*(2*p - 1)
+ 1) - b^2*(m + p*q + (n - q)*(p - 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n +
c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] &&
 PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p
, 0] && RationalQ[m, q] && GtQ[m + p*q + 1, n - q] && NeQ[m + p*(2*n - q) +
 1, 0] && NeQ[m + p*q + (n - q)*(2*p - 1) + 1, 0]
```

Rule 1953

```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(j_.)))/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x
_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol] :> Dist[(x^(q/2)*Sqrt[a + b*x^(n -
q) + c*x^(2*(n - q))])/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)], Int[(x^(m - q/2
)*(A + B*x^(n - q)))/Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))], x], x] /; Fre
eQ[{a, b, c, A, B, m, n, q}, x] && EqQ[j, n - q] && EqQ[r, 2*n - q] && PosQ
[n - q] && (EqQ[m, 1/2] || EqQ[m, -2^(-1)]) && EqQ[n, 3] && EqQ[q, 1]
```

Rubi steps

$$\int x^{3/2}\sqrt{ax+bx^3+cx^5}\,dx = \frac{\sqrt{x}\left(b+3cx^2\right)\sqrt{ax+bx^3+cx^5}}{15c} + \frac{\int \frac{\sqrt{x}\left(-ab-2\left(b^2-3ac\right)x^2\right)}{\sqrt{ax+bx^3+cx^5}}\,dx}{15c}$$

$$= \frac{\sqrt{x}\left(b+3cx^2\right)\sqrt{ax+bx^3+cx^5}}{15c} + \frac{\left(\sqrt{x}\sqrt{a+bx^2+cx^4}\right)\int \frac{-ab-2\left(b^2-3ac\right)x^2}{\sqrt{a+bx^2+cx^4}}\,dx}{15c\sqrt{ax+bx^3+cx^5}}$$

$$= \frac{\sqrt{x}\left(b+3cx^2\right)\sqrt{ax+bx^3+cx^5}}{15c} + \frac{\left(2\sqrt{a}\left(b^2-3ac\right)\sqrt{x}\sqrt{a+bx^2+cx^4}\right)\int \frac{1-\ldots}{\sqrt{a+b\ldots}}}{15c^{3/2}\sqrt{ax+bx^3+cx^5}}$$

$$= -\frac{2\left(b^2-3ac\right)x^{3/2}\left(a+bx^2+cx^4\right)}{15c^{3/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{ax+bx^3+cx^5}} + \frac{\sqrt{x}\left(b+3cx^2\right)\sqrt{ax+bx^3+cx^5}}{15c} + \frac{2\sqrt[4]{\ldots}}{\ldots}$$

Mathematica [C] time = 1.48, size = 486, normalized size = 1.28

$$\sqrt{x}\left(-i\left(b^2-3ac\right)\left(\sqrt{b^2-4ac}-b\right)\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{-2\sqrt{b^2-4ac}+2b+4cx^2}{b-\sqrt{b^2-4ac}}}\,E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right)\middle|\frac{b+\sqrt{b^2\ldots}}{b-\sqrt{b^2\ldots}}\right.\right.\ldots$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*Sqrt[a*x + b*x^3 + c*x^5],x]
```

```
[Out] (Sqrt[x]*(2*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(b + 3*c*x^2)*(a + b*x^2 +
c*x^4) - I*(b^2 - 3*a*c)*(-b + Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*
c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*
c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqr
t[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + I*(
-b^3 + 4*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - 3*a*c*Sqrt[b^2 - 4*a*c])*Sqrt[(b +
 Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b
^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]
*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2
- 4*a*c])]))/(30*c^2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[x*(a + b*x^2 + c*
x^4)])
```

fricas [F] time = 0.88, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{cx^5+bx^3+ax}\,x^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^5 + b*x^3 + a*x)*x^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx^5+bx^3+ax}\,x^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^5 + b*x^3 + a*x)*x^(3/2), x)
```

maple [B] time = 0.05, size = 1042, normalized size = 2.74

$$-\frac{\sqrt{\left(c\,x^4+b\,x^2+a\right)x}\,\left(-6\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\;b\,c^2x^7-6\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\;\sqrt{-4ac+b^2}\;c^2x^7-8\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\;b^2c\,x^5-8\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(c*x^5+b*x^3+a*x)^(1/2),x)
```

```
[Out] -1/30*(x*(c*x^4+b*x^2+a))^(1/2)*(-6*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x^7*b
*c^2-6*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-4*a*c+b^2)^(1/2)*x^7*c^2-8*((-b+
(-4*a*c+b^2)^(1/2))/a)^(1/2)*x^5*b^2*c-8*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*
(-4*a*c+b^2)^(1/2)*x^5*b*c-6*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x^3*a*b*c-6*
((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-4*a*c+b^2)^(1/2)*x^3*a*c-2*((-b+(-4*a*c
+b^2)^(1/2))/a)^(1/2)*x^3*b^3-2*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-4*a*c+b
^2)^(1/2)*x^3*b^2+12*(-2*(x^2*(-4*a*c+b^2)^(1/2)-b*x^2-2*a)/a)^(1/2)*((x^2*
(-4*a*c+b^2)^(1/2)+b*x^2+2*a)/a)^(1/2)*EllipticF(1/2*2^(1/2)*((-b+(-4*a*c+b
^2)^(1/2))/a)^(1/2)*x,1/2*2^(1/2)*(((-4*a*c+b^2)^(1/2)*b-2*a*c+b^2)/a/c)^(1
/2))*a^2*c-3*b^2*a*(-2*(x^2*(-4*a*c+b^2)^(1/2)-b*x^2-2*a)/a)^(1/2)*((x^2*(-
4*a*c+b^2)^(1/2)+b*x^2+2*a)/a)^(1/2)*EllipticF(1/2*2^(1/2)*((-b+(-4*a*c+b^2
)^(1/2))/a)^(1/2)*x,1/2*2^(1/2)*(((-4*a*c+b^2)^(1/2)*b-2*a*c+b^2)/a/c)^(1/2
))+b*a*(-2*(x^2*(-4*a*c+b^2)^(1/2)-b*x^2-2*a)/a)^(1/2)*((x^2*(-4*a*c+b^2)^(
1/2)+b*x^2+2*a)/a)^(1/2)*EllipticF(1/2*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^
(1/2)*x,1/2*2^(1/2)*(((-4*a*c+b^2)^(1/2)*b-2*a*c+b^2)/a/c)^(1/2))*(-4*a*c+b
^2)^(1/2)-12*(-2*(x^2*(-4*a*c+b^2)^(1/2)-b*x^2-2*a)/a)^(1/2)*((x^2*(-4*a*c+
```

```
b^2)^(1/2)+b*x^2+2*a)/a)^(1/2)*EllipticE(1/2*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2
))/a)^(1/2)*x,1/2*2^(1/2)*(((-4*a*c+b^2)^(1/2)*b-2*a*c+b^2)/a/c)^(1/2))*a^2
*c+4*(-2*(x^2*(-4*a*c+b^2)^(1/2)-b*x^2-2*a)/a)^(1/2)*((x^2*(-4*a*c+b^2)^(1/
2)+b*x^2+2*a)/a)^(1/2)*EllipticE(1/2*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1
/2)*x,1/2*2^(1/2)*(((-4*a*c+b^2)^(1/2)*b-2*a*c+b^2)/a/c)^(1/2))*a*b^2-2*((-
b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x*a*b^2-2*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*
(-4*a*c+b^2)^(1/2)*x*a*b)/x^(1/2)/(c*x^4+b*x^2+a)/c/((-b+(-4*a*c+b^2)^(1/2)
)/a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx^5 + bx^3 + ax}\, x^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^5 + b*x^3 + a*x)*x^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^{3/2}\, \sqrt{c\,x^5 + b\,x^3 + a\,x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(a*x + b*x^3 + c*x^5)^(1/2),x)
[Out] int(x^(3/2)*(a*x + b*x^3 + c*x^5)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{\frac{3}{2}} \sqrt{x \left(a + bx^2 + cx^4\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(c*x**5+b*x**3+a*x)**(1/2),x)
[Out] Integral(x**(3/2)*sqrt(x*(a + b*x**2 + c*x**4)), x)
```

3.106 $\int \sqrt{x}\sqrt{ax+bx^3+cx^5}\,dx$

Optimal. Leaf size=129

$$\frac{\left(b+2cx^2\right)\sqrt{ax+bx^3+cx^5}}{8c\sqrt{x}}-\frac{\sqrt{x}\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}\,\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c^{3/2}\sqrt{ax+bx^3+cx^5}}$$

```
[Out] -1/16*(-4*a*c+b^2)*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))*x
^(1/2)*(c*x^4+b*x^2+a)^(1/2)/c^(3/2)/(c*x^5+b*x^3+a*x)^(1/2)+1/8*(2*c*x^2+b
)*(c*x^5+b*x^3+a*x)^(1/2)/c/x^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1918, 1914, 1107, 621, 206}

$$\frac{\left(b+2cx^2\right)\sqrt{ax+bx^3+cx^5}}{8c\sqrt{x}}-\frac{\sqrt{x}\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}\,\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c^{3/2}\sqrt{ax+bx^3+cx^5}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*Sqrt[a*x + b*x^3 + c*x^5],x]
```

```
[Out] ((b + 2*c*x^2)*Sqrt[a*x + b*x^3 + c*x^5])/(8*c*Sqrt[x]) - ((b^2 - 4*a*c)*Sq
rt[x]*Sqrt[a + b*x^2 + c*x^4]*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x
^2 + c*x^4])])/(16*c^(3/2)*Sqrt[a*x + b*x^3 + c*x^5])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1914

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))])/Sqrt[
a*x^q + b*x^n + c*x^(2*n - q)], Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

Rule 1918

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(x^(m - n + q + 1)*(b + 2*c*x^(n - q))*(a*x^q + b*x^n
+ c*x^(2*n - q))^p)/(2*c*(n - q)*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2
*c*(2*p + 1)), Int[x^(m + q)*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x
```

```
] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && !IntegerQ[p]
 && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && Eq
Q[m + p*q + 1, n - q]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{x}\sqrt{ax+bx^3+cx^5}\,dx &= \frac{\left(b+2cx^2\right)\sqrt{ax+bx^3+cx^5}}{8c\sqrt{x}}-\frac{\left(b^2-4ac\right)\int \frac{x^{3/2}}{\sqrt{ax+bx^3+cx^5}}\,dx}{8c}\\
&= \frac{\left(b+2cx^2\right)\sqrt{ax+bx^3+cx^5}}{8c\sqrt{x}}-\frac{\left(\left(b^2-4ac\right)\sqrt{x}\sqrt{a+bx^2+cx^4}\right)\int \frac{x}{\sqrt{a+bx^2+cx^4}}\,dx}{8c\sqrt{ax+bx^3+cx^5}}\\
&= \frac{\left(b+2cx^2\right)\sqrt{ax+bx^3+cx^5}}{8c\sqrt{x}}-\frac{\left(\left(b^2-4ac\right)\sqrt{x}\sqrt{a+bx^2+cx^4}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+bx+\cdots}}\right.}{16c\sqrt{ax+bx^3+cx^5}}\\
&= \frac{\left(b+2cx^2\right)\sqrt{ax+bx^3+cx^5}}{8c\sqrt{x}}-\frac{\left(\left(b^2-4ac\right)\sqrt{x}\sqrt{a+bx^2+cx^4}\right)\mathrm{Subst}\left(\int \frac{1}{4c-x^2}\,d\cdots\right.}{8c\sqrt{ax+bx^3+cx^5}}\\
&= \frac{\left(b+2cx^2\right)\sqrt{ax+bx^3+cx^5}}{8c\sqrt{x}}-\frac{\left(b^2-4ac\right)\sqrt{x}\sqrt{a+bx^2+cx^4}\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2\cdots}}\right.}{16c^{3/2}\sqrt{ax+bx^3+cx^5}}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 126, normalized size = 0.98

$$\frac{\sqrt{x\left(a+bx^2+cx^4\right)}\left(\frac{\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{4c}-\frac{\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{8c^{3/2}}\right)}{2\sqrt{x}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*Sqrt[a*x + b*x^3 + c*x^5],x]
```

```
[Out] (Sqrt[x*(a + b*x^2 + c*x^4)]*(((b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(4*c)
 - ((b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])
])/(8*c^(3/2))))/(2*Sqrt[x]*Sqrt[a + b*x^2 + c*x^4])
```

fricas [A] time = 0.82, size = 232, normalized size = 1.80

$$\left[-\frac{\left(b^2-4\,ac\right)\sqrt{c}\,x\log\left(-\frac{8\,c^2x^5+8\,bcx^3+4\,\sqrt{cx^5+bx^3+ax}\left(2\,cx^2+b\right)\sqrt{c}\,\sqrt{x}+\left(b^2+4\,ac\right)x}{x}\right)-4\,\sqrt{cx^5+bx^3+ax}\left(2\,c^2x^2+bc\right)\sqrt{x}\cdots}{32\,c^2x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/32*((b^2 - 4*a*c)*sqrt(c)*x*log(-(8*c^2*x^5 + 8*b*c*x^3 + 4*sqrt(c*x^5
+ b*x^3 + a*x)*(2*c*x^2 + b)*sqrt(c)*sqrt(x) + (b^2 + 4*a*c)*x)/x) - 4*sqrt
(c*x^5 + b*x^3 + a*x)*(2*c^2*x^2 + b*c)*sqrt(x))/(c^2*x), 1/16*((b^2 - 4*a*
c)*sqrt(-c)*x*arctan(1/2*sqrt(c*x^5 + b*x^3 + a*x)*(2*c*x^2 + b)*sqrt(-c)*s
qrt(x)/(c^2*x^5 + b*c*x^3 + a*c*x)) + 2*sqrt(c*x^5 + b*x^3 + a*x)*(2*c^2*x^
2 + b*c)*sqrt(x))/(c^2*x)]
```

giac [A] time = 0.97, size = 127, normalized size = 0.98

$$\frac{1}{8}\sqrt{cx^4+bx^2+a}\left(2x^2+\frac{b}{c}\right)+\frac{\left(b^2-4ac\right)\log\left(\left|-2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}-b\right|\right)}{16c^{\frac{3}{2}}}-\frac{b^2\log\left(\left|-b+2\sqrt{a}\sqrt{c}\right|\right)-\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(c*x^4 + b*x^2 + a)*(2*x^2 + b/c) + 1/16*(b^2 - 4*a*c)*log(abs(-2*(
sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) - b))/c^(3/2) - 1/16*(b^2*lo
g(abs(-b + 2*sqrt(a)*sqrt(c))) - 4*a*c*log(abs(-b + 2*sqrt(a)*sqrt(c))) + 2
*sqrt(a)*b*sqrt(c))/c^(3/2)
```

maple [A] time = 0.01, size = 157, normalized size = 1.22

$$\frac{\sqrt{\left(cx^4+bx^2+a\right)x}\left(4\sqrt{cx^4+bx^2+a}\,c^{\frac{3}{2}}x^2+4ac\ln\left(\frac{2cx^2+b+2\sqrt{cx^4+bx^2+a}\,\sqrt{c}}{2\sqrt{c}}\right)-b^2\ln\left(\frac{2cx^2+b+2\sqrt{cx^4+bx^2+a}\,\sqrt{c}}{2\sqrt{c}}\right)+\cdots\right.}{16\sqrt{cx^4+bx^2+a}\,c^{\frac{3}{2}}\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(c*x^5+b*x^3+a*x)^(1/2),x)
```

```
[Out] 1/16*((c*x^4+b*x^2+a)*x)^(1/2)/c^(3/2)*(4*x^2*c^(3/2)*(c*x^4+b*x^2+a)^(1/2)
+4*ln(1/2*(2*c*x^2+2*(c*x^4+b*x^2+a)^(1/2)*c^(1/2)+b)/c^(1/2))*a*c-ln(1/2*(
2*c*x^2+2*(c*x^4+b*x^2+a)^(1/2)*c^(1/2)+b)/c^(1/2))*b^2+2*b*(c*x^4+b*x^2+a)
^(1/2)*c^(1/2))/x^(1/2)/(c*x^4+b*x^2+a)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{cx^5+bx^3+ax}\sqrt{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^5 + b*x^3 + a*x)*sqrt(x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\sqrt{x}\,\sqrt{c\,x^5+b\,x^3+a\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(a*x + b*x^3 + c*x^5)^(1/2),x)
```

```
[Out] int(x^(1/2)*(a*x + b*x^3 + c*x^5)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{x}\sqrt{x\left(a+bx^2+cx^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(c*x**5+b*x**3+a*x)**(1/2),x)
```

```
[Out] Integral(sqrt(x)*sqrt(x*(a + b*x**2 + c*x**4)), x)
```

3.107 $\int \frac{\sqrt{ax+bx^3+cx^5}}{\sqrt{x}}\,dx$

Optimal. Leaf size=347

$$\frac{\sqrt[4]{a}\sqrt{x}\left(2\sqrt{a}\sqrt{c}+b\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{6c^{3/4}\sqrt{ax+bx^3+cx^5}}-\frac{\sqrt[4]{a}\,b\sqrt{x}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\cdots}}{3\cdots}$$

```
[Out] 1/3*b*x^(3/2)*(c*x^4+b*x^2+a)/c^(1/2)/(a^(1/2)+x^2*c^(1/2))/(c*x^5+b*x^3+a*
x)^(1/2)+1/3*x^(1/2)*(c*x^5+b*x^3+a*x)^(1/2)-1/3*a^(1/4)*b*(cos(2*arctan(c^
(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2
*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c
^(1/2))*x^(1/2)*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(3/4)/(c*
x^5+b*x^3+a*x)^(1/2)+1/6*a^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)
/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))
),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*(b+2*a^(1/2)*c^(1/
2))*x^(1/2)*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(3/4)/(c*x^5+
b*x^3+a*x)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 347, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1921, 1953, 1197, 1103, 1195}

$$\frac{\sqrt[4]{a}\sqrt{x}\left(2\sqrt{a}\sqrt{c}+b\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{6c^{3/4}\sqrt{ax+bx^3+cx^5}}-\frac{\sqrt[4]{a}\,b\sqrt{x}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\cdots}}{3\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x + b*x^3 + c*x^5]/Sqrt[x],x]
```

```
[Out] (b*x^(3/2)*(a + b*x^2 + c*x^4))/(3*Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[a*x
 + b*x^3 + c*x^5]) + (Sqrt[x]*Sqrt[a*x + b*x^3 + c*x^5])/3 - (a^(1/4)*b*Sqr
t[x]*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^
2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4]
)/(3*c^(3/4)*Sqrt[a*x + b*x^3 + c*x^5]) + (a^(1/4)*(b + 2*Sqrt[a]*Sqrt[c])*
Sqrt[x]*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]
*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))
/4])/(6*c^(3/4)*Sqrt[a*x + b*x^3 + c*x^5])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1921

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(x^(m + 1)*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(m + p*(
2*n - q) + 1), x] + Dist[((n - q)*p)/(m + p*(2*n - q) + 1), Int[x^(m + q)*(
2*a + b*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ
[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^
2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && GtQ[m + p*q
+ 1, -(n - q)] && NeQ[m + p*(2*n - q) + 1, 0]
```

Rule 1953

```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(j_.)))/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x
_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol] :> Dist[(x^(q/2)*Sqrt[a + b*x^(n -
q) + c*x^(2*(n - q))])/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)], Int[(x^(m - q/2
)*(A + B*x^(n - q)))/Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))], x], x] /; Fre
eQ[{a, b, c, A, B, m, n, q}, x] && EqQ[j, n - q] && EqQ[r, 2*n - q] && PosQ
[n - q] && (EqQ[m, 1/2] || EqQ[m, -2^(-1)]) && EqQ[n, 3] && EqQ[q, 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{ax+bx^3+cx^5}}{\sqrt{x}}\,dx &= \frac{1}{3}\sqrt{x}\,\sqrt{ax+bx^3+cx^5}+\frac{1}{3}\int \frac{\sqrt{x}\left(2a+bx^2\right)}{\sqrt{ax+bx^3+cx^5}}\,dx\\
&= \frac{1}{3}\sqrt{x}\,\sqrt{ax+bx^3+cx^5}+\frac{\left(\sqrt{x}\,\sqrt{a+bx^2+cx^4}\right)\int \frac{2a+bx^2}{\sqrt{a+bx^2+cx^4}}\,dx}{3\sqrt{ax+bx^3+cx^5}}\\
&= \frac{1}{3}\sqrt{x}\,\sqrt{ax+bx^3+cx^5}+\frac{\left(\sqrt{a}\left(2\sqrt{a}+\frac{b}{\sqrt{c}}\right)\sqrt{x}\,\sqrt{a+bx^2+cx^4}\right)\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx}{3\sqrt{ax+bx^3+cx^5}}-\frac{\left(\sqrt{a}\cdots\right.}{\cdots}\\
&= \frac{bx^{3/2}\left(a+bx^2+cx^4\right)}{3\sqrt{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{ax+bx^3+cx^5}}+\frac{1}{3}\sqrt{x}\,\sqrt{ax+bx^3+cx^5}-\frac{\sqrt[4]{a}\,b\sqrt{x}\left(\sqrt{a}+\sqrt{c}\,x^2\cdots\right.}{\cdots}
\end{aligned}
$$

Mathematica [C] time = 0.99, size = 452, normalized size = 1.30

$$
\frac{\sqrt{x}\left(-i\left(b\sqrt{b^2-4ac}+4ac-b^2\right)\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{-2\sqrt{b^2-4ac}+2b+4cx^2}{b-\sqrt{b^2-4ac}}}\,F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)\cdots\right.}{12c\sqrt{\cdots}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x + b*x^3 + c*x^5]/Sqrt[x],x]
```

```
[Out] (Sqrt[x]*(4*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(a + b*x^2 + c*x^4) + I*b*(
-b + Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^
2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a
*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b +
```

```
Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - I*(-b^2 + 4*a*c + b*Sqrt[b^2
- 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*S
qrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Elliptic
F[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a
*c])/(b - Sqrt[b^2 - 4*a*c])]))/(12*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[
x*(a + b*x^2 + c*x^4)])
```

fricas [F] time = 0.92, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^5+bx^3+ax}}{\sqrt{x}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)^(1/2)/x^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(c*x^5 + b*x^3 + a*x)/sqrt(x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^5+bx^3+ax}}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)^(1/2)/x^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^5 + b*x^3 + a*x)/sqrt(x), x)
```

maple [A] time = 0.02, size = 508, normalized size = 1.46

$$\frac{\sqrt{\left(c\,x^4+b\,x^2+a\right)x}\,\left(\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,bc\,x^5+\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{-4ac+b^2}\,c\,x^5+\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,b^2x^3+\sqrt{\frac{-b+\sqrt{}}{}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^5+b*x^3+a*x)^(1/2)/x^(1/2),x)
[Out] 1/3*((c*x^4+b*x^2+a)*x)^(1/2)/x^(1/2)*(((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-
4*a*c+b^2)^(1/2)*x^5*c+((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x^5*b*c+((-b+(-4*a
*c+b^2)^(1/2))/a)^(1/2)*(-4*a*c+b^2)^(1/2)*x^3*b+((-b+(-4*a*c+b^2)^(1/2))/a
)^(1/2)*x^3*b^2+a*(-2*(-b*x^2+(-4*a*c+b^2)^(1/2)*x^2-2*a)/a)^(1/2)*((b*x^2+
(-4*a*c+b^2)^(1/2)*x^2+2*a)/a)^(1/2)*EllipticF(1/2*2^(1/2)*((-b+(-4*a*c+b^2
)^(1/2))/a)^(1/2)*x,1/2*2^(1/2)*((-2*a*c+b^2+(-4*a*c+b^2)^(1/2)*b)/a/c)^(1/
2))*(-4*a*c+b^2)^(1/2)+b*a*(-2*(-b*x^2+(-4*a*c+b^2)^(1/2)*x^2-2*a)/a)^(1/2)
*((b*x^2+(-4*a*c+b^2)^(1/2)*x^2+2*a)/a)^(1/2)*EllipticE(1/2*2^(1/2)*((-b+(-
4*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*2^(1/2)*((-2*a*c+b^2+(-4*a*c+b^2)^(1/2)*b)
/a/c)^(1/2))+((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-4*a*c+b^2)^(1/2)*x*a+((-b+
(-4*a*c+b^2)^(1/2))/a)^(1/2)*x*a*b)/(c*x^4+b*x^2+a)/((-b+(-4*a*c+b^2)^(1/2)
)/a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^5+bx^3+ax}}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)^(1/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^5 + b*x^3 + a*x)/sqrt(x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^5+b\,x^3+a\,x}}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^3 + c*x^5)^(1/2)/x^(1/2),x)
```

```
[Out] int((a*x + b*x^3 + c*x^5)^(1/2)/x^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x\left(a+bx^2+cx^4\right)}}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**5+b*x**3+a*x)**(1/2)/x**(1/2),x)
```

```
[Out] Integral(sqrt(x*(a + b*x**2 + c*x**4))/sqrt(x), x)
```

3.108 $\int \frac{\sqrt{ax+bx^3+cx^5}}{x^{3/2}} \, dx$

Optimal. Leaf size=194

$$\frac{\sqrt{ax+bx^3+cx^5}}{2\sqrt{x}}-\frac{\sqrt{a}\sqrt{x}\sqrt{a+bx^2+cx^4}\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{ax+bx^3+cx^5}}+\frac{b\sqrt{x}\sqrt{a+bx^2+cx^4}\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+c}}\right.}{4\sqrt{c}\sqrt{ax+bx^3+cx^5}}$$

```
[Out] -1/2*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))*a^(1/2)*x^(1/2)
*(c*x^4+b*x^2+a)^(1/2)/(c*x^5+b*x^3+a*x)^(1/2)+1/4*b*arctanh(1/2*(2*c*x^2+b
)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))*x^(1/2)*(c*x^4+b*x^2+a)^(1/2)/c^(1/2)/(c*x
^5+b*x^3+a*x)^(1/2)+1/2*(c*x^5+b*x^3+a*x)^(1/2)/x^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1921, 1953, 1251, 843, 621, 206, 724}

$$\frac{\sqrt{ax+bx^3+cx^5}}{2\sqrt{x}}-\frac{\sqrt{a}\sqrt{x}\sqrt{a+bx^2+cx^4}\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{ax+bx^3+cx^5}}+\frac{b\sqrt{x}\sqrt{a+bx^2+cx^4}\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+c}}\right.}{4\sqrt{c}\sqrt{ax+bx^3+cx^5}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x + b*x^3 + c*x^5]/x^(3/2),x]
```

```
[Out] Sqrt[a*x + b*x^3 + c*x^5]/(2*Sqrt[x]) - (Sqrt[a]*Sqrt[x]*Sqrt[a + b*x^2 + c
*x^4]*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(2*Sqrt[a
*x + b*x^3 + c*x^5]) + (b*Sqrt[x]*Sqrt[a + b*x^2 + c*x^4]*ArcTanh[(b + 2*c*
x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(4*Sqrt[c]*Sqrt[a*x + b*x^3 + c*
x^5])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1921

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(x^(m + 1)*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(m + p*(
2*n - q) + 1), x] + Dist[((n - q)*p)/(m + p*(2*n - q) + 1), Int[x^(m + q)*(
2*a + b*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ
[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^
2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && GtQ[m + p*q
+ 1, -(n - q)] && NeQ[m + p*(2*n - q) + 1, 0]
```

Rule 1953

```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(j_.)))/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x
_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol] :> Dist[(x^(q/2)*Sqrt[a + b*x^(n -
q) + c*x^(2*(n - q))])/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)], Int[(x^(m - q/2
)*(A + B*x^(n - q)))/Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))], x], x] /; Fre
eQ[{a, b, c, A, B, m, n, q}, x] && EqQ[j, n - q] && EqQ[r, 2*n - q] && PosQ
[n - q] && (EqQ[m, 1/2] || EqQ[m, -2^(-1)]) && EqQ[n, 3] && EqQ[q, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{ax+bx^3+cx^5}}{x^{3/2}}\,dx &= \frac{\sqrt{ax+bx^3+cx^5}}{2\sqrt{x}}+\frac{1}{2}\int \frac{2a+bx^2}{\sqrt{x}\sqrt{ax+bx^3+cx^5}}\,dx\\
&= \frac{\sqrt{ax+bx^3+cx^5}}{2\sqrt{x}}+\frac{\left(\sqrt{x}\sqrt{a+bx^2+cx^4}\right)\int \frac{2a+bx^2}{x\sqrt{a+bx^2+cx^4}}\,dx}{2\sqrt{ax+bx^3+cx^5}}\\
&= \frac{\sqrt{ax+bx^3+cx^5}}{2\sqrt{x}}+\frac{\left(\sqrt{x}\sqrt{a+bx^2+cx^4}\right)\text{Subst}\left(\int \frac{2a+bx}{x\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)}{4\sqrt{ax+bx^3+cx^5}}\\
&= \frac{\sqrt{ax+bx^3+cx^5}}{2\sqrt{x}}+\frac{\left(a\sqrt{x}\sqrt{a+bx^2+cx^4}\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)}{2\sqrt{ax+bx^3+cx^5}}+\frac{\left(b\sqrt{x}\sqrt{a\right.}{}}{}\\
&= \frac{\sqrt{ax+bx^3+cx^5}}{2\sqrt{x}}-\frac{\left(a\sqrt{x}\sqrt{a+bx^2+cx^4}\right)\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx,x,\frac{2a+bx^2}{\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{ax+bx^3+cx^5}}+\frac{\left(b\sqrt{x}\right.}{}\\
&= \frac{\sqrt{ax+bx^3+cx^5}}{2\sqrt{x}}-\frac{\sqrt{a}\sqrt{x}\sqrt{a+bx^2+cx^4}\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{ax+bx^3+cx^5}}+\frac{b\sqrt{x}\sqrt{a+bx^2+}}{4\sqrt{c}}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 155, normalized size = 0.80

$$\frac{\sqrt{x}\sqrt{a+bx^2+cx^4}\left(2\sqrt{c}\sqrt{a+bx^2+cx^4}-2\sqrt{a}\sqrt{c}\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)+b\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)\right)}{4\sqrt{c}\sqrt{x\left(a+bx^2+cx^4\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x + b*x^3 + c*x^5]/x^(3/2),x]
```

```
[Out] (Sqrt[x]*Sqrt[a + b*x^2 + c*x^4]*(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4] - 2*Sqr
t[a]*Sqrt[c]*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])] + b
*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])]))/(4*Sqrt[c]*Sq
rt[x*(a + b*x^2 + c*x^4)])
```

fricas [A] time = 0.96, size = 666, normalized size = 3.43

$$\left[\frac{b\sqrt{c}\,x\log\left(-\frac{8\,c^2x^5+8\,bcx^3+4\,\sqrt{cx^5+bx^3+ax}\left(2\,cx^2+b\right)\sqrt{c}\,\sqrt{x}+\left(b^2+4\,ac\right)x}{x}\right)+2\,\sqrt{a}\,cx\log\left(-\frac{\left(b^2+4\,ac\right)x^5+8\,abx^3+8\,a^2x-4\,\sqrt{cx^5+bx^3+a}}{x^5}\right.}{8\,cx}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)^(1/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(b*sqrt(c)*x*log(-(8*c^2*x^5 + 8*b*c*x^3 + 4*sqrt(c*x^5 + b*x^3 + a*x)
*(2*c*x^2 + b)*sqrt(c)*sqrt(x) + (b^2 + 4*a*c)*x)/x) + 2*sqrt(a)*c*x*log(-(
(b^2 + 4*a*c)*x^5 + 8*a*b*x^3 + 8*a^2*x - 4*sqrt(c*x^5 + b*x^3 + a*x)*(b*x^
2 + 2*a)*sqrt(a)*sqrt(x))/x^5) + 4*sqrt(c*x^5 + b*x^3 + a*x)*c*sqrt(x))/(c*
x), -1/4*(b*sqrt(-c)*x*arctan(1/2*sqrt(c*x^5 + b*x^3 + a*x)*(2*c*x^2 + b)*s
qrt(-c)*sqrt(x)/(c^2*x^5 + b*c*x^3 + a*c*x)) - sqrt(a)*c*x*log(-((b^2 + 4*a
*c)*x^5 + 8*a*b*x^3 + 8*a^2*x - 4*sqrt(c*x^5 + b*x^3 + a*x)*(b*x^2 + 2*a)*s
qrt(a)*sqrt(x))/x^5) - 2*sqrt(c*x^5 + b*x^3 + a*x)*c*sqrt(x))/(c*x), 1/8*(4
*sqrt(-a)*c*x*arctan(1/2*sqrt(c*x^5 + b*x^3 + a*x)*(b*x^2 + 2*a)*sqrt(-a)*s
qrt(x)/(a*c*x^5 + a*b*x^3 + a^2*x)) + b*sqrt(c)*x*log(-(8*c^2*x^5 + 8*b*c*x
^3 + 4*sqrt(c*x^5 + b*x^3 + a*x)*(2*c*x^2 + b)*sqrt(c)*sqrt(x) + (b^2 + 4*a
*c)*x)/x) + 4*sqrt(c*x^5 + b*x^3 + a*x)*c*sqrt(x))/(c*x), 1/4*(2*sqrt(-a)*c
*x*arctan(1/2*sqrt(c*x^5 + b*x^3 + a*x)*(b*x^2 + 2*a)*sqrt(-a)*sqrt(x)/(a*c
*x^5 + a*b*x^3 + a^2*x)) - b*sqrt(-c)*x*arctan(1/2*sqrt(c*x^5 + b*x^3 + a*x
)*(2*c*x^2 + b)*sqrt(-c)*sqrt(x)/(c^2*x^5 + b*c*x^3 + a*c*x)) + 2*sqrt(c*x^
5 + b*x^3 + a*x)*c*sqrt(x))/(c*x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)^(1/2)/x^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]index.cc index_m operator + Error: Bad Argument Value
```

maple [A] time = 0.01, size = 136, normalized size = 0.70

$$-\frac{\sqrt{\left(c\,x^4+b\,x^2+a\right)x}\,\left(2\sqrt{a}\,\sqrt{c}\,\ln\left(\frac{b\,x^2+2a+2\sqrt{c\,x^4+b\,x^2+a}\,\sqrt{a}}{x^2}\right)-b\ln\left(\frac{2c\,x^2+b+2\sqrt{c\,x^4+b\,x^2+a}\,\sqrt{c}}{2\sqrt{c}}\right)-2\sqrt{c\,x^4+b\,x^2+}\right.}{4\sqrt{c\,x^4+b\,x^2+a}\,\sqrt{c}\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^5+b*x^3+a*x)^(1/2)/x^(3/2),x)
```

```
[Out] -1/4*((c*x^4+b*x^2+a)*x)^(1/2)/x^(1/2)*(2*a^(1/2)*ln((b*x^2+2*a+2*(c*x^4+b*
x^2+a)^(1/2)*a^(1/2))/x^2)*c^(1/2)-2*(c*x^4+b*x^2+a)^(1/2)*c^(1/2)-b*ln(1/2
*(2*c*x^2+b+2*(c*x^4+b*x^2+a)^(1/2)*c^(1/2))/c^(1/2)))/(c*x^4+b*x^2+a)^(1/2
)/c^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^5 + bx^3 + ax}}{x^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)^(1/2)/x^(3/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^5 + b*x^3 + a*x)/x^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x^5 + b\,x^3 + a\,x}}{x^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^3 + c*x^5)^(1/2)/x^(3/2),x)
[Out] int((a*x + b*x^3 + c*x^5)^(1/2)/x^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x\left(a + bx^2 + cx^4\right)}}{x^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**5+b*x**3+a*x)**(1/2)/x**(3/2),x)
[Out] Integral(sqrt(x*(a + b*x**2 + c*x**4))/x**(3/2), x)
```

3.109 $\int x^{3/2} \left(ax+bx^3+cx^5\right)^{3/2} dx$

Optimal. Leaf size=244

$$\frac{\left(128a^2c^2-100ab^2c+15b^4\right)\sqrt{ax+bx^3+cx^5}}{1280c^3\sqrt{x}}-\frac{3b\sqrt{x}\left(b^2-4ac\right)^2\sqrt{a+bx^2+cx^4}\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{512c^{7/2}\sqrt{ax+bx^3+cx^5}}-\frac{x^{3/}}{}$$

```
[Out] 1/80*(8*c*x^2+3*b)*(c*x^5+b*x^3+a*x)^(3/2)*x^(1/2)/c-3/512*b*(-4*a*c+b^2)^2
*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))*x^(1/2)*(c*x^4+b*x^
2+a)^(1/2)/c^(7/2)/(c*x^5+b*x^3+a*x)^(1/2)-1/640*x^(3/2)*(b*(-4*a*c+5*b^2)+
4*c*(-16*a*c+5*b^2)*x^2)*(c*x^5+b*x^3+a*x)^(1/2)/c^2+1/1280*(128*a^2*c^2-10
0*a*b^2*c+15*b^4)*(c*x^5+b*x^3+a*x)^(1/2)/c^3/x^(1/2)
```

Rubi [A] time = 0.36, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1919, 1945, 1949, 12, 1914, 1107, 621, 206}

$$\frac{\left(128a^2c^2-100ab^2c+15b^4\right)\sqrt{ax+bx^3+cx^5}}{1280c^3\sqrt{x}}-\frac{x^{3/2}\left(4cx^2\left(5b^2-16ac\right)+b\left(5b^2-4ac\right)\right)\sqrt{ax+bx^3+cx^5}}{640c^2}-\frac{3b}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a*x + b*x^3 + c*x^5)^(3/2),x]
```

```
[Out] ((15*b^4 - 100*a*b^2*c + 128*a^2*c^2)*Sqrt[a*x + b*x^3 + c*x^5])/(1280*c^3*
Sqrt[x]) - (x^(3/2)*(b*(5*b^2 - 4*a*c) + 4*c*(5*b^2 - 16*a*c)*x^2)*Sqrt[a*x
 + b*x^3 + c*x^5])/(640*c^2) + (Sqrt[x]*(3*b + 8*c*x^2)*(a*x + b*x^3 + c*x^
5)^(3/2))/(80*c) - (3*b*(b^2 - 4*a*c)^2*Sqrt[x]*Sqrt[a + b*x^2 + c*x^4]*Arc
Tanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(512*c^(7/2)*Sqrt[
a*x + b*x^3 + c*x^5])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1914

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))])/Sqrt[
a*x^q + b*x^n + c*x^(2*n - q)], Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
```

```
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

Rule 1919

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(x^(m - n + q + 1)*(b*(n - q)*p + c*(m + p*q + (n - q)
*(2*p - 1) + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(c*(m + p*(2*
n - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1)), x] + Dist[((n - q)*p)/(c*(m
 + p*(2*n - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1)), Int[x^(m - (n - 2*q
))*Simp[-(a*b*(m + p*q - n + q + 1)) + (2*a*c*(m + p*q + (n - q)*(2*p - 1)
+ 1) - b^2*(m + p*q + (n - q)*(p - 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n +
c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] &&
 PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p
, 0] && RationalQ[m, q] && GtQ[m + p*q + 1, n - q] && NeQ[m + p*(2*n - q) +
 1, 0] && NeQ[m + p*q + (n - q)*(2*p - 1) + 1, 0]
```

Rule 1945

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[(x^(m + 1)*(b*B*(n - q)*p +
 A*c*(m + p*q + (n - q)*(2*p + 1) + 1) + B*c*(m + p*q + 2*(n - q)*p + 1)*x^
(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(c*(m + p*(2*n - q) + 1)*(m + p
*q + (n - q)*(2*p + 1) + 1)), x] + Dist[((n - q)*p)/(c*(m + p*(2*n - q) + 1
)*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(m + q)*Simp[2*a*A*c*(m + p*q +
 (n - q)*(2*p + 1) + 1) - a*b*B*(m + p*q + 1) + (2*a*B*c*(m + p*q + 2*(n -
q)*p + 1) + A*b*c*(m + p*q + (n - q)*(2*p + 1) + 1) - b^2*B*(m + p*q + (n -
 q)*p + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /
; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Intege
rQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q]
&& GtQ[m + p*q, -(n - q) - 1] && NeQ[m + p*(2*n - q) + 1, 0] && NeQ[m + p*q
 + (n - q)*(2*p + 1) + 1, 0]
```

Rule 1949

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[(B*x^(m - n + 1)*(a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1))/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), x] -
Dist[1/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(m - n + q)*Simp[a*B*(m
 + p*q - n + q + 1) + (b*B*(m + p*q + (n - q)*p + 1) - A*c*(m + p*q + (n -
q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x]
/; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Integ
erQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[p, -1] && LtQ[p, 0] && R
ationalQ[m, q] && GeQ[m + p*q, n - q - 1] && NeQ[m + p*q + (n - q)*(2*p + 1
) + 1, 0]
```

Rubi steps

$$\int x^{3/2}\left(ax+bx^3+cx^5\right)^{3/2}dx=\frac{\sqrt{x}\left(3b+8cx^2\right)\left(ax+bx^3+cx^5\right)^{3/2}}{80c}+\frac{3\int\sqrt{x}\left(-2ab-\left(5b^2-16ac\right)x^2\right)\sqrt{ax+}}{80c}$$

$$=-\frac{x^{3/2}\left(b\left(5b^2-4ac\right)+4c\left(5b^2-16ac\right)x^2\right)\sqrt{ax+bx^3+cx^5}}{640c^2}+\frac{\sqrt{x}\left(3b+8cx^2\right)}{}$$

$$=\frac{\left(15b^4-100ab^2c+128a^2c^2\right)\sqrt{ax+bx^3+cx^5}}{1280c^3\sqrt{x}}-\frac{x^{3/2}\left(b\left(5b^2-4ac\right)+4c\left(5b^2\right.\right.}{640}$$

$$=\frac{\left(15b^4-100ab^2c+128a^2c^2\right)\sqrt{ax+bx^3+cx^5}}{1280c^3\sqrt{x}}-\frac{x^{3/2}\left(b\left(5b^2-4ac\right)+4c\left(5b^2\right.\right.}{640}$$

$$=\frac{\left(15b^4-100ab^2c+128a^2c^2\right)\sqrt{ax+bx^3+cx^5}}{1280c^3\sqrt{x}}-\frac{x^{3/2}\left(b\left(5b^2-4ac\right)+4c\left(5b^2\right.\right.}{640}$$

$$=\frac{\left(15b^4-100ab^2c+128a^2c^2\right)\sqrt{ax+bx^3+cx^5}}{1280c^3\sqrt{x}}-\frac{x^{3/2}\left(b\left(5b^2-4ac\right)+4c\left(5b^2\right.\right.}{640}$$

$$=\frac{\left(15b^4-100ab^2c+128a^2c^2\right)\sqrt{ax+bx^3+cx^5}}{1280c^3\sqrt{x}}-\frac{x^{3/2}\left(b\left(5b^2-4ac\right)+4c\left(5b^2\right.\right.}{640}$$

$$=\frac{\left(15b^4-100ab^2c+128a^2c^2\right)\sqrt{ax+bx^3+cx^5}}{1280c^3\sqrt{x}}-\frac{x^{3/2}\left(b\left(5b^2-4ac\right)+4c\left(5b^2\right.\right.}{640}$$

Mathematica [A] time = 0.20, size = 192, normalized size = 0.79

$$\frac{\left(x\left(a+bx^2+cx^4\right)\right)^{3/2}\left(-\frac{3b\left(b^2-4ac\right)\left(\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)-2\sqrt{c}\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}\right)}{256c^{7/2}}-\frac{b\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{16c^2}+\right.}{2x^{3/2}\left(a+bx^2+cx^4\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a*x + b*x^3 + c*x^5)^(3/2),x]
```

```
[Out] ((x*(a + b*x^2 + c*x^4))^(3/2)*(-1/16*(b*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^
(3/2))/c^2 + (a + b*x^2 + c*x^4)^(5/2)/(5*c) - (3*b*(b^2 - 4*a*c)*(-2*Sqrt[
c]*(b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4] + (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x
^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])]))/(256*c^(7/2))))/(2*x^(3/2)*(a +
b*x^2 + c*x^4)^(3/2))
```

fricas [A] time = 0.71, size = 396, normalized size = 1.62

$$\left[\frac{15\left(b^5-8\,ab^3c+16\,a^2bc^2\right)\sqrt{c}\,x\log\left(-\frac{8\,c^2x^5+8\,bcx^3-4\,\sqrt{cx^5+bx^3+ax}\left(2\,cx^2+b\right)\sqrt{c}\,\sqrt{x}+\left(b^2+4\,ac\right)x}{x}\right)+4\left(128\,c^5x^8+176\,bc^4\right.}{5120\,c^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^5+b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/5120*(15*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*sqrt(c)*x*log(-(8*c^2*x^5 + 8*
b*c*x^3 - 4*sqrt(c*x^5 + b*x^3 + a*x)*(2*c*x^2 + b)*sqrt(c)*sqrt(x) + (b^2
```

```
+ 4*a*c)*x)/x) + 4*(128*c^5*x^8 + 176*b*c^4*x^6 + 15*b^4*c - 100*a*b^2*c^2
+ 128*a^2*c^3 + 8*(b^2*c^3 + 32*a*c^4)*x^4 - 2*(5*b^3*c^2 - 28*a*b*c^3)*x^2
)*sqrt(c*x^5 + b*x^3 + a*x)*sqrt(x))/(c^4*x), 1/2560*(15*(b^5 - 8*a*b^3*c +
 16*a^2*b*c^2)*sqrt(-c)*x*arctan(1/2*sqrt(c*x^5 + b*x^3 + a*x)*(2*c*x^2 + b
)*sqrt(-c)*sqrt(x)/(c^2*x^5 + b*c*x^3 + a*c*x)) + 2*(128*c^5*x^8 + 176*b*c^
4*x^6 + 15*b^4*c - 100*a*b^2*c^2 + 128*a^2*c^3 + 8*(b^2*c^3 + 32*a*c^4)*x^4
 - 2*(5*b^3*c^2 - 28*a*b*c^3)*x^2)*sqrt(c*x^5 + b*x^3 + a*x)*sqrt(x))/(c^4*
x)]
```

giac [B] time = 2.11, size = 662, normalized size = 2.71

$$\frac{1}{96}\left(2\sqrt{cx^4+bx^2+a}\left(2\left(4x^2+\frac{b}{c}\right)x^2-\frac{3b^2-8ac}{c^2}\right)-\frac{3\left(b^3-4abc\right)\log\left(\left|-2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}-b\right|\right)}{c^{\frac{5}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^5+b*x^3+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] 1/96*(2*sqrt(c*x^4 + b*x^2 + a)*(2*(4*x^2 + b/c)*x^2 - (3*b^2 - 8*a*c)/c^2)
 - 3*(b^3 - 4*a*b*c)*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqr
t(c) - b))/c^(5/2) + (3*b^3*log(abs(-b + 2*sqrt(a)*sqrt(c))) - 12*a*b*c*log
(abs(-b + 2*sqrt(a)*sqrt(c))) + 6*sqrt(a)*b^2*sqrt(c) - 16*a^(3/2)*c^(3/2))
/c^(5/2))*a + 1/768*(2*sqrt(c*x^4 + b*x^2 + a)*(2*(4*(6*x^2 + b/c)*x^2 - (5
*b^2*c - 12*a*c^2)/c^3)*x^2 + (15*b^3 - 52*a*b*c)/c^3) + 3*(5*b^4 - 24*a*b^
2*c + 16*a^2*c^2)*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c
) - b))/c^(7/2) - (15*b^4*log(abs(-b + 2*sqrt(a)*sqrt(c))) - 72*a*b^2*c*log
(abs(-b + 2*sqrt(a)*sqrt(c))) + 48*a^2*c^2*log(abs(-b + 2*sqrt(a)*sqrt(c)))
 + 30*sqrt(a)*b^3*sqrt(c) - 104*a^(3/2)*b*c^(3/2))/c^(7/2))*b + 1/7680*(2*s
qrt(c*x^4 + b*x^2 + a)*(2*(4*(6*(8*x^2 + b/c)*x^2 - (7*b^2*c^2 - 16*a*c^3)/
c^4)*x^2 + (35*b^3*c - 116*a*b*c^2)/c^4)*x^2 - (105*b^4 - 460*a*b^2*c + 256
*a^2*c^2)/c^4) - 15*(7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)*log(abs(-2*(sqrt(c)
*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) - b))/c^(9/2) + (105*b^5*log(abs(-b
 + 2*sqrt(a)*sqrt(c))) - 600*a*b^3*c*log(abs(-b + 2*sqrt(a)*sqrt(c))) + 720
*a^2*b*c^2*log(abs(-b + 2*sqrt(a)*sqrt(c))) + 210*sqrt(a)*b^4*sqrt(c) - 920
*a^(3/2)*b^2*c^(3/2) + 512*a^(5/2)*c^(5/2))/c^(9/2))*c
```

maple [A] time = 0.01, size = 369, normalized size = 1.51

$$-\frac{\sqrt{\left(c\,x^4+b\,x^2+a\right)x}\;\left(-256\sqrt{c\,x^4+b\,x^2+a}\;c^{\frac{9}{2}}x^8-352\sqrt{c\,x^4+b\,x^2+a}\;b\,c^{\frac{7}{2}}x^6-512\sqrt{c\,x^4+b\,x^2+a}\;a\,c^{\frac{7}{2}}x^4-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(c*x^5+b*x^3+a*x)^(3/2),x)
```

```
[Out] -1/2560*((c*x^4+b*x^2+a)*x)^(1/2)/c^(7/2)*(-256*x^8*c^(9/2)*(c*x^4+b*x^2+a)
^(1/2)-352*x^6*b*c^(7/2)*(c*x^4+b*x^2+a)^(1/2)-512*x^4*a*c^(7/2)*(c*x^4+b*x
^2+a)^(1/2)-16*x^4*b^2*c^(5/2)*(c*x^4+b*x^2+a)^(1/2)-112*x^2*a*b*c^(5/2)*(c
*x^4+b*x^2+a)^(1/2)+20*x^2*b^3*c^(3/2)*(c*x^4+b*x^2+a)^(1/2)+240*ln(1/2*(2*
c*x^2+b+2*(c*x^4+b*x^2+a)^(1/2)*c^(1/2))/c^(1/2))*a^2*b*c^2-120*ln(1/2*(2*c
*x^2+b+2*(c*x^4+b*x^2+a)^(1/2)*c^(1/2))/c^(1/2))*a*b^3*c+15*ln(1/2*(2*c*x^2
+b+2*(c*x^4+b*x^2+a)^(1/2)*c^(1/2))/c^(1/2))*b^5-256*a^2*c^(5/2)*(c*x^4+b*x
^2+a)^(1/2)+200*a*b^2*c^(3/2)*(c*x^4+b*x^2+a)^(1/2)-30*b^4*c^(1/2)*(c*x^4+b
*x^2+a)^(1/2))/x^(1/2)/(c*x^4+b*x^2+a)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(cx^5+bx^3+ax\right)^{\frac{3}{2}}x^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^5+b*x^3+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^5 + b*x^3 + a*x)^(3/2)*x^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^{3/2}\left(c\,x^5+b\,x^3+a\,x\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(a*x + b*x^3 + c*x^5)^(3/2),x)
```

```
[Out] int(x^(3/2)*(a*x + b*x^3 + c*x^5)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(c*x**5+b*x**3+a*x)**(3/2),x)
```

```
[Out] Timed out
```

3.110 $\int \sqrt{x}\left(ax+bx^3+cx^5\right)^{3/2}\,dx$

Optimal. Leaf size=487

$$\frac{\sqrt[4]{a}\sqrt{x}\left(84a^2c^2-57ab^2c+4\sqrt{a}\,b\sqrt{c}\left(b^2-6ac\right)+8b^4\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\cdots\right.\right.}{630c^{11/4}\sqrt{ax+bx^3+cx^5}}$$

```
[Out] 1/63*(7*c*x^2+3*b)*(c*x^5+b*x^3+a*x)^(3/2)/c/x^(1/2)+1/315*(84*a^2*c^2-57*a
*b^2*c+8*b^4)*x^(3/2)*(c*x^4+b*x^2+a)/c^(5/2)/(a^(1/2)+x^2*c^(1/2))/(c*x^5+
b*x^3+a*x)^(1/2)-1/315*(b*(-9*a*c+4*b^2)+6*c*(-7*a*c+2*b^2)*x^2)*x^(1/2)*(c
*x^5+b*x^3+a*x)^(1/2)/c^2-1/315*a^(1/4)*(84*a^2*c^2-57*a*b^2*c+8*b^4)*(cos(
2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*Elli
pticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^
(1/2)+x^2*c^(1/2))*x^(1/2)*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/
c^(11/4)/(c*x^5+b*x^3+a*x)^(1/2)+1/630*a^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1
/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1
/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*(8*b
^4-57*a*b^2*c+84*a^2*c^2+4*b*(-6*a*c+b^2)*a^(1/2)*c^(1/2))*x^(1/2)*((c*x^4+
b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(11/4)/(c*x^5+b*x^3+a*x)^(1/2)
```

Rubi [A] time = 0.46, antiderivative size = 487, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1919, 1945, 1953, 1197, 1103, 1195}

$$\frac{x^{3/2}\left(84a^2c^2-57ab^2c+8b^4\right)\left(a+bx^2+cx^4\right)}{315c^{5/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{ax+bx^3+cx^5}}+\frac{\sqrt[4]{a}\sqrt{x}\left(84a^2c^2-57ab^2c+4\sqrt{a}\,b\sqrt{c}\left(b^2-6ac\right)+8b^4\right)\left(\sqrt{a}+\sqrt{c}\,x^2\cdots\right.}{630c^{11/4}\sqrt{ax+bx^3+\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a*x + b*x^3 + c*x^5)^(3/2),x]
```

```
[Out] ((8*b^4 - 57*a*b^2*c + 84*a^2*c^2)*x^(3/2)*(a + b*x^2 + c*x^4))/(315*c^(5/2
)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[a*x + b*x^3 + c*x^5]) - (Sqrt[x]*(b*(4*b^2 -
 9*a*c) + 6*c*(2*b^2 - 7*a*c)*x^2)*Sqrt[a*x + b*x^3 + c*x^5])/(315*c^2) + (
(3*b + 7*c*x^2)*(a*x + b*x^3 + c*x^5)^(3/2))/(63*c*Sqrt[x]) - (a^(1/4)*(8*b
^4 - 57*a*b^2*c + 84*a^2*c^2)*Sqrt[x]*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x
^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/
4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(315*c^(11/4)*Sqrt[a*x + b*x^3 + c*x^5])
 + (a^(1/4)*(8*b^4 - 57*a*b^2*c + 84*a^2*c^2 + 4*Sqrt[a]*b*Sqrt[c]*(b^2 - 6
*a*c))*Sqrt[x]*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] +
Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sq
rt[c]))/4])/(630*c^(11/4)*Sqrt[a*x + b*x^3 + c*x^5])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
```

```
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1919

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(x^(m - n + q + 1)*(b*(n - q)*p + c*(m + p*q + (n - q)
*(2*p - 1) + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(c*(m + p*(2*
n - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1)), x] + Dist[((n - q)*p)/(c*(m
 + p*(2*n - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1)), Int[x^(m - (n - 2*q
))*Simp[-(a*b*(m + p*q - n + q + 1)) + (2*a*c*(m + p*q + (n - q)*(2*p - 1)
+ 1) - b^2*(m + p*q + (n - q)*(p - 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n +
c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] &&
 PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p
, 0] && RationalQ[m, q] && GtQ[m + p*q + 1, n - q] && NeQ[m + p*(2*n - q) +
 1, 0] && NeQ[m + p*q + (n - q)*(2*p - 1) + 1, 0]
```

Rule 1945

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[(x^(m + 1)*(b*B*(n - q)*p +
 A*c*(m + p*q + (n - q)*(2*p + 1) + 1) + B*c*(m + p*q + 2*(n - q)*p + 1)*x^
(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(c*(m + p*(2*n - q) + 1)*(m + p
*q + (n - q)*(2*p + 1) + 1)), x] + Dist[((n - q)*p)/(c*(m + p*(2*n - q) + 1
)*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(m + q)*Simp[2*a*A*c*(m + p*q +
 (n - q)*(2*p + 1) + 1) - a*b*B*(m + p*q + 1) + (2*a*B*c*(m + p*q + 2*(n -
q)*p + 1) + A*b*c*(m + p*q + (n - q)*(2*p + 1) + 1) - b^2*B*(m + p*q + (n -
 q)*p + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /
; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Intege
rQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q]
&& GtQ[m + p*q, -(n - q) - 1] && NeQ[m + p*(2*n - q) + 1, 0] && NeQ[m + p*q
 + (n - q)*(2*p + 1) + 1, 0]
```

Rule 1953

```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(j_.)))/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x
_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol] :> Dist[(x^(q/2)*Sqrt[a + b*x^(n -
q) + c*x^(2*(n - q))])/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)], Int[(x^(m - q/2
)*(A + B*x^(n - q)))/Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))], x], x] /; Fre
eQ[{a, b, c, A, B, m, n, q}, x] && EqQ[j, n - q] && EqQ[r, 2*n - q] && PosQ
[n - q] && (EqQ[m, 1/2] || EqQ[m, -2^(-1)]) && EqQ[n, 3] && EqQ[q, 1]
```

Rubi steps

$$\int \sqrt{x}\left(ax+bx^3+cx^5\right)^{3/2}dx = \frac{\left(3b+7cx^2\right)\left(ax+bx^3+cx^5\right)^{3/2}}{63c\sqrt{x}} + \frac{\int \frac{\left(-ab-2\left(2b^2-7ac\right)x^2\right)\sqrt{ax+bx^3+cx^5}}{\sqrt{x}}dx}{21c}$$

$$= -\frac{\sqrt{x}\left(b\left(4b^2-9ac\right)+6c\left(2b^2-7ac\right)x^2\right)\sqrt{ax+bx^3+cx^5}}{315c^2} + \frac{\left(3b+7cx^2\right)\left(ax+b\right.}{63c\sqrt{x}}$$

$$= -\frac{\sqrt{x}\left(b\left(4b^2-9ac\right)+6c\left(2b^2-7ac\right)x^2\right)\sqrt{ax+bx^3+cx^5}}{315c^2} + \frac{\left(3b+7cx^2\right)\left(ax+b\right.}{63c\sqrt{x}}$$

$$= -\frac{\sqrt{x}\left(b\left(4b^2-9ac\right)+6c\left(2b^2-7ac\right)x^2\right)\sqrt{ax+bx^3+cx^5}}{315c^2} + \frac{\left(3b+7cx^2\right)\left(ax+b\right.}{63c\sqrt{x}}$$

$$= \frac{\left(8b^4-57ab^2c+84a^2c^2\right)x^{3/2}\left(a+bx^2+cx^4\right)}{315c^{5/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{ax+bx^3+cx^5}} - \frac{\sqrt{x}\left(b\left(4b^2-9ac\right)+6c\left(2b^2-7a\right.\right.}{315c^2}$$

Mathematica [C] time = 2.29, size = 609, normalized size = 1.25

$$\frac{\sqrt{x}\left(i\left(84a^2c^2-57ab^2c+8b^4\right)\left(\sqrt{b^2-4ac}-b\right)\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{-2\sqrt{b^2-4ac}+2b+4cx^2}{b-\sqrt{b^2-4ac}}}\,E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a*x + b*x^3 + c*x^5)^(3/2),x]
```

```
[Out] (Sqrt[x]*(4*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(-4*b^4*x^2 - b^3*c*x^4 + 5
3*b^2*c^2*x^6 + 85*b*c^3*x^8 + 35*c^4*x^10 + a^2*c*(24*b + 77*c*x^2) + a*(-
4*b^3 + 27*b^2*c*x^2 + 151*b*c^2*x^4 + 112*c^3*x^6)) + I*(8*b^4 - 57*a*b^2*
c + 84*a^2*c^2)*(-b + Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*
x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b
 - Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4
*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - I*(-8*b^5 +
65*a*b^3*c - 132*a^2*b*c^2 + 8*b^4*Sqrt[b^2 - 4*a*c] - 57*a*b^2*c*Sqrt[b^2
- 4*a*c] + 84*a^2*c^2*Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*
x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b
 - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4
*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]))/(1260*c^3*Sq
rt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[x*(a + b*x^2 + c*x^4)])
```

fricas [F] time = 0.89, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(cx^5+bx^3+ax\right)^{\frac{3}{2}}\sqrt{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)^(3/2)*x^(1/2),x, algorithm="fricas")
```

```
[Out] integral((c*x^5 + b*x^3 + a*x)^(3/2)*sqrt(x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(cx^5+bx^3+ax\right)^{\frac{3}{2}}\sqrt{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)^(3/2)*x^(1/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^5 + b*x^3 + a*x)^(3/2)*sqrt(x), x)
```

maple [B] time = 0.02, size = 1878, normalized size = 3.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^5+b*x^3+a*x)^(3/2)*x^(1/2),x)
```

```
[Out] -1/315*((c*x^4+b*x^2+a)*x)^(1/2)/x^(1/2)*(-85*((-b+(-4*a*c+b^2)^(1/2))/a)^(
1/2)*(-4*a*c+b^2)^(1/2)*x^9*b*c^3-2*EllipticF(1/2*2^(1/2)*((-b+(-4*a*c+b^2)
^(1/2))/a)^(1/2)*x,1/2*2^(1/2)*((-2*a*c+b^2+(-4*a*c+b^2)^(1/2)*b)/a/c)^(1/2
))*(-2*(-b*x^2+(-4*a*c+b^2)^(1/2)*x^2-2*a)/a)^(1/2)*((b*x^2+(-4*a*c+b^2)^(1
/2)*x^2+2*a)/a)^(1/2)*(-4*a*c+b^2)^(1/2)*a*b^3-35*((-b+(-4*a*c+b^2)^(1/2))/
a)^(1/2)*(-4*a*c+b^2)^(1/2)*x^11*c^4-35*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x
^11*b*c^4-85*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x^9*b^2*c^3-53*((-b+(-4*a*c+
b^2)^(1/2))/a)^(1/2)*x^7*b^3*c^2+((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x^5*b^4*
c+4*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-4*a*c+b^2)^(1/2)*x^3*b^4+4*((-b+(-4
*a*c+b^2)^(1/2))/a)^(1/2)*x*a*b^4+4*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x^3*b
^5-45*EllipticF(1/2*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*2^(1/2)
*((-2*a*c+b^2+(-4*a*c+b^2)^(1/2)*b)/a/c)^(1/2))*(-2*(-b*x^2+(-4*a*c+b^2)^(1
/2)*x^2-2*a)/a)^(1/2)*((b*x^2+(-4*a*c+b^2)^(1/2)*x^2+2*a)/a)^(1/2)*a^2*b^2*
c+57*EllipticE(1/2*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*2^(1/2)*
((-2*a*c+b^2+(-4*a*c+b^2)^(1/2)*b)/a/c)^(1/2))*(-2*(-b*x^2+(-4*a*c+b^2)^(1/
2)*x^2-2*a)/a)^(1/2)*((b*x^2+(-4*a*c+b^2)^(1/2)*x^2+2*a)/a)^(1/2)*a^2*b^2*c
-151*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-4*a*c+b^2)^(1/2)*x^5*a*b*c^2-27*((
-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-4*a*c+b^2)^(1/2)*x^3*a*b^2*c-24*((-b+(-4*
a*c+b^2)^(1/2))/a)^(1/2)*(-4*a*c+b^2)^(1/2)*x*a^2*b*c+12*EllipticF(1/2*2^(1
/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*2^(1/2)*((-2*a*c+b^2+(-4*a*c+b^
2)^(1/2)*b)/a/c)^(1/2))*(-2*(-b*x^2+(-4*a*c+b^2)^(1/2)*x^2-2*a)/a)^(1/2)*((
b*x^2+(-4*a*c+b^2)^(1/2)*x^2+2*a)/a)^(1/2)*(-4*a*c+b^2)^(1/2)*a^2*b*c-112*(
(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-4*a*c+b^2)^(1/2)*x^7*a*c^3-53*((-b+(-4*a
*c+b^2)^(1/2))/a)^(1/2)*(-4*a*c+b^2)^(1/2)*x^7*b^2*c^2-112*((-b+(-4*a*c+b^2
)^(1/2))/a)^(1/2)*x^7*a*b*c^3+((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-4*a*c+b^2
)^(1/2)*x^5*b^3*c-151*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x^5*a*b^2*c^2-77*((
-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-4*a*c+b^2)^(1/2)*x^3*a^2*c^2-77*((-b+(-4*
a*c+b^2)^(1/2))/a)^(1/2)*x^3*a^2*b*c^2+84*EllipticF(1/2*2^(1/2)*((-b+(-4*a*
c+b^2)^(1/2))/a)^(1/2)*x,1/2*2^(1/2)*((-2*a*c+b^2+(-4*a*c+b^2)^(1/2)*b)/a/c
)^(1/2))*(-2*(-b*x^2+(-4*a*c+b^2)^(1/2)*x^2-2*a)/a)^(1/2)*((b*x^2+(-4*a*c+b
^2)^(1/2)*x^2+2*a)/a)^(1/2)*a^3*c^2+6*EllipticF(1/2*2^(1/2)*((-b+(-4*a*c+b^
2)^(1/2))/a)^(1/2)*x,1/2*2^(1/2)*((-2*a*c+b^2+(-4*a*c+b^2)^(1/2)*b)/a/c)^(1
/2))*(-2*(-b*x^2+(-4*a*c+b^2)^(1/2)*x^2-2*a)/a)^(1/2)*((b*x^2+(-4*a*c+b^2)^
(1/2)*x^2+2*a)/a)^(1/2)*a*b^4-84*EllipticE(1/2*2^(1/2)*((-b+(-4*a*c+b^2)^(1
/2))/a)^(1/2)*x,1/2*2^(1/2)*((-2*a*c+b^2+(-4*a*c+b^2)^(1/2)*b)/a/c)^(1/2))*
(-2*(-b*x^2+(-4*a*c+b^2)^(1/2)*x^2-2*a)/a)^(1/2)*((b*x^2+(-4*a*c+b^2)^(1/2)
*x^2+2*a)/a)^(1/2)*a^3*c^2-8*EllipticE(1/2*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))
/a)^(1/2)*x,1/2*2^(1/2)*((-2*a*c+b^2+(-4*a*c+b^2)^(1/2)*b)/a/c)^(1/2))*(-2*
(-b*x^2+(-4*a*c+b^2)^(1/2)*x^2-2*a)/a)^(1/2)*((b*x^2+(-4*a*c+b^2)^(1/2)*x^2
+2*a)/a)^(1/2)*a*b^4+4*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-4*a*c+b^2)^(1/2)
*x*a*b^3-24*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x*a^2*b^2*c-27*((-b+(-4*a*c+b
^2)^(1/2))/a)^(1/2)*x^3*a*b^3*c)/(c*x^4+b*x^2+a)/c^2/((-b+(-4*a*c+b^2)^(1/2
))/a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^5 + bx^3 + ax\right)^{\frac{3}{2}} \sqrt{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)^(3/2)*x^(1/2),x, algorithm="maxima")
[Out] integrate((c*x^5 + b*x^3 + a*x)^(3/2)*sqrt(x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{x}\,\left(c\,x^5+b\,x^3+a\,x\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(a*x + b*x^3 + c*x^5)^(3/2),x)
[Out] int(x^(1/2)*(a*x + b*x^3 + c*x^5)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x}\left(x\left(a+bx^2+cx^4\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**5+b*x**3+a*x)**(3/2)*x**(1/2),x)
[Out] Integral(sqrt(x)*(x*(a + b*x**2 + c*x**4))**(3/2), x)
```

3.111 $\int \frac{(ax+bx^3+cx^5)^{3/2}}{\sqrt{x}} \, dx$

Optimal. Leaf size=177

$$\frac{3\sqrt{x}\left(b^2-4ac\right)^2\sqrt{a+bx^2+cx^4}\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{256c^{5/2}\sqrt{ax+bx^3+cx^5}}-\frac{3\left(b^2-4ac\right)\left(b+2cx^2\right)\sqrt{ax+bx^3+cx^5}}{128c^2\sqrt{x}}+\frac{(b+2cx}{}$$

```
[Out] 1/16*(2*c*x^2+b)*(c*x^5+b*x^3+a*x)^(3/2)/c/x^(3/2)+3/256*(-4*a*c+b^2)^2*arc
tanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))*x^(1/2)*(c*x^4+b*x^2+a)
^(1/2)/c^(5/2)/(c*x^5+b*x^3+a*x)^(1/2)-3/128*(-4*a*c+b^2)*(2*c*x^2+b)*(c*x^
5+b*x^3+a*x)^(1/2)/c^2/x^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1918, 1914, 1107, 621, 206}

$$-\frac{3\left(b^2-4ac\right)\left(b+2cx^2\right)\sqrt{ax+bx^3+cx^5}}{128c^2\sqrt{x}}+\frac{3\sqrt{x}\left(b^2-4ac\right)^2\sqrt{a+bx^2+cx^4}\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{256c^{5/2}\sqrt{ax+bx^3+cx^5}}+\frac{(b+2c}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^3 + c*x^5)^(3/2)/Sqrt[x],x]
```

```
[Out] (-3*(b^2 - 4*a*c)*(b + 2*c*x^2)*Sqrt[a*x + b*x^3 + c*x^5])/(128*c^2*Sqrt[x]
) + ((b + 2*c*x^2)*(a*x + b*x^3 + c*x^5)^(3/2))/(16*c*x^(3/2)) + (3*(b^2 -
4*a*c)^2*Sqrt[x]*Sqrt[a + b*x^2 + c*x^4]*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*S
qrt[a + b*x^2 + c*x^4])])/(256*c^(5/2)*Sqrt[a*x + b*x^3 + c*x^5])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1914

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))])/Sqrt[
a*x^q + b*x^n + c*x^(2*n - q)], Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

Rule 1918

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(x^(m - n + q + 1)*(b + 2*c*x^(n - q))*(a*x^q + b*x^n
+ c*x^(2*n - q))^p)/(2*c*(n - q)*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2
*c*(2*p + 1)), Int[x^(m + q)*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x
] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && !IntegerQ[p]
 && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && Eq
Q[m + p*q + 1, n - q]
```

Rubi steps

$$\int \frac{\left(ax+bx^3+cx^5\right)^{3/2}}{\sqrt{x}}\,dx = \frac{\left(b+2cx^2\right)\left(ax+bx^3+cx^5\right)^{3/2}}{16cx^{3/2}} - \frac{\left(3\left(b^2-4ac\right)\right)\int \sqrt{x}\sqrt{ax+bx^3+cx^5}\,dx}{16c}$$

$$= -\frac{3\left(b^2-4ac\right)\left(b+2cx^2\right)\sqrt{ax+bx^3+cx^5}}{128c^2\sqrt{x}} + \frac{\left(b+2cx^2\right)\left(ax+bx^3+cx^5\right)^{3/2}}{16cx^{3/2}} + \frac{\left(3\left(b^2\right.\right.}{}$$

$$= -\frac{3\left(b^2-4ac\right)\left(b+2cx^2\right)\sqrt{ax+bx^3+cx^5}}{128c^2\sqrt{x}} + \frac{\left(b+2cx^2\right)\left(ax+bx^3+cx^5\right)^{3/2}}{16cx^{3/2}} + \frac{\left(3\left(b^2\right.\right.}{}$$

$$= -\frac{3\left(b^2-4ac\right)\left(b+2cx^2\right)\sqrt{ax+bx^3+cx^5}}{128c^2\sqrt{x}} + \frac{\left(b+2cx^2\right)\left(ax+bx^3+cx^5\right)^{3/2}}{16cx^{3/2}} + \frac{\left(3\left(b^2\right.\right.}{}$$

$$= -\frac{3\left(b^2-4ac\right)\left(b+2cx^2\right)\sqrt{ax+bx^3+cx^5}}{128c^2\sqrt{x}} + \frac{\left(b+2cx^2\right)\left(ax+bx^3+cx^5\right)^{3/2}}{16cx^{3/2}} + \frac{\left(3\left(b^2\right.\right.}{}$$

$$= -\frac{3\left(b^2-4ac\right)\left(b+2cx^2\right)\sqrt{ax+bx^3+cx^5}}{128c^2\sqrt{x}} + \frac{\left(b+2cx^2\right)\left(ax+bx^3+cx^5\right)^{3/2}}{16cx^{3/2}} + \frac{3\left(b^2\right.}{}$$

Mathematica [A] time = 0.11, size = 152, normalized size = 0.86

$$\frac{\sqrt{x}\sqrt{a+bx^2+cx^4}\left(2\sqrt{c}\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}\left(4c\left(5a+2cx^4\right)-3b^2+8bcx^2\right)+3\left(b^2-4ac\right)^2\tanh^{-1}\left(\frac{}{2\sqrt{c}}\right.\right.}{256c^{5/2}\sqrt{x\left(a+bx^2+cx^4\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^3 + c*x^5)^(3/2)/Sqrt[x],x]
```

```
[Out] (Sqrt[x]*Sqrt[a + b*x^2 + c*x^4]*(2*Sqrt[c]*(b + 2*c*x^2)*Sqrt[a + b*x^2 +
c*x^4]*(-3*b^2 + 8*b*c*x^2 + 4*c*(5*a + 2*c*x^4)) + 3*(b^2 - 4*a*c)^2*ArcTa
nh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])]))/(256*c^(5/2)*Sqrt[x
*(a + b*x^2 + c*x^4)])
```

fricas [A] time = 0.84, size = 332, normalized size = 1.88

$$\left[\frac{3\left(b^4-8ab^2c+16a^2c^2\right)\sqrt{c}\,x\log\left(-\frac{8c^2x^5+8bcx^3+4\sqrt{cx^5+bx^3+ax}\left(2cx^2+b\right)\sqrt{c}\sqrt{x}+\left(b^2+4ac\right)x}{x}\right)+4\left(16c^4x^6+24bc^3x^4-3b\right.}{512c^3x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)^(3/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] [1/512*(3*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(c)*x*log(-(8*c^2*x^5 + 8*b*c*
x^3 + 4*sqrt(c*x^5 + b*x^3 + a*x)*(2*c*x^2 + b)*sqrt(c)*sqrt(x) + (b^2 + 4*
a*c)*x)/x) + 4*(16*c^4*x^6 + 24*b*c^3*x^4 - 3*b^3*c + 20*a*b*c^2 + 2*(b^2*c
^2 + 20*a*c^3)*x^2)*sqrt(c*x^5 + b*x^3 + a*x)*sqrt(x))/(c^3*x), -1/256*(3*(
b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(-c)*x*arctan(1/2*sqrt(c*x^5 + b*x^3 + a*
x)*(2*c*x^2 + b)*sqrt(-c)*sqrt(x)/(c^2*x^5 + b*c*x^3 + a*c*x)) - 2*(16*c^4*
x^6 + 24*b*c^3*x^4 - 3*b^3*c + 20*a*b*c^2 + 2*(b^2*c^2 + 20*a*c^3)*x^2)*sqr
t(c*x^5 + b*x^3 + a*x)*sqrt(x))/(c^3*x)]
```

giac [B] time = 1.58, size = 518, normalized size = 2.93

$$\frac{1}{16}\left(2\sqrt{cx^4+bx^2+a}\left(2x^2+\frac{b}{c}\right)+\frac{\left(b^2-4ac\right)\log\left(\left|-2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}-b\right|\right)}{c^{\frac{3}{2}}}-\frac{b^2\log\left(\left|-b+2\sqrt{\right.\right.}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)^(3/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] 1/16*(2*sqrt(c*x^4 + b*x^2 + a)*(2*x^2 + b/c) + (b^2 - 4*a*c)*log(abs(-2*(s
qrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) - b))/c^(3/2) - (b^2*log(abs(
-b + 2*sqrt(a)*sqrt(c))) - 4*a*c*log(abs(-b + 2*sqrt(a)*sqrt(c))) + 2*sqrt(
a)*b*sqrt(c))/c^(3/2))*a + 1/96*(2*sqrt(c*x^4 + b*x^2 + a)*(2*(4*x^2 + b/c)
*x^2 - (3*b^2 - 8*a*c)/c^2) - 3*(b^3 - 4*a*b*c)*log(abs(-2*(sqrt(c)*x^2 - s
qrt(c*x^4 + b*x^2 + a))*sqrt(c) - b))/c^(5/2) + (3*b^3*log(abs(-b + 2*sqrt(
a)*sqrt(c))) - 12*a*b*c*log(abs(-b + 2*sqrt(a)*sqrt(c))) + 6*sqrt(a)*b^2*sq
rt(c) - 16*a^(3/2)*c^(3/2))/c^(5/2))*b + 1/768*(2*sqrt(c*x^4 + b*x^2 + a)*(
2*(4*(6*x^2 + b/c)*x^2 - (5*b^2*c - 12*a*c^2)/c^3)*x^2 + (15*b^3 - 52*a*b*c
)/c^3) + 3*(5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*log(abs(-2*(sqrt(c)*x^2 - sqrt
(c*x^4 + b*x^2 + a))*sqrt(c) - b))/c^(7/2) - (15*b^4*log(abs(-b + 2*sqrt(a)
*sqrt(c))) - 72*a*b^2*c*log(abs(-b + 2*sqrt(a)*sqrt(c))) + 48*a^2*c^2*log(a
bs(-b + 2*sqrt(a)*sqrt(c))) + 30*sqrt(a)*b^3*sqrt(c) - 104*a^(3/2)*b*c^(3/2
))/c^(7/2))*c
```

maple [A] time = 0.02, size = 295, normalized size = 1.67

$$\frac{\sqrt{\left(c\,x^4+b\,x^2+a\right)x}\,\left(32\sqrt{c\,x^4+b\,x^2+a}\;c^{\frac{7}{2}}x^6+48\sqrt{c\,x^4+b\,x^2+a}\;b\,c^{\frac{5}{2}}x^4+80\sqrt{c\,x^4+b\,x^2+a}\;a\,c^{\frac{5}{2}}x^2+4\sqrt{\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^5+b*x^3+a*x)^(3/2)/x^(1/2),x)
```

```
[Out] 1/256*((c*x^4+b*x^2+a)*x)^(1/2)/c^(5/2)*(32*x^6*c^(7/2)*(c*x^4+b*x^2+a)^(1/
2)+48*x^4*b*c^(5/2)*(c*x^4+b*x^2+a)^(1/2)+80*x^2*a*c^(5/2)*(c*x^4+b*x^2+a)^
(1/2)+4*x^2*b^2*c^(3/2)*(c*x^4+b*x^2+a)^(1/2)+48*ln(1/2*(2*c*x^2+b+2*(c*x^4
+b*x^2+a)^(1/2)*c^(1/2))/c^(1/2))*a^2*c^2-24*ln(1/2*(2*c*x^2+b+2*(c*x^4+b*x
^2+a)^(1/2)*c^(1/2))/c^(1/2))*a*b^2*c+3*ln(1/2*(2*c*x^2+b+2*(c*x^4+b*x^2+a)
^(1/2)*c^(1/2))/c^(1/2))*b^4+40*a*b*c^(3/2)*(c*x^4+b*x^2+a)^(1/2)-6*b^3*c^(
1/2)*(c*x^4+b*x^2+a)^(1/2))/x^(1/2)/(c*x^4+b*x^2+a)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^5+bx^3+ax\right)^{\frac{3}{2}}}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)^(3/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^5 + b*x^3 + a*x)^(3/2)/sqrt(x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,x^5+b\,x^3+a\,x\right)^{3/2}}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^3 + c*x^5)^(3/2)/x^(1/2),x)
[Out] int((a*x + b*x^3 + c*x^5)^(3/2)/x^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x\left(a+bx^2+cx^4\right)\right)^{\frac{3}{2}}}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**5+b*x**3+a*x)**(3/2)/x**(1/2),x)
[Out] Integral((x*(a + b*x**2 + c*x**4))**(3/2)/sqrt(x), x)
```

3.112 $\int \frac{\left(ax+bx^3+cx^5\right)^{3/2}}{x^{3/2}}\,dx$

Optimal. Leaf size=425

$$-\frac{\sqrt[4]{a}\sqrt{x}\left(\sqrt{a}\sqrt{c}\left(b^2-20ac\right)+2b\left(b^2-8ac\right)\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{70c^{7/4}\sqrt{ax+bx^3+cx^5}}+\cdots 2$$

```
[Out] 1/7*(c*x^5+b*x^3+a*x)^(3/2)/x^(1/2)-2/35*b*(-8*a*c+b^2)*x^(3/2)*(c*x^4+b*x^
2+a)/c^(3/2)/(a^(1/2)+x^2*c^(1/2))/(c*x^5+b*x^3+a*x)^(1/2)+1/35*(3*b*c*x^2+
10*a*c+b^2)*x^(1/2)*(c*x^5+b*x^3+a*x)^(1/2)/c+2/35*a^(1/4)*b*(-8*a*c+b^2)*(
cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*
EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))
*(a^(1/2)+x^2*c^(1/2))*x^(1/2)*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1
/2)/c^(7/4)/(c*x^5+b*x^3+a*x)^(1/2)-1/70*a^(1/4)*(cos(2*arctan(c^(1/4)*x/a^
(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^
(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*(2
*b*(-8*a*c+b^2)+(-20*a*c+b^2)*a^(1/2)*c^(1/2))*x^(1/2)*((c*x^4+b*x^2+a)/(a^
(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(7/4)/(c*x^5+b*x^3+a*x)^(1/2)
```

Rubi [A] time = 0.45, antiderivative size = 425, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1921, 1945, 1953, 1197, 1103, 1195}

$$-\frac{2bx^{3/2}\left(b^2-8ac\right)\left(a+bx^2+cx^4\right)}{35c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{ax+bx^3+cx^5}}-\frac{\sqrt[4]{a}\sqrt{x}\left(\sqrt{a}\sqrt{c}\left(b^2-20ac\right)+2b\left(b^2-8ac\right)\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+\cdots}{\left(\sqrt{a}+\sqrt{c}\cdots\right.}}}{70c^{7/4}\sqrt{ax+bx^3+cx^5}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^3 + c*x^5)^(3/2)/x^(3/2),x]
```

```
[Out] (-2*b*(b^2 - 8*a*c)*x^(3/2)*(a + b*x^2 + c*x^4))/(35*c^(3/2)*(Sqrt[a] + Sqr
t[c]*x^2)*Sqrt[a*x + b*x^3 + c*x^5]) + (Sqrt[x]*(b^2 + 10*a*c + 3*b*c*x^2)*
Sqrt[a*x + b*x^3 + c*x^5])/(35*c) + (a*x + b*x^3 + c*x^5)^(3/2)/(7*Sqrt[x])
 + (2*a^(1/4)*b*(b^2 - 8*a*c)*Sqrt[x]*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x
^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/
4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(35*c^(7/4)*Sqrt[a*x + b*x^3 + c*x^5]) -
 (a^(1/4)*(Sqrt[a]*Sqrt[c]*(b^2 - 20*a*c) + 2*b*(b^2 - 8*a*c))*Sqrt[x]*(Sqr
t[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ell
ipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(70*c^(
7/4)*Sqrt[a*x + b*x^3 + c*x^5])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
```

```
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1921

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(x^(m + 1)*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(m + p*(
2*n - q) + 1), x] + Dist[((n - q)*p)/(m + p*(2*n - q) + 1), Int[x^(m + q)*(
2*a + b*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ
[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^
2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && GtQ[m + p*q
+ 1, -(n - q)] && NeQ[m + p*(2*n - q) + 1, 0]
```

Rule 1945

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[(x^(m + 1)*(b*B*(n - q)*p +
 A*c*(m + p*q + (n - q)*(2*p + 1) + 1) + B*c*(m + p*q + 2*(n - q)*p + 1)*x^
(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(c*(m + p*(2*n - q) + 1)*(m + p
*q + (n - q)*(2*p + 1) + 1)), x] + Dist[((n - q)*p)/(c*(m + p*(2*n - q) + 1
)*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(m + q)*Simp[2*a*A*c*(m + p*q +
 (n - q)*(2*p + 1) + 1) - a*b*B*(m + p*q + 1) + (2*a*B*c*(m + p*q + 2*(n -
q)*p + 1) + A*b*c*(m + p*q + (n - q)*(2*p + 1) + 1) - b^2*B*(m + p*q + (n -
 q)*p + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /
; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Intege
rQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q]
&& GtQ[m + p*q, -(n - q) - 1] && NeQ[m + p*(2*n - q) + 1, 0] && NeQ[m + p*q
 + (n - q)*(2*p + 1) + 1, 0]
```

Rule 1953

```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(j_.)))/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x
_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol] :> Dist[(x^(q/2)*Sqrt[a + b*x^(n -
q) + c*x^(2*(n - q))])/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)], Int[(x^(m - q/2
)*(A + B*x^(n - q)))/Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))], x], x] /; Fre
eQ[{a, b, c, A, B, m, n, q}, x] && EqQ[j, n - q] && EqQ[r, 2*n - q] && PosQ
[n - q] && (EqQ[m, 1/2] || EqQ[m, -2^(-1)]) && EqQ[n, 3] && EqQ[q, 1]
```

Rubi steps

$$\int \frac{\left(ax+bx^3+cx^5\right)^{3/2}}{x^{3/2}}\,dx = \frac{\left(ax+bx^3+cx^5\right)^{3/2}}{7\sqrt{x}} + \frac{3}{7}\int \frac{\left(2a+bx^2\right)\sqrt{ax+bx^3+cx^5}}{\sqrt{x}}\,dx$$

$$= \frac{\sqrt{x}\left(b^2+10ac+3bcx^2\right)\sqrt{ax+bx^3+cx^5}}{35c} + \frac{\left(ax+bx^3+cx^5\right)^{3/2}}{7\sqrt{x}} + \int \frac{\sqrt{x}\left(-a\left(b^2-20a\right.\right.}{\sqrt{ax+}}$$

$$= \frac{\sqrt{x}\left(b^2+10ac+3bcx^2\right)\sqrt{ax+bx^3+cx^5}}{35c} + \frac{\left(ax+bx^3+cx^5\right)^{3/2}}{7\sqrt{x}} + \left(\sqrt{x}\sqrt{a+bx^2}\right.$$

$$= \frac{\sqrt{x}\left(b^2+10ac+3bcx^2\right)\sqrt{ax+bx^3+cx^5}}{35c} + \frac{\left(ax+bx^3+cx^5\right)^{3/2}}{7\sqrt{x}} + \left(2\sqrt{a}\,b\left(b^2-8\right.\right.$$

$$= -\frac{2b\left(b^2-8ac\right)x^{3/2}\left(a+bx^2+cx^4\right)}{35c^{3/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{ax+bx^3+cx^5}} + \frac{\sqrt{x}\left(b^2+10ac+3bcx^2\right)\sqrt{ax+bx^3+c}}{35c}$$

Mathematica [C] time = 1.74, size = 540, normalized size = 1.27

$$\frac{\sqrt{x}\left(2cx\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\left(15a^2c+a\left(b^2+23bcx^2+20c^2x^4\right)+x^2\left(b^3+9b^2cx^2+13bc^2x^4+5c^3x^6\right)\right)+i\left(-20a^2c^2+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^3 + c*x^5)^(3/2)/x^(3/2),x]
```

```
[Out] (Sqrt[x]*(2*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(15*a^2*c + a*(b^2 + 23*b*c
*x^2 + 20*c^2*x^4) + x^2*(b^3 + 9*b^2*c*x^2 + 13*b*c^2*x^4 + 5*c^3*x^6)) -
I*b*(b^2 - 8*a*c)*(-b + Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*
c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/
(b - Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 -
 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + I*(-b^4 +
9*a*b^2*c - 20*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 8*a*b*c*Sqrt[b^2 - 4*a*c])
*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b
- 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSi
nh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b -
 Sqrt[b^2 - 4*a*c])]))/(70*c^2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[x*(a +
b*x^2 + c*x^4)])
```

fricas [F] time = 0.88, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\sqrt{cx^5+bx^3+ax}\left(cx^4+bx^2+a\right)}{\sqrt{x}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)^(3/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^5 + b*x^3 + a*x)*(c*x^4 + b*x^2 + a)/sqrt(x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^5+bx^3+ax\right)^{\frac{3}{2}}}{x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)^(3/2)/x^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^5 + b*x^3 + a*x)^(3/2)/x^(3/2), x)
```

maple [B] time = 0.02, size = 1394, normalized size = 3.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^5+b*x^3+a*x)^(3/2)/x^(3/2),x)
```

```
[Out] -1/70*((c*x^4+b*x^2+a)*x)^(1/2)*(-10*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x^9*
b*c^3-10*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-4*a*c+b^2)^(1/2)*x^9*c^3-26*((
-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x^7*b^2*c^2-26*((-b+(-4*a*c+b^2)^(1/2))/a)^
(1/2)*(-4*a*c+b^2)^(1/2)*x^7*b*c^2-40*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x^5
*a*b*c^2-40*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-4*a*c+b^2)^(1/2)*x^5*a*c^2-
18*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x^5*b^3*c-18*((-b+(-4*a*c+b^2)^(1/2))/
a)^(1/2)*(-4*a*c+b^2)^(1/2)*x^5*b^2*c-46*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*
x^3*a*b^2*c-46*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-4*a*c+b^2)^(1/2)*x^3*a*b
*c-2*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x^3*b^4-2*((-b+(-4*a*c+b^2)^(1/2))/a
)^(1/2)*(-4*a*c+b^2)^(1/2)*x^3*b^3+12*(-2*(-b*x^2+(-4*a*c+b^2)^(1/2)*x^2-2*
a)/a)^(1/2)*((b*x^2+(-4*a*c+b^2)^(1/2)*x^2+2*a)/a)^(1/2)*EllipticF(1/2*2^(1
/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*2^(1/2)*((-2*a*c+b^2+(-4*a*c+b^
2)^(1/2)*b)/a/c)^(1/2))*a^2*b*c-20*(-2*(-b*x^2+(-4*a*c+b^2)^(1/2)*x^2-2*a)/
a)^(1/2)*((b*x^2+(-4*a*c+b^2)^(1/2)*x^2+2*a)/a)^(1/2)*EllipticF(1/2*2^(1/2)
*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*2^(1/2)*((-2*a*c+b^2+(-4*a*c+b^2)^
(1/2)*b)/a/c)^(1/2))*(-4*a*c+b^2)^(1/2)*a^2*c-3*(-2*(-b*x^2+(-4*a*c+b^2)^(1
/2)*x^2-2*a)/a)^(1/2)*((b*x^2+(-4*a*c+b^2)^(1/2)*x^2+2*a)/a)^(1/2)*Elliptic
F(1/2*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*2^(1/2)*((-2*a*c+b^2+
(-4*a*c+b^2)^(1/2)*b)/a/c)^(1/2))*a*b^3+(-2*(-b*x^2+(-4*a*c+b^2)^(1/2)*x^2-
2*a)/a)^(1/2)*((b*x^2+(-4*a*c+b^2)^(1/2)*x^2+2*a)/a)^(1/2)*EllipticF(1/2*2^
(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*2^(1/2)*((-2*a*c+b^2+(-4*a*c+
b^2)^(1/2)*b)/a/c)^(1/2))*(-4*a*c+b^2)^(1/2)*a*b^2-32*(-2*(-b*x^2+(-4*a*c+b
^2)^(1/2)*x^2-2*a)/a)^(1/2)*((b*x^2+(-4*a*c+b^2)^(1/2)*x^2+2*a)/a)^(1/2)*El
lipticE(1/2*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*2^(1/2)*((-2*a*
c+b^2+(-4*a*c+b^2)^(1/2)*b)/a/c)^(1/2))*a^2*b*c+4*(-2*(-b*x^2+(-4*a*c+b^2)^
(1/2)*x^2-2*a)/a)^(1/2)*((b*x^2+(-4*a*c+b^2)^(1/2)*x^2+2*a)/a)^(1/2)*Ellipt
icE(1/2*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*2^(1/2)*((-2*a*c+b^
2+(-4*a*c+b^2)^(1/2)*b)/a/c)^(1/2))*a*b^3-30*((-b+(-4*a*c+b^2)^(1/2))/a)^(1
/2)*x*a^2*b*c-30*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-4*a*c+b^2)^(1/2)*x*a^2
*c-2*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x*a*b^3-2*((-b+(-4*a*c+b^2)^(1/2))/a
)^(1/2)*(-4*a*c+b^2)^(1/2)*x*a*b^2)/x^(1/2)/(c*x^4+b*x^2+a)/c/((-b+(-4*a*c+
b^2)^(1/2))/a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^5 + bx^3 + ax\right)^{\frac{3}{2}}}{x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)^(3/2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^5 + b*x^3 + a*x)^(3/2)/x^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c\,x^5+b\,x^3+a\,x\right)}^{3/2}}{x^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^3 + c*x^5)^(3/2)/x^(3/2),x)
```

```
[Out] int((a*x + b*x^3 + c*x^5)^(3/2)/x^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x\left(a+bx^2+cx^4\right)\right)^{\frac{3}{2}}}{x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**5+b*x**3+a*x)**(3/2)/x**(3/2),x)
```

```
[Out] Integral((x*(a + b*x**2 + c*x**4))**(3/2)/x**(3/2), x)
```

3.113 $\int \frac{x^{3/2}}{\sqrt{ax+bx^3+cx^5}}\,dx$

Optimal. Leaf size=82

$$\frac{\sqrt{x}\sqrt{a+bx^2+cx^4}\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{c}\sqrt{ax+bx^3+cx^5}}$$

```
[Out] 1/2*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))*x^(1/2)*(c*x^4+b
*x^2+a)^(1/2)/c^(1/2)/(c*x^5+b*x^3+a*x)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1914, 1107, 621, 206}

$$\frac{\sqrt{x}\sqrt{a+bx^2+cx^4}\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{c}\sqrt{ax+bx^3+cx^5}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/Sqrt[a*x + b*x^3 + c*x^5],x]
```

```
[Out] (Sqrt[x]*Sqrt[a + b*x^2 + c*x^4]*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a +
b*x^2 + c*x^4])])/(2*Sqrt[c]*Sqrt[a*x + b*x^3 + c*x^5])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1914

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))])/Sqrt[
a*x^q + b*x^n + c*x^(2*n - q)], Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

Rubi steps

$$\int \frac{x^{3/2}}{\sqrt{ax+bx^3+cx^5}}\,dx = \frac{\left(\sqrt{x}\sqrt{a+bx^2+cx^4}\right)\int \frac{x}{\sqrt{a+bx^2+cx^4}}\,dx}{\sqrt{ax+bx^3+cx^5}}$$
$$= \frac{\left(\sqrt{x}\sqrt{a+bx^2+cx^4}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)}{2\sqrt{ax+bx^3+cx^5}}$$
$$= \frac{\left(\sqrt{x}\sqrt{a+bx^2+cx^4}\right)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx,x,\frac{b+2cx^2}{\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{ax+bx^3+cx^5}}$$
$$= \frac{\sqrt{x}\sqrt{a+bx^2+cx^4}\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{c}\sqrt{ax+bx^3+cx^5}}$$

Mathematica [A] time = 0.02, size = 82, normalized size = 1.00

$$\frac{\sqrt{x}\sqrt{a+bx^2+cx^4}\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{c}\sqrt{x\left(a+bx^2+cx^4\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/Sqrt[a*x + b*x^3 + c*x^5],x]
```

```
[Out] (Sqrt[x]*Sqrt[a + b*x^2 + c*x^4]*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a +
b*x^2 + c*x^4])])/(2*Sqrt[c]*Sqrt[x*(a + b*x^2 + c*x^4)])
```

fricas [A] time = 0.93, size = 135, normalized size = 1.65

$$\left[\frac{\log\left(-\frac{8\,c^2x^5+8\,bcx^3+4\,\sqrt{cx^5+bx^3+ax}\left(2\,cx^2+b\right)\sqrt{c}\,\sqrt{x}+\left(b^2+4\,ac\right)x}{x}\right)}{4\,\sqrt{c}},-\frac{\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^5+bx^3+ax}\left(2\,cx^2+b\right)\sqrt{-c}\,\sqrt{x}}{2\left(c^2x^5+bcx^3+acx\right)}\right)}{2\,c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*log(-(8*c^2*x^5 + 8*b*c*x^3 + 4*sqrt(c*x^5 + b*x^3 + a*x)*(2*c*x^2 + b
)*sqrt(c)*sqrt(x) + (b^2 + 4*a*c)*x)/x)/sqrt(c), -1/2*sqrt(-c)*arctan(1/2*s
qrt(c*x^5 + b*x^3 + a*x)*(2*c*x^2 + b)*sqrt(-c)*sqrt(x)/(c^2*x^5 + b*c*x^3
+ a*c*x))/c]
```

giac [A] time = 0.63, size = 60, normalized size = 0.73

$$-\frac{\log\left(\left|-2\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}-b\right|\right)}{2\,\sqrt{c}}+\frac{\log\left(\left|-b+2\,\sqrt{a}\,\sqrt{c}\right|\right)}{2\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) - b))/sqrt(
c) + 1/2*log(abs(-b + 2*sqrt(a)*sqrt(c)))/sqrt(c)
```

maple [A] time = 0.01, size = 72, normalized size = 0.88

$$\frac{\sqrt{\left(c\,x^{4}+b\,x^{2}+a\right)x}\;\ln\left(\frac{2c\,x^{2}+b+2\sqrt{c\,x^{4}+b\,x^{2}+a}\;\sqrt{c}}{2\sqrt{c}}\right)}{2\sqrt{c\,x^{4}+b\,x^{2}+a}\;\sqrt{c}\;\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(c*x^5+b*x^3+a*x)^(1/2),x)
```

```
[Out] 1/2/x^(1/2)*((c*x^4+b*x^2+a)*x)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*ln(1/2*(2*c*x^2
+b+2*(c*x^4+b*x^2+a)^(1/2)*c^(1/2))/c^(1/2))/c^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{3}{2}}}{\sqrt{cx^{5}+bx^{3}+ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(3/2)/sqrt(c*x^5 + b*x^3 + a*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{3/2}}{\sqrt{c\,x^{5}+b\,x^{3}+a\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(a*x + b*x^3 + c*x^5)^(1/2),x)
```

```
[Out] int(x^(3/2)/(a*x + b*x^3 + c*x^5)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{3}{2}}}{\sqrt{x\left(a+bx^{2}+cx^{4}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(c*x**5+b*x**3+a*x)**(1/2),x)
```

```
[Out] Integral(x**(3/2)/sqrt(x*(a + b*x**2 + c*x**4)), x)
```

3.114 $\int \frac{\sqrt{x}}{\sqrt{ax+bx^3+cx^5}}\,dx$

Optimal. Leaf size=121

$$\frac{\sqrt{x}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{ax+bx^3+cx^5}}$$

```
[Out] 1/2*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/
4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(
1/2))*(a^(1/2)+x^2*c^(1/2))*x^(1/2)*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^
2)^(1/2)/a^(1/4)/c^(1/4)/(c*x^5+b*x^3+a*x)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1914, 1103}

$$\frac{\sqrt{x}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{ax+bx^3+cx^5}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/Sqrt[a*x + b*x^3 + c*x^5],x]
```

```
[Out] (Sqrt[x]*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c
]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c])
)/4])/(2*a^(1/4)*c^(1/4)*Sqrt[a*x + b*x^3 + c*x^5])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1914

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))])/Sqrt[
a*x^q + b*x^n + c*x^(2*n - q)], Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{x}}{\sqrt{ax+bx^3+cx^5}}\,dx &= \frac{\left(\sqrt{x}\sqrt{a+bx^2+cx^4}\right)\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx}{\sqrt{ax+bx^3+cx^5}}\\ &= \frac{\sqrt{x}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{ax+bx^3+cx^5}}\end{aligned}$$

Mathematica [C] time = 0.12, size = 193, normalized size = 1.60

$$-\frac{i\sqrt{x}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\,F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\sqrt{x\left(a+bx^2+cx^4\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/Sqrt[a*x + b*x^3 + c*x^5],x]
```

```
[Out] ((-I)*Sqrt[x]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]
)]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*
Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 -
 4*a*c])])/(Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[x*(a + b*x^2 + c*x
^4)])
```

fricas [F] time = 0.89, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x}}{\sqrt{cx^5+bx^3+ax}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x)/sqrt(c*x^5 + b*x^3 + a*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x}}{\sqrt{cx^5+bx^3+ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x)/sqrt(c*x^5 + b*x^3 + a*x), x)
```

maple [A] time = 0.02, size = 177, normalized size = 1.46

$$\frac{\sqrt{\left(c\,x^4+b\,x^2+a\right)x}\,\sqrt{-\frac{2\left(-b\,x^2+\sqrt{-4ac+b^2}\,x^2-2a\right)}{a}}\,\sqrt{\frac{b\,x^2+\sqrt{-4ac+b^2}\,x^2+2a}{a}}\,\operatorname{EllipticF}\left(\frac{\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,x}{2},\frac{\sqrt{2}\,\sqrt{\frac{-2ac+b^2+\sqrt{-4ac+b^2}\,b}{ac}}}{2}\right)}{2\left(c\,x^4+b\,x^2+a\right)\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(c*x^5+b*x^3+a*x)^(1/2),x)
```

```
[Out] 1/2/x^(1/2)*((c*x^4+b*x^2+a)*x)^(1/2)/(c*x^4+b*x^2+a)/((-b+(-4*a*c+b^2)^(1/
2))/a)^(1/2)*(-2*(-b*x^2+(-4*a*c+b^2)^(1/2)*x^2-2*a)/a)^(1/2)*((b*x^2+(-4*a
*c+b^2)^(1/2)*x^2+2*a)/a)^(1/2)*EllipticF(1/2*2^(1/2)*((-b+(-4*a*c+b^2)^(1/
2))/a)^(1/2)*x,1/2*2^(1/2)*((-2*a*c+b^2+(-4*a*c+b^2)^(1/2)*b)/a/c)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x}}{\sqrt{cx^5+bx^3+ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(x)/sqrt(c*x^5 + b*x^3 + a*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{x}}{\sqrt{c\,x^5+b\,x^3+a\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a*x + b*x^3 + c*x^5)^(1/2),x)
[Out] int(x^(1/2)/(a*x + b*x^3 + c*x^5)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x}}{\sqrt{x\left(a+bx^2+cx^4\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(c*x**5+b*x**3+a*x)**(1/2),x)
[Out] Integral(sqrt(x)/sqrt(x*(a + b*x**2 + c*x**4)), x)
```

3.115 $\int \frac{1}{\sqrt{x}\sqrt{ax+bx^3+cx^5}}\,dx$

Optimal. Leaf size=51

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{x}\left(2a+bx^2\right)}{2\sqrt{a}\sqrt{ax+bx^3+cx^5}}\right)}{2\sqrt{a}}$$

[Out] -1/2*arctanh(1/2*(b*x^2+2*a)*x^(1/2)/a^(1/2)/(c*x^5+b*x^3+a*x)^(1/2))/a^(1/2)

Rubi [A] time = 0.03, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1913, 206}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{x}\left(2a+bx^2\right)}{2\sqrt{a}\sqrt{ax+bx^3+cx^5}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*Sqrt[a*x + b*x^3 + c*x^5]),x]
```

```
[Out] -ArcTanh[(Sqrt[x]*(2*a + b*x^2))/(2*Sqrt[a]*Sqrt[a*x + b*x^3 + c*x^5])]/(2*Sqrt[a])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 1913

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol] :> Dist[-2/(n - q), Subst[Int[1/(4*a - x^2), x], x, (x^(m + 1)*(2*a + b*x^(n - q)))/Sqrt[a*x^q + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, m, n, q, r}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && NeQ[b^2 - 4*a*c, 0] && EqQ[m, q/2 - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x}\sqrt{ax+bx^3+cx^5}}\,dx &= -\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{\sqrt{x}\left(2a+bx^2\right)}{\sqrt{ax+bx^3+cx^5}}\right)\\ &= -\frac{\tanh^{-1}\left(\frac{\sqrt{x}\left(2a+bx^2\right)}{2\sqrt{a}\sqrt{ax+bx^3+cx^5}}\right)}{2\sqrt{a}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 1.63

$$-\frac{\sqrt{x}\sqrt{a+bx^2+cx^4}\,\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}\sqrt{x\left(a+bx^2+cx^4\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*Sqrt[a*x + b*x^3 + c*x^5]),x]
```

```
[Out] -1/2*(Sqrt[x]*Sqrt[a + b*x^2 + c*x^4]*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt
[a + b*x^2 + c*x^4])])/(Sqrt[a]*Sqrt[x*(a + b*x^2 + c*x^4)])
```

fricas [A] time = 0.83, size = 137, normalized size = 2.69

$$\left[\frac{\log\left(-\frac{\left(b^2+4\,ac\right)x^5+8\,abx^3+8\,a^2x-4\,\sqrt{cx^5+bx^3+ax}\left(bx^2+2\,a\right)\sqrt{a}\,\sqrt{x}}{x^5}\right)}{4\,\sqrt{a}}, \frac{\sqrt{-a}\,\arctan\left(\frac{\sqrt{cx^5+bx^3+ax}\left(bx^2+2\,a\right)\sqrt{-a}\,\sqrt{x}}{2\left(acx^5+abx^3+a^2x\right)}\right)}{2\,a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*log(-((b^2 + 4*a*c)*x^5 + 8*a*b*x^3 + 8*a^2*x - 4*sqrt(c*x^5 + b*x^3 +
 a*x)*(b*x^2 + 2*a)*sqrt(a)*sqrt(x))/x^5)/sqrt(a), 1/2*sqrt(-a)*arctan(1/2*
sqrt(c*x^5 + b*x^3 + a*x)*(b*x^2 + 2*a)*sqrt(-a)*sqrt(x)/(a*c*x^5 + a*b*x^3
 + a^2*x))/a]
```

giac [A] time = 0.54, size = 56, normalized size = 1.10

$$\frac{\arctan\left(-\frac{\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))/sqrt(-a))/sqrt(-a) - arctan
(sqrt(a)/sqrt(-a))/sqrt(-a)
```

maple [A] time = 0.02, size = 72, normalized size = 1.41

$$-\frac{\sqrt{\left(c\,x^4+b\,x^2+a\right)x}\;\ln\left(\frac{b\,x^2+2a+2\sqrt{c\,x^4+b\,x^2+a}\;\sqrt{a}}{x^2}\right)}{2\sqrt{c\,x^4+b\,x^2+a}\;\sqrt{a}\;\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(c*x^5+b*x^3+a*x)^(1/2),x)
```

```
[Out] -1/2/x^(1/2)*((c*x^4+b*x^2+a)*x)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/a^(1/2)*ln((b*
x^2+2*a+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/2))/x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^5+bx^3+ax}\,\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^5 + b*x^3 + a*x)*sqrt(x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{x}\;\sqrt{c\,x^5+b\,x^3+a\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(a*x + b*x^3 + c*x^5)^(1/2)),x)
```

```
[Out] int(1/(x^(1/2)*(a*x + b*x^3 + c*x^5)^(1/2)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x}\,\sqrt{x\left(a+bx^2+cx^4\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(c*x**5+b*x**3+a*x)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(x)*sqrt(x*(a + b*x**2 + c*x**4))), x)
```

3.116 $\int \frac{1}{x^{3/2}\sqrt{ax+bx^3+cx^5}}\,dx$

Optimal. Leaf size=330

$$\frac{\sqrt[4]{c}\sqrt{x}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2a^{3/4}\sqrt{ax+bx^3+cx^5}}-\frac{\sqrt[4]{c}\sqrt{x}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\right.}{a^{3/4}\sqrt{ax+bx^3+}}$$

```
[Out] x^(3/2)*(c*x^4+b*x^2+a)*c^(1/2)/a/(a^(1/2)+x^2*c^(1/2))/(c*x^5+b*x^3+a*x)^(
1/2)-(c*x^5+b*x^3+a*x)^(1/2)/a/x^(3/2)-c^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1
/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1
/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*x^(1
/2)*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(3/4)/(c*x^5+b*x^3+a*
x)^(1/2)+1/2*c^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arcta
n(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a
^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*x^(1/2)*((c*x^4+b*x^2+a)/(a^(1
/2)+x^2*c^(1/2))^2)^(1/2)/a^(3/4)/(c*x^5+b*x^3+a*x)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1929, 12, 1914, 1139, 1103, 1195}

$$\frac{\sqrt[4]{c}\sqrt{x}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2a^{3/4}\sqrt{ax+bx^3+cx^5}}-\frac{\sqrt[4]{c}\sqrt{x}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\right.}{a^{3/4}\sqrt{ax+bx^3+}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*Sqrt[a*x + b*x^3 + c*x^5]),x]
```

```
[Out] (Sqrt[c]*x^(3/2)*(a + b*x^2 + c*x^4))/(a*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[a*x +
 b*x^3 + c*x^5]) - Sqrt[a*x + b*x^3 + c*x^5]/(a*x^(3/2)) - (c^(1/4)*Sqrt[x]
*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2
]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(a
^(3/4)*Sqrt[a*x + b*x^3 + c*x^5]) + (c^(1/4)*Sqrt[x]*(Sqrt[a] + Sqrt[c]*x^2
)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c
^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(3/4)*Sqrt[a*x + b*x
^3 + c*x^5])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1139

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[1/q, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[1/q, I
nt[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1914

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))])/Sqrt[
a*x^q + b*x^n + c*x^(2*n - q)], Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

Rule 1929

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(x^(m - q + 1)*(a*x^q + b*x^n + c*x^(2*n - q))^(p + 1)
)/(a*(m + p*q + 1)), x] - Dist[1/(a*(m + p*q + 1)), Int[x^(m + n - q)*(b*(m
 + p*q + (n - q)*(p + 1) + 1) + c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n -
q))*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c}, x] && EqQ
[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ
[n, 0] && GeQ[p, -1] && LtQ[p, 0] && RationalQ[m, q] && LtQ[m + p*q + 1, 0]
```

Rubi steps

$$\int \frac{1}{x^{3/2}\sqrt{ax+bx^3+cx^5}}\,dx = -\frac{\sqrt{ax+bx^3+cx^5}}{ax^{3/2}}+\frac{\int \frac{cx^{5/2}}{\sqrt{ax+bx^3+cx^5}}\,dx}{a}$$

$$= -\frac{\sqrt{ax+bx^3+cx^5}}{ax^{3/2}}+\frac{c\int \frac{x^{5/2}}{\sqrt{ax+bx^3+cx^5}}\,dx}{a}$$

$$= -\frac{\sqrt{ax+bx^3+cx^5}}{ax^{3/2}}+\frac{\left(c\sqrt{x}\sqrt{a+bx^2+cx^4}\right)\int \frac{x^2}{\sqrt{a+bx^2+cx^4}}\,dx}{a\sqrt{ax+bx^3+cx^5}}$$

$$= -\frac{\sqrt{ax+bx^3+cx^5}}{ax^{3/2}}+\frac{\left(\sqrt{c}\sqrt{x}\sqrt{a+bx^2+cx^4}\right)\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx}{\sqrt{a}\sqrt{ax+bx^3+cx^5}}-\frac{\left(\sqrt{c}\sqrt{x}\sqrt{a+bx^2 \cdots\right.}{\sqrt{a}\sqrt{a \cdots}}$$

$$= \frac{\sqrt{c}\,x^{3/2}\left(a+bx^2+cx^4\right)}{a\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{ax+bx^3+cx^5}}-\frac{\sqrt{ax+bx^3+cx^5}}{ax^{3/2}}-\frac{\sqrt[4]{c}\sqrt{x}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a \cdots}{(\sqrt{\cdots}}}}{a^{3/ \cdots}}$$

Mathematica [C] time = 0.47, size = 303, normalized size = 0.92

$$\frac{-4\left(a+bx^2+cx^4\right)+\frac{i\sqrt{2}x\left(\sqrt{b^2-4ac}-b\right)\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\left(E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)-F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{\cdots}{b\cdots}}\cdots\right.\right.\right.}{\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}}}{4a\sqrt{x}\sqrt{x\left(a+bx^2+cx^4\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*Sqrt[a*x + b*x^3 + c*x^5]),x]
```

```
[Out] (-4*(a + b*x^2 + c*x^4) + (I*Sqrt[2]*(-b + Sqrt[b^2 - 4*a*c])*x*Sqrt[(b + S
qrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b
- Sqrt[b^2 - 4*a*c])]*(EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4
*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - EllipticF[I*
ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])
/(b - Sqrt[b^2 - 4*a*c])]))/Sqrt[c/(b + Sqrt[b^2 - 4*a*c])])/(4*a*Sqrt[x]*S
qrt[x*(a + b*x^2 + c*x^4)])
```

fricas [F] time = 0.91, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^5 + bx^3 + ax}\,\sqrt{x}}{cx^7 + bx^5 + ax^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^5 + b*x^3 + a*x)*sqrt(x)/(c*x^7 + b*x^5 + a*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^5 + bx^3 + ax}\,x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^5 + b*x^3 + a*x)*x^(3/2)), x)
```

maple [A] time = 0.02, size = 508, normalized size = 1.54

$$\left(-\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,bc\,x^4-\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{-4ac+b^2}\,c\,x^4+\sqrt{-\frac{2\left(-b\,x^2+\sqrt{-4ac+b^2}\,x^2-2a\right)}{a}}\,\sqrt{\frac{b\,x^2+\sqrt{-4ac+b^2}\,x^2+2a}{a}}\,ac\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(c*x^5+b*x^3+a*x)^(1/2),x)
```

```
[Out] (-((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-4*a*c+b^2)^(1/2)*x^4*c-((-b+(-4*a*c+b
^2)^(1/2))/a)^(1/2)*x^4*b*c-c*(-2*(-b*x^2+(-4*a*c+b^2)^(1/2)*x^2-2*a)/a)^(1
/2)*((b*x^2+(-4*a*c+b^2)^(1/2)*x^2+2*a)/a)^(1/2)*a*x*EllipticF(1/2*2^(1/2)*
((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*2^(1/2)*((-2*a*c+b^2+(-4*a*c+b^2)^(
1/2)*b)/a/c)^(1/2))+c*(-2*(-b*x^2+(-4*a*c+b^2)^(1/2)*x^2-2*a)/a)^(1/2)*((b*
x^2+(-4*a*c+b^2)^(1/2)*x^2+2*a)/a)^(1/2)*a*x*EllipticE(1/2*2^(1/2)*((-b+(-4
*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*2^(1/2)*((-2*a*c+b^2+(-4*a*c+b^2)^(1/2)*b)/
a/c)^(1/2))-((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-4*a*c+b^2)^(1/2)*x^2*b-((-b
+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x^2*b^2-((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-4
*a*c+b^2)^(1/2)*a-((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*a*b)/x^(3/2)*((c*x^4+b*
x^2+a)*x)^(1/2)/(c*x^4+b*x^2+a)/a/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)/(b+(-4*
a*c+b^2)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^5 + bx^3 + ax}\,x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(c*x^5 + b*x^3 + a*x)*x^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^{3/2}\,\sqrt{c\,x^5+b\,x^3+a\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(a*x + b*x^3 + c*x^5)^(1/2)),x)
[Out] int(1/(x^(3/2)*(a*x + b*x^3 + c*x^5)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{3}{2}}\sqrt{x\left(a+bx^2+cx^4\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(c*x**5+b*x**3+a*x)**(1/2),x)
[Out] Integral(1/(x**(3/2)*sqrt(x*(a + b*x**2 + c*x**4))), x)
```

3.117 $\int \frac{x^{3/2}}{\left(ax+bx^3+cx^5\right)^{3/2}}\,dx$

Optimal. Leaf size=391

$$\frac{b\sqrt[4]{c}\,\sqrt{x}\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right)}{a^{3/4}\left(b^2-4ac\right)\sqrt{ax+bx^3+cx^5}}-\frac{\sqrt[4]{c}\,\sqrt{x}\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\right.}{2a^{3/4}\left(b-2\sqrt{a}\,\sqrt{c}\right)\sqrt{a}}$$

```
[Out] x^(3/2)*(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/(c*x^5+b*x^3+a*x)^(1/2)-b*x^(3/2
)*(c*x^4+b*x^2+a)*c^(1/2)/a/(-4*a*c+b^2)/(a^(1/2)+x^2*c^(1/2))/(c*x^5+b*x^3
+a*x)^(1/2)+b*c^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arct
an(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/
a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*x^(1/2)*((c*x^4+b*x^2+a)/(a^(
1/2)+x^2*c^(1/2))^2)^(1/2)/a^(3/4)/(-4*a*c+b^2)/(c*x^5+b*x^3+a*x)^(1/2)-1/2
*c^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/
a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/
2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*x^(1/2)*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1
/2))^2)^(1/2)/a^(3/4)/(-2*a^(1/2)*c^(1/2)+b)/(c*x^5+b*x^3+a*x)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 391, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1924, 1953, 1197, 1103, 1195}

$$\frac{b\sqrt[4]{c}\,\sqrt{x}\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right)}{a^{3/4}\left(b^2-4ac\right)\sqrt{ax+bx^3+cx^5}}-\frac{\sqrt[4]{c}\,\sqrt{x}\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\right.}{2a^{3/4}\left(b-2\sqrt{a}\,\sqrt{c}\right)\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(a*x + b*x^3 + c*x^5)^(3/2),x]
```

```
[Out] (x^(3/2)*(b^2 - 2*a*c + b*c*x^2))/(a*(b^2 - 4*a*c)*Sqrt[a*x + b*x^3 + c*x^5
]) - (b*Sqrt[c]*x^(3/2)*(a + b*x^2 + c*x^4))/(a*(b^2 - 4*a*c)*(Sqrt[a] + Sq
rt[c]*x^2)*Sqrt[a*x + b*x^3 + c*x^5]) + (b*c^(1/4)*Sqrt[x]*(Sqrt[a] + Sqrt[
c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*Arc
Tan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(a^(3/4)*(b^2 - 4*a
*c)*Sqrt[a*x + b*x^3 + c*x^5]) - (c^(1/4)*Sqrt[x]*(Sqrt[a] + Sqrt[c]*x^2)*S
qrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1
/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(3/4)*(b - 2*Sqrt[a]*Sq
rt[c])*Sqrt[a*x + b*x^3 + c*x^5])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1924

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> -Simp[(x^(m - q + 1)*(b^2 - 2*a*c + b*c*x^(n - q))*(a*x^q +
 b*x^n + c*x^(2*n - q))^(p + 1))/(a*(n - q)*(p + 1)*(b^2 - 4*a*c)), x] + Di
st[1/(a*(n - q)*(p + 1)*(b^2 - 4*a*c)), Int[x^(m - q)*(b^2*(m + p*q + (n -
q)*(p + 1) + 1) - 2*a*c*(m + p*q + 2*(n - q)*(p + 1) + 1) + b*c*(m + p*q +
(n - q)*(2*p + 3) + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p + 1),
x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] &&  !Intege
rQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p, -1] && RationalQ[m, q]
 && LtQ[m + p*q + 1, n - q]
```

Rule 1953

```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(j_.)))/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x
_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol] :> Dist[(x^(q/2)*Sqrt[a + b*x^(n -
q) + c*x^(2*(n - q))])/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)], Int[(x^(m - q/2
)*(A + B*x^(n - q)))/Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))], x], x] /; Fre
eQ[{a, b, c, A, B, m, n, q}, x] && EqQ[j, n - q] && EqQ[r, 2*n - q] && PosQ
[n - q] && (EqQ[m, 1/2] || EqQ[m, -2^(-1)]) && EqQ[n, 3] && EqQ[q, 1]
```

Rubi steps

$$\int \frac{x^{3/2}}{\left(ax+bx^3+cx^5\right)^{3/2}}\,dx = \frac{x^{3/2}\left(b^2-2ac+bcx^2\right)}{a\left(b^2-4ac\right)\sqrt{ax+bx^3+cx^5}} - \frac{\int \frac{\sqrt{x}\left(2ac+bcx^2\right)}{\sqrt{ax+bx^3+cx^5}}\,dx}{a\left(b^2-4ac\right)}$$

$$= \frac{x^{3/2}\left(b^2-2ac+bcx^2\right)}{a\left(b^2-4ac\right)\sqrt{ax+bx^3+cx^5}} - \frac{\left(\sqrt{x}\sqrt{a+bx^2+cx^4}\right)\int \frac{2ac+bcx^2}{\sqrt{a+bx^2+cx^4}}\,dx}{a\left(b^2-4ac\right)\sqrt{ax+bx^3+cx^5}}$$

$$= \frac{x^{3/2}\left(b^2-2ac+bcx^2\right)}{a\left(b^2-4ac\right)\sqrt{ax+bx^3+cx^5}} + \frac{\left(b\sqrt{c}\sqrt{x}\sqrt{a+bx^2+cx^4}\right)\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+bx^2+cx^4}}\,dx}{\sqrt{a}\left(b^2-4ac\right)\sqrt{ax+bx^3+cx^5}} - \frac{\left(\left(b+\right.\right.}{}$$

$$= \frac{x^{3/2}\left(b^2-2ac+bcx^2\right)}{a\left(b^2-4ac\right)\sqrt{ax+bx^3+cx^5}} - \frac{b\sqrt{c}\,x^{3/2}\left(a+bx^2+cx^4\right)}{a\left(b^2-4ac\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{ax+bx^3+cx^5}} + \frac{b\sqrt[4]{c}}{}$$

Mathematica [C] time = 1.03, size = 463, normalized size = 1.18

$$-\frac{\sqrt{x}\left(-4x\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\left(-2ac+b^2+bcx^2\right)-i\left(b\sqrt{b^2-4ac}+4ac-b^2\right)\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{-2\sqrt{b^2-4ac}+2b+4cx^2}{b-\sqrt{b^2-4ac}}}F\left(i\right.\right.}{4a\left(b^2-\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(a*x + b*x^3 + c*x^5)^(3/2),x]
```

```
[Out] -1/4*(Sqrt[x]*(-4*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(b^2 - 2*a*c + b*c*x^2)
 + I*b*(-b + Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b +
 Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b
^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x
], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - I*(-b^2 + 4*a*c + b*S
qrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*
a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*
EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b
^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]))/(a*(b^2 - 4*a*c)*Sqrt[c/(b + Sqrt[b
^2 - 4*a*c])]*Sqrt[x*(a + b*x^2 + c*x^4)])
```

fricas [F] time = 0.86, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^5+bx^3+ax}\,\sqrt{x}}{c^2x^9+2\,bcx^7+\left(b^2+2\,ac\right)x^5+2\,abx^3+a^2x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^5+b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^5 + b*x^3 + a*x)*sqrt(x)/(c^2*x^9 + 2*b*c*x^7 + (b^2 + 2*
a*c)*x^5 + 2*a*b*x^3 + a^2*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{3}{2}}}{\left(cx^5+bx^3+ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^5+b*x^3+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^(3/2)/(c*x^5 + b*x^3 + a*x)^(3/2), x)
```

maple [A] time = 0.02, size = 533, normalized size = 1.36

$$\frac{\sqrt{\left(c\,x^4+b\,x^2+a\right)x}\,\left(-\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\;b^2c\,x^3-\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\;\sqrt{-4ac+b^2}\;bc\,x^3+2\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\;abcx+\sqrt{\vphantom{a}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(c*x^5+b*x^3+a*x)^(3/2),x)
```

```
[Out] ((c*x^4+b*x^2+a)*x)^(1/2)*(-((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x^3*b^2*c-((-
b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-4*a*c+b^2)^(1/2)*x^3*b*c+c*(-2*(-b*x^2+(-4
*a*c+b^2)^(1/2)*x^2-2*a)/a)^(1/2)*((b*x^2+(-4*a*c+b^2)^(1/2)*x^2+2*a)/a)^(1
/2)*EllipticF(1/2*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*2^(1/2)*(
(-2*a*c+b^2+(-4*a*c+b^2)^(1/2)*b)/a/c)^(1/2))*a*(-4*a*c+b^2)^(1/2)+b*c*(-2*
(-b*x^2+(-4*a*c+b^2)^(1/2)*x^2-2*a)/a)^(1/2)*((b*x^2+(-4*a*c+b^2)^(1/2)*x^2
+2*a)/a)^(1/2)*a*EllipticE(1/2*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x,
1/2*2^(1/2)*((-2*a*c+b^2+(-4*a*c+b^2)^(1/2)*b)/a/c)^(1/2))+2*((-b+(-4*a*c+b
^2)^(1/2))/a)^(1/2)*x*a*b*c+2*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-4*a*c+b^2
)^(1/2)*x*a*c-((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x*b^3-((-b+(-4*a*c+b^2)^(1/
2))/a)^(1/2)*(-4*a*c+b^2)^(1/2)*x*b^2)/x^(1/2)/(c*x^4+b*x^2+a)/a/(4*a*c-b^2
)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{3}{2}}}{\left(cx^5 + bx^3 + ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^5+b*x^3+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(3/2)/(c*x^5 + b*x^3 + a*x)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{3/2}}{\left(c\,x^5 + b\,x^3 + a\,x\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(a*x + b*x^3 + c*x^5)^(3/2),x)
```

```
[Out] int(x^(3/2)/(a*x + b*x^3 + c*x^5)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(c*x**5+b*x**3+a*x)**(3/2),x)
```

```
[Out] Timed out
```

3.118 $\int \frac{\sqrt{x}}{\left(ax+bx^3+cx^5\right)^{3/2}}\,dx$

Optimal. Leaf size=103

$$\frac{\sqrt{x}\left(-2ac+b^2+bcx^2\right)}{a\left(b^2-4ac\right)\sqrt{ax+bx^3+cx^5}}-\frac{\tanh^{-1}\left(\frac{\sqrt{x}\left(2a+bx^2\right)}{2\sqrt{a}\sqrt{ax+bx^3+cx^5}}\right)}{2a^{3/2}}$$

```
[Out] -1/2*arctanh(1/2*(b*x^2+2*a)*x^(1/2)/a^(1/2)/(c*x^5+b*x^3+a*x)^(1/2))/a^(3/
2)+(b*c*x^2-2*a*c+b^2)*x^(1/2)/a/(-4*a*c+b^2)/(c*x^5+b*x^3+a*x)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1922, 1913, 206}

$$\frac{\sqrt{x}\left(-2ac+b^2+bcx^2\right)}{a\left(b^2-4ac\right)\sqrt{ax+bx^3+cx^5}}-\frac{\tanh^{-1}\left(\frac{\sqrt{x}\left(2a+bx^2\right)}{2\sqrt{a}\sqrt{ax+bx^3+cx^5}}\right)}{2a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(a*x + b*x^3 + c*x^5)^(3/2),x]
```

```
[Out] (Sqrt[x]*(b^2 - 2*a*c + b*c*x^2))/(a*(b^2 - 4*a*c)*Sqrt[a*x + b*x^3 + c*x^5
]) - ArcTanh[(Sqrt[x]*(2*a + b*x^2))/(2*Sqrt[a]*Sqrt[a*x + b*x^3 + c*x^5])]
/(2*a^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1913

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[-2/(n - q), Subst[Int[1/(4*a - x^2), x], x, (x^(m + 1)*
(2*a + b*x^(n - q)))/Sqrt[a*x^q + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, m,
 n, q, r}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && NeQ[b^2 - 4*a*c, 0] && E
qQ[m, q/2 - 1]
```

Rule 1922

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> -Simp[(x^(m - q + 1)*(b^2 - 2*a*c + b*c*x^(n - q))*(a*x^q +
 b*x^n + c*x^(2*n - q))^(p + 1))/(a*(n - q)*(p + 1)*(b^2 - 4*a*c)), x] + Di
st[(2*a*c - b^2*(p + 2))/(a*(p + 1)*(b^2 - 4*a*c)), Int[x^(m - q)*(a*x^q +
b*x^n + c*x^(2*n - q))^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n
 - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
&& LtQ[p, -1] && RationalQ[m, p, q] && EqQ[m + p*q + 1, -((n - q)*(2*p + 3)
)]
```

Rubi steps

$$\int \frac{\sqrt{x}}{\left(ax+bx^3+cx^5\right)^{3/2}}\,dx=\frac{\sqrt{x}\left(b^2-2ac+bcx^2\right)}{a\left(b^2-4ac\right)\sqrt{ax+bx^3+cx^5}}+\frac{\int \frac{1}{\sqrt{x}\sqrt{ax+bx^3+cx^5}}\,dx}{a}$$

$$=\frac{\sqrt{x}\left(b^2-2ac+bcx^2\right)}{a\left(b^2-4ac\right)\sqrt{ax+bx^3+cx^5}}-\frac{\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx,x,\frac{\sqrt{x}\left(2a+bx^2\right)}{\sqrt{ax+bx^3+cx^5}}\right)}{a}$$

$$=\frac{\sqrt{x}\left(b^2-2ac+bcx^2\right)}{a\left(b^2-4ac\right)\sqrt{ax+bx^3+cx^5}}-\frac{\tanh^{-1}\left(\frac{\sqrt{x}\left(2a+bx^2\right)}{2\sqrt{a}\sqrt{ax+bx^3+cx^5}}\right)}{2a^{3/2}}$$

Mathematica [A] time = 0.08, size = 126, normalized size = 1.22

$$\frac{\sqrt{x}\left(\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)-2\sqrt{a}\left(-2ac+b^2+bcx^2\right)\right)}{2a^{3/2}\left(4ac-b^2\right)\sqrt{x\left(a+bx^2+cx^4\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(a*x + b*x^3 + c*x^5)^(3/2),x]
```

```
[Out] (Sqrt[x]*(-2*Sqrt[a]*(b^2 - 2*a*c + b*c*x^2) + (b^2 - 4*a*c)*Sqrt[a + b*x^2
 + c*x^4]*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])]))/(2*a
^(3/2)*(-b^2 + 4*a*c)*Sqrt[x*(a + b*x^2 + c*x^4)])
```

fricas [B] time = 0.80, size = 424, normalized size = 4.12

$$\left[\frac{\left(\left(b^2c-4\,ac^2\right)x^5+\left(b^3-4\,abc\right)x^3+\left(ab^2-4\,a^2c\right)x\right)\sqrt{a}\log\left(-\frac{\left(b^2+4\,ac\right)x^5+8\,abx^3+8\,a^2x-4\,\sqrt{cx^5+bx^3+ax}\left(bx^2+2\,a\right)\sqrt{a}\sqrt{x}}{x^5}\right)+4}{4\left(\left(a^2b^2c-4\,a^3c^2\right)x^5+\left(a^2b^3-4\,a^3bc\right)x^3+\left(a^3b^2-4\,a^4c\right)x\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^5+b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((b^2*c - 4*a*c^2)*x^5 + (b^3 - 4*a*b*c)*x^3 + (a*b^2 - 4*a^2*c)*x)*s
qrt(a)*log(-((b^2 + 4*a*c)*x^5 + 8*a*b*x^3 + 8*a^2*x - 4*sqrt(c*x^5 + b*x^3
 + a*x)*(b*x^2 + 2*a)*sqrt(a)*sqrt(x))/x^5) + 4*sqrt(c*x^5 + b*x^3 + a*x)*(
a*b*c*x^2 + a*b^2 - 2*a^2*c)*sqrt(x))/((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b
^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x), 1/2*(((b^2*c - 4*a*c^2)*x^5 +
 (b^3 - 4*a*b*c)*x^3 + (a*b^2 - 4*a^2*c)*x)*sqrt(-a)*arctan(1/2*sqrt(c*x^5
+ b*x^3 + a*x)*(b*x^2 + 2*a)*sqrt(-a)*sqrt(x)/(a*c*x^5 + a*b*x^3 + a^2*x))
+ 2*sqrt(c*x^5 + b*x^3 + a*x)*(a*b*c*x^2 + a*b^2 - 2*a^2*c)*sqrt(x))/((a^2*
b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x)
]
```

giac [B] time = 0.66, size = 193, normalized size = 1.87

$$\frac{\frac{abcx^2}{a^2b^2-4\,a^3c}+\frac{ab^2-2\,a^2c}{a^2b^2-4\,a^3c}}{\sqrt{cx^4+bx^2+a}}-\frac{ab^2\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)-4\,a^2c\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)+\sqrt{-a}\sqrt{a}\,b^2-2\,\sqrt{-a}\,a^{\frac{3}{2}}c}{\sqrt{-a}\,a^2b^2-4\,\sqrt{-a}\,a^3c}+\frac{\arctan\left(-\frac{\sqrt{c}x^2-\sqrt{cx^4+bx^2}}{\sqrt{-a}}\right.}{\sqrt{-a}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^5+b*x^3+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] (a*b*c*x^2/(a^2*b^2 - 4*a^3*c) + (a*b^2 - 2*a^2*c)/(a^2*b^2 - 4*a^3*c))/sqr
t(c*x^4 + b*x^2 + a) - (a*b^2*arctan(sqrt(a)/sqrt(-a)) - 4*a^2*c*arctan(sqr
t(a)/sqrt(-a)) + sqrt(-a)*sqrt(a)*b^2 - 2*sqrt(-a)*a^(3/2)*c)/(sqrt(-a)*a^2
*b^2 - 4*sqrt(-a)*a^3*c) + arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))/
sqrt(-a))/(sqrt(-a)*a)
```

maple [B] time = 0.02, size = 179, normalized size = 1.74

$$-\frac{\sqrt{\left(c\,x^{4}+b\,x^{2}+a\right)x}\,\left(2\sqrt{a}\,bc\,x^{2}+4\sqrt{c\,x^{4}+b\,x^{2}+a}\,ac\ln\left(\frac{b\,x^{2}+2a+2\sqrt{c\,x^{4}+b\,x^{2}+a}\,\sqrt{a}}{x^{2}}\right)-\sqrt{c\,x^{4}+b\,x^{2}+a}\,b^{2}\ln\right.}{2\left(c\,x^{4}+b\,x^{2}+a\right)\left(4ac-b^{2}\right)a^{\frac{3}{2}}\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(c*x^5+b*x^3+a*x)^(3/2),x)
```

```
[Out] -1/2*((c*x^4+b*x^2+a)*x)^(1/2)/a^(3/2)*(2*x^2*b*c*a^(1/2)+4*ln((b*x^2+2*a+2
*(c*x^4+b*x^2+a)^(1/2)*a^(1/2))/x^2)*a*c*(c*x^4+b*x^2+a)^(1/2)-ln((b*x^2+2*
a+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/2))/x^2)*b^2*(c*x^4+b*x^2+a)^(1/2)-4*a^(3/2)
*c+2*b^2*a^(1/2))/x^(1/2)/(c*x^4+b*x^2+a)/(4*a*c-b^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x}}{\left(cx^{5}+bx^{3}+ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^5+b*x^3+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x)/(c*x^5 + b*x^3 + a*x)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{x}}{\left(c\,x^{5}+b\,x^{3}+a\,x\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a*x + b*x^3 + c*x^5)^(3/2),x)
```

```
[Out] int(x^(1/2)/(a*x + b*x^3 + c*x^5)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x}}{\left(x\left(a+bx^{2}+cx^{4}\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(c*x**5+b*x**3+a*x)**(3/2),x)
```

```
[Out] Integral(sqrt(x)/(x*(a + b*x**2 + c*x**4))**(3/2), x)
```

3.119 $\int \frac{1}{\sqrt{x}\left(ax+bx^3+cx^5\right)^{3/2}}\,dx$

Optimal. Leaf size=468

$$\frac{\sqrt[4]{c}\sqrt{x}\left(\sqrt{a}\,b\sqrt{c}-6ac+2b^2\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2a^{7/4}\left(b^2-4ac\right)\sqrt{ax+bx^3+cx^5}}-\frac{2\sqrt[4]{c}\sqrt{x}\left(b^2-3ac\right)\left(\cdots\right.}{\cdots}$$

```
[Out] 2*(-3*a*c+b^2)*x^(3/2)*(c*x^4+b*x^2+a)*c^(1/2)/a^2/(-4*a*c+b^2)/(a^(1/2)+x^
2*c^(1/2))/(c*x^5+b*x^3+a*x)^(1/2)+(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/x^(1/
2)/(c*x^5+b*x^3+a*x)^(1/2)-2*(-3*a*c+b^2)*(c*x^5+b*x^3+a*x)^(1/2)/a^2/(-4*a
*c+b^2)/x^(3/2)-2*c^(1/4)*(-3*a*c+b^2)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)
^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^
(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*x^(1/2)*((c*
x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(7/4)/(-4*a*c+b^2)/(c*x^5+b*x
^3+a*x)^(1/2)+1/2*c^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*
arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(
2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*(2*b^2-6*a*c+b*a^(1/2)*c^
(1/2))*x^(1/2)*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(7/4)/(-4*
a*c+b^2)/(c*x^5+b*x^3+a*x)^(1/2)
```

Rubi [A] time = 0.41, antiderivative size = 468, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1924, 1951, 1953, 1197, 1103, 1195}

$$\frac{2\sqrt{c}\,x^{3/2}\left(b^2-3ac\right)\left(a+bx^2+cx^4\right)}{a^2\left(b^2-4ac\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{ax+bx^3+cx^5}}-\frac{2\left(b^2-3ac\right)\sqrt{ax+bx^3+cx^5}}{a^2x^{3/2}\left(b^2-4ac\right)}+\frac{\sqrt[4]{c}\sqrt{x}\left(\sqrt{a}\,b\sqrt{c}-6ac+2b^2\right)\left(\sqrt{a}\cdots\right.}{2a^{7/4}\left(\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(a*x + b*x^3 + c*x^5)^(3/2)),x]
```

```
[Out] (b^2 - 2*a*c + b*c*x^2)/(a*(b^2 - 4*a*c)*Sqrt[x]*Sqrt[a*x + b*x^3 + c*x^5])
 + (2*Sqrt[c]*(b^2 - 3*a*c)*x^(3/2)*(a + b*x^2 + c*x^4))/(a^2*(b^2 - 4*a*c)
*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[a*x + b*x^3 + c*x^5]) - (2*(b^2 - 3*a*c)*Sqrt
[a*x + b*x^3 + c*x^5])/(a^2*(b^2 - 4*a*c)*x^(3/2)) - (2*c^(1/4)*(b^2 - 3*a*
c)*Sqrt[x]*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt
[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c
]))/4])/(a^(7/4)*(b^2 - 4*a*c)*Sqrt[a*x + b*x^3 + c*x^5]) + (c^(1/4)*(2*b^2
 + Sqrt[a]*b*Sqrt[c] - 6*a*c)*Sqrt[x]*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x
^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/
4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(7/4)*(b^2 - 4*a*c)*Sqrt[a*x + b*x^
3 + c*x^5])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
```

```
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1924

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> -Simp[(x^(m - q + 1)*(b^2 - 2*a*c + b*c*x^(n - q))*(a*x^q +
 b*x^n + c*x^(2*n - q))^(p + 1))/(a*(n - q)*(p + 1)*(b^2 - 4*a*c)), x] + Di
st[1/(a*(n - q)*(p + 1)*(b^2 - 4*a*c)), Int[x^(m - q)*(b^2*(m + p*q + (n -
q)*(p + 1) + 1) - 2*a*c*(m + p*q + 2*(n - q)*(p + 1) + 1) + b*c*(m + p*q +
(n - q)*(2*p + 3) + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p + 1),
x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] &&  !Intege
rQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p, -1] && RationalQ[m, q]
 && LtQ[m + p*q + 1, n - q]
```

Rule 1951

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[(A*x^(m - q + 1)*(a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1))/(a*(m + p*q + 1)), x] + Dist[1/(a*(m + p*q +
 1)), Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p
+ 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b*
x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q
] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
&& RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n - q
)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]
```

Rule 1953

```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(j_.)))/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x
_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol] :> Dist[(x^(q/2)*Sqrt[a + b*x^(n -
q) + c*x^(2*(n - q))])/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)], Int[(x^(m - q/2
)*(A + B*x^(n - q)))/Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))], x], x] /; Fre
eQ[{a, b, c, A, B, m, n, q}, x] && EqQ[j, n - q] && EqQ[r, 2*n - q] && PosQ
[n - q] && (EqQ[m, 1/2] || EqQ[m, -2^(-1)]) && EqQ[n, 3] && EqQ[q, 1]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\left(ax+bx^3+cx^5\right)^{3/2}}\,dx = \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)\sqrt{x}\sqrt{ax+bx^3+cx^5}} - \frac{\int \frac{-2b^2+6ac-bcx^2}{x^{3/2}\sqrt{ax+bx^3+cx^5}}\,dx}{a\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)\sqrt{x}\sqrt{ax+bx^3+cx^5}} - \frac{2\left(b^2-3ac\right)\sqrt{ax+bx^3+cx^5}}{a^2\left(b^2-4ac\right)x^{3/2}} + \frac{\int \frac{\sqrt{x}\left(abc+2c(\right.}{\sqrt{ax+bx}}}{a^2\left(b^2\right.}$$

$$= \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)\sqrt{x}\sqrt{ax+bx^3+cx^5}} - \frac{2\left(b^2-3ac\right)\sqrt{ax+bx^3+cx^5}}{a^2\left(b^2-4ac\right)x^{3/2}} + \frac{\left(\sqrt{x}\sqrt{a+b}\right.}{a^2\left(b^2\right.}$$

$$= \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)\sqrt{x}\sqrt{ax+bx^3+cx^5}} - \frac{2\left(b^2-3ac\right)\sqrt{ax+bx^3+cx^5}}{a^2\left(b^2-4ac\right)x^{3/2}} - \frac{\left(2\sqrt{c}\left(b^2-\right.\right.}{a}$$

$$= \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)\sqrt{x}\sqrt{ax+bx^3+cx^5}} + \frac{2\sqrt{c}\left(b^2-3ac\right)x^{3/2}\left(a+bx^2+cx^4\right)}{a^2\left(b^2-4ac\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{ax+bx^3+cx^5}}$$

Mathematica [C] time = 1.35, size = 519, normalized size = 1.11

$$-\frac{2\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\left(-4a^2c+a\left(b^2-7bcx^2-6c^2x^4\right)+2b^2x^2\left(b+cx^2\right)\right)-ix\left(b^2-3ac\right)\left(\sqrt{b^2-4ac}-b\right)\sqrt{\frac{\sqrt{b^2-4ac}+b+2}{\sqrt{b^2-4ac}+b}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(a*x + b*x^3 + c*x^5)^(3/2)),x]
```

```
[Out] -1/2*(2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*(-4*a^2*c + 2*b^2*x^2*(b + c*x^2) +
 a*(b^2 - 7*b*c*x^2 - 6*c^2*x^4)) - I*(b^2 - 3*a*c)*(-b + Sqrt[b^2 - 4*a*c]
)*x*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2
*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE[I*Ar
cSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(
b - Sqrt[b^2 - 4*a*c])] + I*(-b^3 + 4*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - 3*a*c
*Sqrt[b^2 - 4*a*c])*x*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2
- 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c
])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sq
rt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(a^2*(b^2 - 4*a*c)*Sqrt[c/(b + S
qrt[b^2 - 4*a*c])]*Sqrt[x]*Sqrt[x*(a + b*x^2 + c*x^4)])
```

fricas [F] time = 1.04, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^5+bx^3+ax}\,\sqrt{x}}{c^2x^{11}+2\,bcx^9+\left(b^2+2\,ac\right)x^7+2\,abx^5+a^2x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^5+b*x^3+a*x)^(3/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^5 + b*x^3 + a*x)*sqrt(x)/(c^2*x^11 + 2*b*c*x^9 + (b^2 + 2
*a*c)*x^7 + 2*a*b*x^5 + a^2*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^5+bx^3+ax\right)^{\frac{3}{2}}\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^5+b*x^3+a*x)^(3/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^5 + b*x^3 + a*x)^(3/2)*sqrt(x)), x)
```

maple [B] time = 0.04, size = 1136, normalized size = 2.43

$$-\frac{\sqrt{\left(c\,x^{4}+b\,x^{2}+a\right)x}\,\left(12\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}\,a b\,c^{2}x^{4}-4\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}\,b^{3}c\,x^{4}+12\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}\,\sqrt{-4ac+b^{2}}\,a\,c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^5+b*x^3+a*x)^(3/2)/x^(1/2),x)
```

```
[Out] -1/2*((c*x^4+b*x^2+a)*x)^(1/2)/x^(3/2)*(12*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2
)*(-4*a*c+b^2)^(1/2)*x^4*a*c^2-4*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-4*a*c+
b^2)^(1/2)*x^4*b^2*c+12*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x^4*a*b*c^2-4*((-
b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x^4*b^3*c+a*b*c*(-2*(-b*x^2+(-4*a*c+b^2)^(1/
2)*x^2-2*a)/a)^(1/2)*((b*x^2+(-4*a*c+b^2)^(1/2)*x^2+2*a)/a)^(1/2)*EllipticF
(1/2*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*2^(1/2)*((-2*a*c+b^2+(
-4*a*c+b^2)^(1/2)*b)/a/c)^(1/2))*x*(-4*a*c+b^2)^(1/2)+12*(-2*(-b*x^2+(-4*a*
c+b^2)^(1/2)*x^2-2*a)/a)^(1/2)*((b*x^2+(-4*a*c+b^2)^(1/2)*x^2+2*a)/a)^(1/2)
*EllipticF(1/2*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*2^(1/2)*((-2
*a*c+b^2+(-4*a*c+b^2)^(1/2)*b)/a/c)^(1/2))*x*a^2*c^2-3*a*b^2*c*(-2*(-b*x^2+
(-4*a*c+b^2)^(1/2)*x^2-2*a)/a)^(1/2)*((b*x^2+(-4*a*c+b^2)^(1/2)*x^2+2*a)/a)
^(1/2)*EllipticF(1/2*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*2^(1/2
)*((-2*a*c+b^2+(-4*a*c+b^2)^(1/2)*b)/a/c)^(1/2))*x-12*(-2*(-b*x^2+(-4*a*c+b
^2)^(1/2)*x^2-2*a)/a)^(1/2)*((b*x^2+(-4*a*c+b^2)^(1/2)*x^2+2*a)/a)^(1/2)*El
lipticE(1/2*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*2^(1/2)*((-2*a*
c+b^2+(-4*a*c+b^2)^(1/2)*b)/a/c)^(1/2))*x*a^2*c^2+4*(-2*(-b*x^2+(-4*a*c+b^2
)^(1/2)*x^2-2*a)/a)^(1/2)*((b*x^2+(-4*a*c+b^2)^(1/2)*x^2+2*a)/a)^(1/2)*Elli
pticE(1/2*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x,1/2*2^(1/2)*((-2*a*c+
b^2+(-4*a*c+b^2)^(1/2)*b)/a/c)^(1/2))*x*a*b^2*c+14*((-b+(-4*a*c+b^2)^(1/2))
/a)^(1/2)*(-4*a*c+b^2)^(1/2)*x^2*a*b*c-4*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*
(-4*a*c+b^2)^(1/2)*x^2*b^3+14*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x^2*a*b^2*c
-4*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x^2*b^4+8*((-b+(-4*a*c+b^2)^(1/2))/a)^
(1/2)*(-4*a*c+b^2)^(1/2)*a^2*c-2*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-4*a*c+
b^2)^(1/2)*a*b^2+8*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*a^2*b*c-2*((-b+(-4*a*c
+b^2)^(1/2))/a)^(1/2)*a*b^3)/(c*x^4+b*x^2+a)/a^2/(4*a*c-b^2)/((-b+(-4*a*c+b
^2)^(1/2))/a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^{5}+bx^{3}+ax\right)^{\frac{3}{2}}\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^5+b*x^3+a*x)^(3/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^5 + b*x^3 + a*x)^(3/2)*sqrt(x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{x}\,\left(c\,x^{5}+b\,x^{3}+a\,x\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(a*x + b*x^3 + c*x^5)^(3/2)),x)
```

```
[Out] int(1/(x^(1/2)*(a*x + b*x^3 + c*x^5)^(3/2)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x}\left(x\left(a+bx^2+cx^4\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**5+b*x**3+a*x)**(3/2)/x**(1/2),x)
```

```
[Out] Integral(1/(sqrt(x)*(x*(a + b*x**2 + c*x**4))**(3/2)), x)
```

3.120 $\int \frac{1}{x^{3/2}\left(ax+bx^3+cx^5\right)^{3/2}}\,dx$

Optimal. Leaf size=154

$$\frac{3b\tanh^{-1}\left(\frac{\sqrt{x}\left(2a+bx^2\right)}{2\sqrt{a}\sqrt{ax+bx^3+cx^5}}\right)}{4a^{5/2}}-\frac{\left(3b^2-8ac\right)\sqrt{ax+bx^3+cx^5}}{2a^2x^{5/2}\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx^2}{ax^{3/2}\left(b^2-4ac\right)\sqrt{ax+bx^3+cx^5}}$$

```
[Out] 3/4*b*arctanh(1/2*(b*x^2+2*a)*x^(1/2)/a^(1/2)/(c*x^5+b*x^3+a*x)^(1/2))/a^(5
/2)+(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/x^(3/2)/(c*x^5+b*x^3+a*x)^(1/2)-1/2*
(-8*a*c+3*b^2)*(c*x^5+b*x^3+a*x)^(1/2)/a^2/(-4*a*c+b^2)/x^(5/2)
```

Rubi [A] time = 0.17, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1924, 1951, 12, 1913, 206}

$$-\frac{\left(3b^2-8ac\right)\sqrt{ax+bx^3+cx^5}}{2a^2x^{5/2}\left(b^2-4ac\right)}+\frac{3b\tanh^{-1}\left(\frac{\sqrt{x}\left(2a+bx^2\right)}{2\sqrt{a}\sqrt{ax+bx^3+cx^5}}\right)}{4a^{5/2}}+\frac{-2ac+b^2+bcx^2}{ax^{3/2}\left(b^2-4ac\right)\sqrt{ax+bx^3+cx^5}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(a*x + b*x^3 + c*x^5)^(3/2)),x]
```

```
[Out] (b^2 - 2*a*c + b*c*x^2)/(a*(b^2 - 4*a*c)*x^(3/2)*Sqrt[a*x + b*x^3 + c*x^5])
 - ((3*b^2 - 8*a*c)*Sqrt[a*x + b*x^3 + c*x^5])/(2*a^2*(b^2 - 4*a*c)*x^(5/2)
) + (3*b*ArcTanh[(Sqrt[x]*(2*a + b*x^2))/(2*Sqrt[a]*Sqrt[a*x + b*x^3 + c*x^
5])])/(4*a^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1913

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[-2/(n - q), Subst[Int[1/(4*a - x^2), x], x, (x^(m + 1)*
(2*a + b*x^(n - q)))/Sqrt[a*x^q + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, m,
 n, q, r}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && NeQ[b^2 - 4*a*c, 0] && E
qQ[m, q/2 - 1]
```

Rule 1924

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> -Simp[(x^(m - q + 1)*(b^2 - 2*a*c + b*c*x^(n - q))*(a*x^q +
 b*x^n + c*x^(2*n - q))^(p + 1))/(a*(n - q)*(p + 1)*(b^2 - 4*a*c)), x] + Di
st[1/(a*(n - q)*(p + 1)*(b^2 - 4*a*c)), Int[x^(m - q)*(b^2*(m + p*q + (n -
q)*(p + 1) + 1) - 2*a*c*(m + p*q + 2*(n - q)*(p + 1) + 1) + b*c*(m + p*q +
(n - q)*(2*p + 3) + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p + 1),
x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] &&  !Intege
rQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p, -1] && RationalQ[m, q]
```

```
&& LtQ[m + p*q + 1, n - q]
```

Rule 1951

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[(A*x^(m - q + 1)*(a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1))/(a*(m + p*q + 1)), x] + Dist[1/(a*(m + p*q +
 1)), Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p
+ 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b*
x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q
] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
&& RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n - q
)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]
```

Rubi steps

$$\int \frac{1}{x^{3/2}\left(ax+bx^3+cx^5\right)^{3/2}}\,dx = \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)x^{3/2}\sqrt{ax+bx^3+cx^5}} - \frac{\int \frac{-3b^2+8ac-2bcx^2}{x^{5/2}\sqrt{ax+bx^3+cx^5}}\,dx}{a\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)x^{3/2}\sqrt{ax+bx^3+cx^5}} - \frac{\left(3b^2-8ac\right)\sqrt{ax+bx^3+cx^5}}{2a^2\left(b^2-4ac\right)x^{5/2}} + \frac{\int -\frac{3b\left(b^2-\right.}{\sqrt{x}\sqrt{ax+b}}}{2a^2\left(b^2-\right.}$$

$$= \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)x^{3/2}\sqrt{ax+bx^3+cx^5}} - \frac{\left(3b^2-8ac\right)\sqrt{ax+bx^3+cx^5}}{2a^2\left(b^2-4ac\right)x^{5/2}} - \frac{(3b)\int \frac{}{\sqrt{x}\sqrt{a}}}{2a}$$

$$= \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)x^{3/2}\sqrt{ax+bx^3+cx^5}} - \frac{\left(3b^2-8ac\right)\sqrt{ax+bx^3+cx^5}}{2a^2\left(b^2-4ac\right)x^{5/2}} + \frac{(3b)\,\mathrm{Subst}\Big(}{}$$

$$= \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)x^{3/2}\sqrt{ax+bx^3+cx^5}} - \frac{\left(3b^2-8ac\right)\sqrt{ax+bx^3+cx^5}}{2a^2\left(b^2-4ac\right)x^{5/2}} + \frac{3b\tanh^{-1}\Big(}{4}$$

Mathematica [A] time = 0.07, size = 160, normalized size = 1.04

$$\frac{2\sqrt{a}\left(-4a^2c+a\left(b^2-10bcx^2-8c^2x^4\right)+3b^2x^2\left(b+cx^2\right)\right)-3bx^2\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}\,\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+c}}\right.}{4a^{5/2}x^{3/2}\left(4ac-b^2\right)\sqrt{x\left(a+bx^2+cx^4\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(a*x + b*x^3 + c*x^5)^(3/2)),x]
```

```
[Out] (2*Sqrt[a]*(-4*a^2*c + 3*b^2*x^2*(b + c*x^2) + a*(b^2 - 10*b*c*x^2 - 8*c^2*
x^4)) - 3*b*(b^2 - 4*a*c)*x^2*Sqrt[a + b*x^2 + c*x^4]*ArcTanh[(2*a + b*x^2)
/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(4*a^(5/2)*(-b^2 + 4*a*c)*x^(3/2)*Sq
rt[x*(a + b*x^2 + c*x^4)])
```

fricas [A] time = 0.95, size = 508, normalized size = 3.30

$$\left[\frac{3\left(\left(b^3c-4abc^2\right)x^7+\left(b^4-4ab^2c\right)x^5+\left(ab^3-4a^2bc\right)x^3\right)\sqrt{a}\log\left(-\frac{\left(b^2+4ac\right)x^5+8abx^3+8a^2x+4\sqrt{cx^5+bx^3+ax}\left(bx^2+2a\right)\sqrt{a}\sqrt{}}{x^5}\right.}{8\left(\left(a^3b^2c-4a^4c^2\right)x^7+\left(a^3b^3-4a^4bc\right)x^5+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(c*x^5+b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*((b^3*c - 4*a*b*c^2)*x^7 + (b^4 - 4*a*b^2*c)*x^5 + (a*b^3 - 4*a^2*b
*c)*x^3)*sqrt(a)*log(-((b^2 + 4*a*c)*x^5 + 8*a*b*x^3 + 8*a^2*x + 4*sqrt(c*x
^5 + b*x^3 + a*x)*(b*x^2 + 2*a)*sqrt(a)*sqrt(x))/x^5) - 4*sqrt(c*x^5 + b*x^
3 + a*x)*((3*a*b^2*c - 8*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (3*a*b^3 - 10*a
^2*b*c)*x^2)*sqrt(x))/((a^3*b^2*c - 4*a^4*c^2)*x^7 + (a^3*b^3 - 4*a^4*b*c)*
x^5 + (a^4*b^2 - 4*a^5*c)*x^3), -1/4*(3*((b^3*c - 4*a*b*c^2)*x^7 + (b^4 - 4
*a*b^2*c)*x^5 + (a*b^3 - 4*a^2*b*c)*x^3)*sqrt(-a)*arctan(1/2*sqrt(c*x^5 + b
*x^3 + a*x)*(b*x^2 + 2*a)*sqrt(-a)*sqrt(x)/(a*c*x^5 + a*b*x^3 + a^2*x)) + 2
*sqrt(c*x^5 + b*x^3 + a*x)*((3*a*b^2*c - 8*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c
 + (3*a*b^3 - 10*a^2*b*c)*x^2)*sqrt(x))/((a^3*b^2*c - 4*a^4*c^2)*x^7 + (a^3
*b^3 - 4*a^4*b*c)*x^5 + (a^4*b^2 - 4*a^5*c)*x^3)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(c*x^5+b*x^3+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.02, size = 220, normalized size = 1.43

$$\frac{\sqrt{\left(c\,x^4+b\,x^2+a\right)x}\,\left(-16a^{\frac{3}{2}}c^2x^4+6\sqrt{a}\;b^2c\,x^4+12\sqrt{c\,x^4+b\,x^2+a}\;abc\,x^2\ln\left(\frac{b\,x^2+2a+2\sqrt{c\,x^4+b\,x^2+a}\;\sqrt{a}}{x^2}\right)-3\sqrt{c}\right.}{4\left(c\,x^4+b\,x^2+a\right)\left(4ac-b^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(c*x^5+b*x^3+a*x)^(3/2),x)
```

```
[Out] 1/4*((c*x^4+b*x^2+a)*x)^(1/2)/a^(5/2)*(-16*x^4*a^(3/2)*c^2+6*x^4*b^2*c*a^(1
/2)+12*ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/2))/x^2)*x^2*a*b*c*(c*x^4
+b*x^2+a)^(1/2)-3*ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/2))/x^2)*x^2*b
^3*(c*x^4+b*x^2+a)^(1/2)-20*a^(3/2)*x^2*b*c+6*x^2*b^3*a^(1/2)-8*a^(5/2)*c+2
*a^(3/2)*b^2)/x^(5/2)/(c*x^4+b*x^2+a)/(4*a*c-b^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^5+bx^3+ax\right)^{\frac{3}{2}}x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(c*x^5+b*x^3+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^5 + b*x^3 + a*x)^(3/2)*x^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{3/2}\,{\left(c\,x^5+b\,x^3+a\,x\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(a*x + b*x^3 + c*x^5)^(3/2)),x)
```

```
[Out] int(1/(x^(3/2)*(a*x + b*x^3 + c*x^5)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{3}{2}} \left(x \left(a + bx^{2} + cx^{4}\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(c*x**5+b*x**3+a*x)**(3/2),x)
[Out] Integral(1/(x**(3/2)*(x*(a + b*x**2 + c*x**4))**(3/2)), x)
```

3.121 $\int \frac{x^{\frac{3}{2}(-1+n)}}{\left(ax^{-1+n}+bx^n+cx^{1+n}\right)^{3/2}}\,dx$

Optimal. Leaf size=51

$$-\frac{2x^{\frac{n-1}{2}}(b+2cx)}{\left(b^2-4ac\right)\sqrt{ax^{n-1}+bx^n+cx^{n+1}}}$$

```
[Out] -2*x^(-1/2+1/2*n)*(2*c*x+b)/(-4*a*c+b^2)/(a*x^(-1+n)+b*x^n+c*x^(1+n))^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {1915}

$$-\frac{2x^{\frac{n-1}{2}}(b+2cx)}{\left(b^2-4ac\right)\sqrt{ax^{n-1}+bx^n+cx^{n+1}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^((3*(-1 + n))/2)/(a*x^(-1 + n) + b*x^n + c*x^(1 + n))^(3/2),x]
```

```
[Out] (-2*x^((-1 + n)/2)*(b + 2*c*x))/((b^2 - 4*a*c)*Sqrt[a*x^(-1 + n) + b*x^n +
c*x^(1 + n)])
```

Rule 1915

```
Int[(x_)^(m_.)/((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(3/
2), x_Symbol] :> Simp[(-2*x^((n - 1)/2)*(b + 2*c*x))/((b^2 - 4*a*c)*Sqrt[a*
x^(n - 1) + b*x^n + c*x^(n + 1)]), x] /; FreeQ[{a, b, c, n}, x] && EqQ[m, (
3*(n - 1))/2] && EqQ[q, n - 1] && EqQ[r, n + 1] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^{\frac{3}{2}(-1+n)}}{\left(ax^{-1+n}+bx^n+cx^{1+n}\right)^{3/2}}\,dx=-\frac{2x^{\frac{1}{2}(-1+n)}(b+2cx)}{\left(b^2-4ac\right)\sqrt{ax^{-1+n}+bx^n+cx^{1+n}}}$$

Mathematica [A] time = 0.09, size = 46, normalized size = 0.90

$$-\frac{2x^{\frac{n-1}{2}}(b+2cx)}{\left(b^2-4ac\right)\sqrt{x^{n-1}(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^((3*(-1 + n))/2)/(a*x^(-1 + n) + b*x^n + c*x^(1 + n))^(3/2),x]
```

```
[Out] (-2*x^((-1 + n)/2)*(b + 2*c*x))/((b^2 - 4*a*c)*Sqrt[x^(-1 + n)*(a + x*(b +
c*x))])
```

fricas [A] time = 0.70, size = 83, normalized size = 1.63

$$-\frac{2\left(2\,cx^2+bx\right)\sqrt{\frac{\left(cx^2+bx+a\right)x^{n+1}}{x^2}}}{\left(ab^2-4\,a^2c+\left(b^2c-4\,ac^2\right)x^2+\left(b^3-4\,abc\right)x\right)x^{\frac{1}{2}\,n+\frac{1}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3/2+3/2*n)/(a*x^(-1+n)+b*x^n+c*x^(1+n))^(3/2),x, algorithm="f
ricas")
```

```
[Out] -2*(2*c*x^2 + b*x)*sqrt((c*x^2 + b*x + a)*x^(n + 1)/x^2)/((a*b^2 - 4*a^2*c
+ (b^2*c - 4*a*c^2)*x^2 + (b^3 - 4*a*b*c)*x)*x^(1/2*n + 1/2))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{3}{2}n-\frac{3}{2}}}{\left(cx^{n+1} + ax^{n-1} + bx^{n}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3/2+3/2*n)/(a*x^(-1+n)+b*x^n+c*x^(1+n))^(3/2),x, algorithm="g
iac")
```

```
[Out] integrate(x^(3/2*n - 3/2)/(c*x^(n + 1) + a*x^(n - 1) + b*x^n)^(3/2), x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{3n}{2}-\frac{3}{2}}}{\left(a\,x^{n-1} + b\,x^{n} + c\,x^{n+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2*n-3/2)/(a*x^(n-1)+b*x^n+c*x^(n+1))^(3/2),x)
```

```
[Out] int(x^(3/2*n-3/2)/(a*x^(n-1)+b*x^n+c*x^(n+1))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{3}{2}n-\frac{3}{2}}}{\left(cx^{n+1} + ax^{n-1} + bx^{n}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3/2+3/2*n)/(a*x^(-1+n)+b*x^n+c*x^(1+n))^(3/2),x, algorithm="m
axima")
```

```
[Out] integrate(x^(3/2*n - 3/2)/(c*x^(n + 1) + a*x^(n - 1) + b*x^n)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{\frac{3\,n}{2}-\frac{3}{2}}}{{\left(b\,x^n+a\,x^{n-1}+c\,x^{n+1}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^((3*n)/2 - 3/2)/(b*x^n + a*x^(n - 1) + c*x^(n + 1))^(3/2),x)
```

```
[Out] int(x^((3*n)/2 - 3/2)/(b*x^n + a*x^(n - 1) + c*x^(n + 1))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-3/2+3/2*n)/(a*x**(-1+n)+b*x**n+c*x**(1+n))**(3/2),x)
```

```
[Out] Timed out
```

3.122 $\int \frac{x(d+ex^2)}{\sqrt{ax+bx^3+cx^5}}\,dx$

Optimal. Leaf size=287

$$\frac{2dx^2\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3\sqrt{ax+bx^3+cx^5}}+\frac{2ex^4\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+}}}{7\sqrt{}}$$

```
[Out] 2/3*d*x^2*AppellF1(3/4,1/2,1/2,7/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^2
/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^
2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(c*x^5+b*x^3+a*x)^(1/2)+2/7*e*x^4*AppellF1(
7/4,1/2,1/2,11/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^2/(b+(-4*a*c+b^2)^(
1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^2/(b+(-4*a*c+b^2)^
(1/2)))^(1/2)/(c*x^5+b*x^3+a*x)^(1/2)
```

Rubi [A] time = 0.40, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1954, 1335, 1141, 510}

$$\frac{2dx^2\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3\sqrt{ax+bx^3+cx^5}}+\frac{2ex^4\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+}}}{7\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d + e*x^2))/Sqrt[a*x + b*x^3 + c*x^5],x]
```

```
[Out] (2*d*x^2*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b
+ Sqrt[b^2 - 4*a*c])]*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b - Sqrt[b^2
 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(3*Sqrt[a*x + b*x^3 + c*x^
5]) + (2*e*x^4*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^
2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x^2)/(b - S
qrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(7*Sqrt[a*x + b*x^3
 + c*x^5])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 1335

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rule 1954

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(k_.) + (c_.)*(x_)^(n_.))^(p_
)*((A_) + (B_.)*(x_)^(q_)), x_Symbol] :> Dist[(a*x^j + b*x^k + c*x^n)^p/(x^
(j*p)*(a + b*x^(k - j) + c*x^(2*(k - j)))^p), Int[x^(m + j*p)*(A + B*x^(k -
 j))*(a + b*x^(k - j) + c*x^(2*(k - j)))^p, x], x] /; FreeQ[{a, b, c, A, B,
 j, k, m, p}, x] && EqQ[q, k - j] && EqQ[n, 2*k - j] &&  !IntegerQ[p] && Po
sQ[k - j]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x\left(d+ex^2\right)}{\sqrt{ax+bx^3+cx^5}}\,dx &= \frac{\left(\sqrt{x}\sqrt{a+bx^2+cx^4}\right)\int \frac{\sqrt{x}\left(d+ex^2\right)}{\sqrt{a+bx^2+cx^4}}\,dx}{\sqrt{ax+bx^3+cx^5}}\\
&= \frac{\left(\sqrt{x}\sqrt{a+bx^2+cx^4}\right)\int \left(\frac{d\sqrt{x}}{\sqrt{a+bx^2+cx^4}}+\frac{ex^{5/2}}{\sqrt{a+bx^2+cx^4}}\right)dx}{\sqrt{ax+bx^3+cx^5}}\\
&= \frac{\left(d\sqrt{x}\sqrt{a+bx^2+cx^4}\right)\int \frac{\sqrt{x}}{\sqrt{a+bx^2+cx^4}}\,dx}{\sqrt{ax+bx^3+cx^5}}+\frac{\left(e\sqrt{x}\sqrt{a+bx^2+cx^4}\right)\int \frac{x^{5/2}}{\sqrt{a+bx^2+cx^4}}\,dx}{\sqrt{ax+bx^3+cx^5}}\\
&= \frac{\left(d\sqrt{x}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\right)\int \frac{\sqrt{x}}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx}{\sqrt{ax+bx^3+cx^5}}+\frac{\left(e\sqrt{x}\sqrt{1+\right.}{}\\
&= \frac{2dx^2\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3\sqrt{ax+bx^3+cx^5}}+\frac{2ex^4\sqrt{1}}{}
\end{aligned}
$$

Mathematica [A] time = 5.12, size = 239, normalized size = 0.83

$$
\frac{2\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\left(7dx^2F_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)+3ex^4F_1\left(\frac{7}{4};\frac{1}{2},\frac{1}{2};\frac{11}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right.\right.}{21\sqrt{x\left(a+bx^2+cx^4\right)}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(d + e*x^2))/Sqrt[a*x + b*x^3 + c*x^5],x]
```

```
[Out] (2*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b
+ Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*(7*d*x^2*AppellF1[3
/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt
[b^2 - 4*a*c])] + 3*e*x^4*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x^2)/(b + Sqr
t[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/(21*Sqrt[x*(a + b*x^
2 + c*x^4)])
```

fricas [F] time = 0.80, size = 0, normalized size = 0.00

$$
\text{integral}\left(\frac{\sqrt{cx^5+bx^3+ax}\left(ex^2+d\right)}{cx^4+bx^2+a}, x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^5 + b*x^3 + a*x)*(e*x^2 + d)/(c*x^4 + b*x^2 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex^2+d)x}{\sqrt{cx^5+bx^3+ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="giac")
[Out] integrate((e*x^2 + d)*x/sqrt(c*x^5 + b*x^3 + a*x), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(e\,x^2+d)\,x}{\sqrt{c\,x^5+b\,x^3+ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d)/(c*x^5+b*x^3+a*x)^(1/2),x)
[Out] int(x*(e*x^2+d)/(c*x^5+b*x^3+a*x)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex^2+d)x}{\sqrt{cx^5+bx^3+ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="maxima")
[Out] integrate((e*x^2 + d)*x/sqrt(c*x^5 + b*x^3 + a*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,(e\,x^2+d)}{\sqrt{c\,x^5+b\,x^3+a\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(d + e*x^2))/(a*x + b*x^3 + c*x^5)^(1/2),x)
[Out] int((x*(d + e*x^2))/(a*x + b*x^3 + c*x^5)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\,(d+ex^2)}{\sqrt{x\,(a+bx^2+cx^4)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d)/(c*x**5+b*x**3+a*x)**(1/2),x)
[Out] Integral(x*(d + e*x**2)/sqrt(x*(a + b*x**2 + c*x**4)), x)
```

3.123 $\int \frac{1}{\sqrt{3x^2-3x^4+x^6}}\,dx$

Optimal. Leaf size=45

$$-\frac{\tanh^{-1}\left(\frac{x(6-3x^2)}{2\sqrt{3}\sqrt{x^6-3x^4+3x^2}}\right)}{2\sqrt{3}}$$

```
[Out] -1/6*arctanh(1/6*x*(-3*x^2+6)*3^(1/2)/(x^6-3*x^4+3*x^2)^(1/2))*3^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1904, 206}

$$-\frac{\tanh^{-1}\left(\frac{x(6-3x^2)}{2\sqrt{3}\sqrt{x^6-3x^4+3x^2}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3*x^2 - 3*x^4 + x^6],x]
```

```
[Out] -ArcTanh[(x*(6 - 3*x^2))/(2*Sqrt[3]*Sqrt[3*x^2 - 3*x^4 + x^6])]/(2*Sqrt[3])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1904

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, (x*(2*a + b*x^(n - 2)))/
Sqrt[a*x^2 + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n
 - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{3x^2-3x^4+x^6}}\,dx &= -\text{Subst}\left(\int \frac{1}{12-x^2}\,dx, x, \frac{x\left(6-3x^2\right)}{\sqrt{3x^2-3x^4+x^6}}\right)\\
&= -\frac{\tanh^{-1}\left(\frac{x(6-3x^2)}{2\sqrt{3}\sqrt{3x^2-3x^4+x^6}}\right)}{2\sqrt{3}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 73, normalized size = 1.62

$$-\frac{x\sqrt{x^4-3x^2+3}\,\tanh^{-1}\left(\frac{6-3x^2}{2\sqrt{3}\sqrt{x^4-3x^2+3}}\right)}{2\sqrt{3}\sqrt{x^2\left(x^4-3x^2+3\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[3*x^2 - 3*x^4 + x^6],x]
```

```
[Out] -1/2*(x*Sqrt[3 - 3*x^2 + x^4]*ArcTanh[(6 - 3*x^2)/(2*Sqrt[3]*Sqrt[3 - 3*x^2
 + x^4])])/(Sqrt[3]*Sqrt[x^2*(3 - 3*x^2 + x^4)])
```

fricas [A] time = 0.87, size = 55, normalized size = 1.22

$$\frac{1}{6}\,\sqrt{3}\,\log\left(-\frac{3\,x^3+2\,\sqrt{3}\left(x^3-2\,x\right)+2\,\sqrt{x^6-3\,x^4+3\,x^2}\left(\sqrt{3}+2\right)-6\,x}{x^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6-3*x^4+3*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*log(-(3*x^3 + 2*sqrt(3)*(x^3 - 2*x) + 2*sqrt(x^6 - 3*x^4 + 3*x^
2)*(sqrt(3) + 2) - 6*x)/x^3)
```

giac [A] time = 0.44, size = 60, normalized size = 1.33

$$\frac{\sqrt{3}\,\log\left(x^2+\sqrt{3}-\sqrt{x^4-3\,x^2+3}\right)-\sqrt{3}\,\log\left(-x^2+\sqrt{3}+\sqrt{x^4-3\,x^2+3}\right)}{6\,\mathrm{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6-3*x^4+3*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*(sqrt(3)*log(x^2 + sqrt(3) - sqrt(x^4 - 3*x^2 + 3)) - sqrt(3)*log(-x^2
+ sqrt(3) + sqrt(x^4 - 3*x^2 + 3)))/sgn(x)
```

maple [A] time = 0.01, size = 58, normalized size = 1.29

$$\frac{\sqrt{x^4-3x^2+3}\,\sqrt{3}\,x\,\mathrm{arctanh}\left(\frac{\left(x^2-2\right)\sqrt{3}}{2\sqrt{x^4-3x^2+3}}\right)}{6\sqrt{x^6-3x^4+3x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6-3*x^4+3*x^2)^(1/2),x)
```

```
[Out] 1/6/(x^6-3*x^4+3*x^2)^(1/2)*x*(x^4-3*x^2+3)^(1/2)*3^(1/2)*arctanh(1/2*(x^2-
2)*3^(1/2)/(x^4-3*x^2+3)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{x^6-3\,x^4+3\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6-3*x^4+3*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(x^6 - 3*x^4 + 3*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{1}{\sqrt{x^6-3\,x^4+3\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2 - 3*x^4 + x^6)^(1/2),x)
```

```
[Out] int(1/(3*x^2 - 3*x^4 + x^6)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{x^6-3x^4+3x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**6-3*x**4+3*x**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(x**6 - 3*x**4 + 3*x**2), x)
```

3.124 $\int \frac{1}{\sqrt{x^2(3-3x^2+x^4)}}\,dx$

Optimal. Leaf size=45

$$-\frac{\tanh^{-1}\left(\frac{x(6-3x^2)}{2\sqrt{3}\sqrt{x^6-3x^4+3x^2}}\right)}{2\sqrt{3}}$$

```
[Out] -1/6*arctanh(1/6*x*(-3*x^2+6)*3^(1/2)/(x^6-3*x^4+3*x^2)^(1/2))*3^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1996, 1904, 206}

$$-\frac{\tanh^{-1}\left(\frac{x(6-3x^2)}{2\sqrt{3}\sqrt{x^6-3x^4+3x^2}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[x^2*(3 - 3*x^2 + x^4)],x]
```

```
[Out] -ArcTanh[(x*(6 - 3*x^2))/(2*Sqrt[3]*Sqrt[3*x^2 - 3*x^4 + x^6])]/(2*Sqrt[3])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1904

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, (x*(2*a + b*x^(n - 2)))/
Sqrt[a*x^2 + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n
 - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1996

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedTrinomialQ[u, x] && !GeneralizedTrinomialMatchQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{x^2\left(3-3x^2+x^4\right)}}\,dx &= \int \frac{1}{\sqrt{3x^2-3x^4+x^6}}\,dx\\
&= -\text{Subst}\left(\int \frac{1}{12-x^2}\,dx, x, \frac{x\left(6-3x^2\right)}{\sqrt{3x^2-3x^4+x^6}}\right)\\
&= -\frac{\tanh^{-1}\left(\frac{x(6-3x^2)}{2\sqrt{3}\sqrt{3x^2-3x^4+x^6}}\right)}{2\sqrt{3}}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 73, normalized size = 1.62

$$-\frac{x\sqrt{x^4-3x^2+3}\tanh^{-1}\left(\frac{6-3x^2}{2\sqrt{3}\sqrt{x^4-3x^2+3}}\right)}{2\sqrt{3}\sqrt{x^2\left(x^4-3x^2+3\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[x^2*(3 - 3*x^2 + x^4)],x]
```

```
[Out] -1/2*(x*Sqrt[3 - 3*x^2 + x^4]*ArcTanh[(6 - 3*x^2)/(2*Sqrt[3]*Sqrt[3 - 3*x^2
 + x^4])])/(Sqrt[3]*Sqrt[x^2*(3 - 3*x^2 + x^4)])
```

fricas [A] time = 0.80, size = 55, normalized size = 1.22

$$\frac{1}{6}\sqrt{3}\log\left(-\frac{3x^3+2\sqrt{3}\left(x^3-2x\right)+2\sqrt{x^6-3x^4+3x^2}\left(\sqrt{3}+2\right)-6x}{x^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2*(x^4-3*x^2+3))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*log(-(3*x^3 + 2*sqrt(3)*(x^3 - 2*x) + 2*sqrt(x^6 - 3*x^4 + 3*x^
2)*(sqrt(3) + 2) - 6*x)/x^3)
```

giac [A] time = 0.72, size = 60, normalized size = 1.33

$$\frac{\sqrt{3}\log\left(x^2+\sqrt{3}-\sqrt{x^4-3x^2+3}\right)-\sqrt{3}\log\left(-x^2+\sqrt{3}+\sqrt{x^4-3x^2+3}\right)}{6\operatorname{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2*(x^4-3*x^2+3))^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*(sqrt(3)*log(x^2 + sqrt(3) - sqrt(x^4 - 3*x^2 + 3)) - sqrt(3)*log(-x^2
+ sqrt(3) + sqrt(x^4 - 3*x^2 + 3)))/sgn(x)
```

maple [A] time = 0.01, size = 58, normalized size = 1.29

$$\frac{\sqrt{x^4-3x^2+3}\,\sqrt{3}\,x\,\operatorname{arctanh}\left(\frac{\left(x^2-2\right)\sqrt{3}}{2\sqrt{x^4-3x^2+3}}\right)}{6\sqrt{\left(x^4-3x^2+3\right)x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(x^4-3*x^2+3))^(1/2),x)
```

```
[Out] 1/6/(x^2*(x^4-3*x^2+3))^(1/2)*x*(x^4-3*x^2+3)^(1/2)*3^(1/2)*arctanh(1/2*(x^
2-2)*3^(1/2)/(x^4-3*x^2+3)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{\left(x^4-3x^2+3\right)x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2*(x^4-3*x^2+3))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt((x^4 - 3*x^2 + 3)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{x^2\,\left(x^4-3\,x^2+3\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(x^4 - 3*x^2 + 3))^(1/2),x)
[Out] int(1/(x^2*(x^4 - 3*x^2 + 3))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^2\left(x^4-3x^2+3\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2*(x**4-3*x**2+3))**(1/2),x)
[Out] Integral(1/sqrt(x**2*(x**4 - 3*x**2 + 3)), x)
```

3.125 $\int \frac{1}{\sqrt{1-\left(1-x^2\right)^3}}\,dx$

Optimal. Leaf size=45

$$-\frac{\tanh^{-1}\left(\frac{x\left(6-3x^2\right)}{2\sqrt{3}\sqrt{x^6-3x^4+3x^2}}\right)}{2\sqrt{3}}$$

```
[Out] -1/6*arctanh(1/6*x*(-3*x^2+6)*3^(1/2)/(x^6-3*x^4+3*x^2)^(1/2))*3^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1996, 1904, 206}

$$-\frac{\tanh^{-1}\left(\frac{x\left(6-3x^2\right)}{2\sqrt{3}\sqrt{x^6-3x^4+3x^2}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[1 - (1 - x^2)^3],x]
```

```
[Out] -ArcTanh[(x*(6 - 3*x^2))/(2*Sqrt[3]*Sqrt[3*x^2 - 3*x^4 + x^6])]/(2*Sqrt[3])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1904

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, (x*(2*a + b*x^(n - 2)))/
Sqrt[a*x^2 + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n
 - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1996

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedTrinomialQ[u, x] &&  !GeneralizedTrinomialMatchQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{1-\left(1-x^2\right)^3}}\,dx &= \int \frac{1}{\sqrt{3x^2-3x^4+x^6}}\,dx\\
&= -\text{Subst}\left(\int \frac{1}{12-x^2}\,dx, x, \frac{x\left(6-3x^2\right)}{\sqrt{3x^2-3x^4+x^6}}\right)\\
&= -\frac{\tanh^{-1}\left(\frac{x\left(6-3x^2\right)}{2\sqrt{3}\sqrt{3x^2-3x^4+x^6}}\right)}{2\sqrt{3}}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 73, normalized size = 1.62

$$-\frac{x\sqrt{x^4-3x^2+3}\tanh^{-1}\left(\frac{6-3x^2}{2\sqrt{3}\sqrt{x^4-3x^2+3}}\right)}{2\sqrt{3}\sqrt{x^2\left(x^4-3x^2+3\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 - (1 - x^2)^3],x]
```

```
[Out] -1/2*(x*Sqrt[3 - 3*x^2 + x^4]*ArcTanh[(6 - 3*x^2)/(2*Sqrt[3]*Sqrt[3 - 3*x^2
 + x^4])])/(Sqrt[3]*Sqrt[x^2*(3 - 3*x^2 + x^4)])
```

fricas [A] time = 0.87, size = 55, normalized size = 1.22

$$\frac{1}{6}\sqrt{3}\log\left(-\frac{3x^3+2\sqrt{3}\left(x^3-2x\right)+2\sqrt{x^6-3x^4+3x^2}\left(\sqrt{3}+2\right)-6x}{x^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(-x^2+1)^3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*log(-(3*x^3 + 2*sqrt(3)*(x^3 - 2*x) + 2*sqrt(x^6 - 3*x^4 + 3*x^
2)*(sqrt(3) + 2) - 6*x)/x^3)
```

giac [A] time = 0.49, size = 60, normalized size = 1.33

$$\frac{\sqrt{3}\log\left(x^2+\sqrt{3}-\sqrt{x^4-3x^2+3}\right)-\sqrt{3}\log\left(-x^2+\sqrt{3}+\sqrt{x^4-3x^2+3}\right)}{6\,\mathrm{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(-x^2+1)^3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*(sqrt(3)*log(x^2 + sqrt(3) - sqrt(x^4 - 3*x^2 + 3)) - sqrt(3)*log(-x^2
+ sqrt(3) + sqrt(x^4 - 3*x^2 + 3)))/sgn(x)
```

maple [A] time = 0.00, size = 58, normalized size = 1.29

$$\frac{\sqrt{x^4-3x^2+3}\,\sqrt{3}\,x\,\mathrm{arctanh}\left(\frac{\left(x^2-2\right)\sqrt{3}}{2\sqrt{x^4-3x^2+3}}\right)}{6\sqrt{x^6-3x^4+3x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-(-x^2+1)^3)^(1/2),x)
```

```
[Out] 1/6/(x^6-3*x^4+3*x^2)^(1/2)*x*(x^4-3*x^2+3)^(1/2)*3^(1/2)*arctanh(1/2*(x^2-
2)*3^(1/2)/(x^4-3*x^2+3)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\left(x^2-1\right)^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(-x^2+1)^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt((x^2 - 1)^3 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{\left(x^2-1\right)^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 - 1)^3 + 1)^(1/2),x)
```

```
[Out] int(1/((x^2 - 1)^3 + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{1-\left(1-x^2\right)^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(-x**2+1)**3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(1 - (1 - x**2)**3), x)
```

3.126 $\int \sqrt{3x^2-3x^4+x^6}\,dx$

Optimal. Leaf size=86

$$-\frac{\sqrt{x^6-3x^4+3x^2}\left(3-2x^2\right)}{8x}-\frac{3\sqrt{x^6-3x^4+3x^2}\,\sinh^{-1}\left(\frac{3-2x^2}{\sqrt{3}}\right)}{16x\sqrt{x^4-3x^2+3}}$$

```
[Out] -1/8*(-2*x^2+3)*(x^6-3*x^4+3*x^2)^(1/2)/x-3/16*arcsinh(1/3*(-2*x^2+3)*3^(1/
2))*(x^6-3*x^4+3*x^2)^(1/2)/x/(x^4-3*x^2+3)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1903, 1107, 612, 619, 215}

$$-\frac{\sqrt{x^6-3x^4+3x^2}\left(3-2x^2\right)}{8x}-\frac{3\sqrt{x^6-3x^4+3x^2}\,\sinh^{-1}\left(\frac{3-2x^2}{\sqrt{3}}\right)}{16x\sqrt{x^4-3x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3*x^2 - 3*x^4 + x^6],x]
```

```
[Out] -((3 - 2*x^2)*Sqrt[3*x^2 - 3*x^4 + x^6])/(8*x) - (3*Sqrt[3*x^2 - 3*x^4 + x^
6]*ArcSinh[(3 - 2*x^2)/Sqrt[3]])/(16*x*Sqrt[3 - 3*x^2 + x^4])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1903

```
Int[Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol]
 :> Dist[Sqrt[a*x^q + b*x^n + c*x^(2*n - q)]/(x^(q/2)*Sqrt[a + b*x^(n - q)
+ c*x^(2*(n - q))]), Int[x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))], x
], x] /; FreeQ[{a, b, c, n, q}, x] && EqQ[r, 2*n - q] && PosQ[n - q]
```

Rubi steps

$$\int \sqrt{3x^2-3x^4+x^6}\,dx = \frac{\sqrt{3x^2-3x^4+x^6}\int x\sqrt{3-3x^2+x^4}\,dx}{x\sqrt{3-3x^2+x^4}}$$
$$= \frac{\sqrt{3x^2-3x^4+x^6}\,\text{Subst}\left(\int \sqrt{3-3x+x^2}\,dx, x, x^2\right)}{2x\sqrt{3-3x^2+x^4}}$$
$$= -\frac{(3-2x^2)\sqrt{3x^2-3x^4+x^6}}{8x} + \frac{\left(3\sqrt{3x^2-3x^4+x^6}\right)\text{Subst}\left(\int \frac{1}{\sqrt{3-3x+x^2}}\,dx, x, x^2\right)}{16x\sqrt{3-3x^2+x^4}}$$
$$= -\frac{(3-2x^2)\sqrt{3x^2-3x^4+x^6}}{8x} + \frac{\left(\sqrt{3}\sqrt{3x^2-3x^4+x^6}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{3}}}\,dx, x, -3+2x^2\right.}{16x\sqrt{3-3x^2+x^4}}$$
$$= -\frac{(3-2x^2)\sqrt{3x^2-3x^4+x^6}}{8x} - \frac{3\sqrt{3x^2-3x^4+x^6}\sinh^{-1}\left(\frac{3-2x^2}{\sqrt{3}}\right)}{16x\sqrt{3-3x^2+x^4}}$$

Mathematica [A] time = 0.03, size = 70, normalized size = 0.81

$$\frac{x\left(4x^6-18x^4+30x^2+3\sqrt{x^4-3x^2+3}\sinh^{-1}\left(\frac{2x^2-3}{\sqrt{3}}\right)-18\right)}{16\sqrt{x^2(x^4-3x^2+3)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3*x^2 - 3*x^4 + x^6],x]
```

```
[Out] (x*(-18 + 30*x^2 - 18*x^4 + 4*x^6 + 3*Sqrt[3 - 3*x^2 + x^4]*ArcSinh[(-3 + 2
*x^2)/Sqrt[3]]))/(16*Sqrt[x^2*(3 - 3*x^2 + x^4)])
```

fricas [A] time = 0.85, size = 70, normalized size = 0.81

$$-\frac{12\,x\log\left(-\frac{2\,x^3-3\,x-2\,\sqrt{x^6-3\,x^4+3\,x^2}}{x}\right)-8\,\sqrt{x^6-3\,x^4+3\,x^2}\left(2\,x^2-3\right)-9\,x}{64\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-3*x^4+3*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/64*(12*x*log(-(2*x^3 - 3*x - 2*sqrt(x^6 - 3*x^4 + 3*x^2))/x) - 8*sqrt(x^
6 - 3*x^4 + 3*x^2)*(2*x^2 - 3) - 9*x)/x
```

giac [A] time = 0.39, size = 69, normalized size = 0.80

$$\frac{1}{16}\left(2\,\sqrt{x^4-3\,x^2+3}\left(2\,x^2-3\right)-3\,\log\left(-2\,x^2+2\,\sqrt{x^4-3\,x^2+3}+3\right)\right)\mathrm{sgn}(x)+\frac{3}{16}\left(2\,\sqrt{3}+\log\left(2\,\sqrt{3}+3\right)\right)\mathrm{sgn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-3*x^4+3*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/16*(2*sqrt(x^4 - 3*x^2 + 3)*(2*x^2 - 3) - 3*log(-2*x^2 + 2*sqrt(x^4 - 3*x
^2 + 3) + 3))*sgn(x) + 3/16*(2*sqrt(3) + log(2*sqrt(3) + 3))*sgn(x)
```

maple [A] time = 0.01, size = 81, normalized size = 0.94

$$\frac{\sqrt{x^6-3x^4+3x^2}\left(4\sqrt{x^4-3x^2+3}\,x^2+3\,\mathrm{arcsinh}\left(\frac{\sqrt{3}\,(2x^2-3)}{3}\right)-6\sqrt{x^4-3x^2+3}\right)}{16\sqrt{x^4-3x^2+3}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6-3*x^4+3*x^2)^(1/2),x)
```

```
[Out] 1/16*(x^6-3*x^4+3*x^2)^(1/2)*(4*(x^4-3*x^2+3)^(1/2)*x^2+3*arcsinh(1/3*3^(1/
2)*(2*x^2-3))-6*(x^4-3*x^2+3)^(1/2))/x/(x^4-3*x^2+3)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^6 - 3\,x^4 + 3\,x^2}\; dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-3*x^4+3*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^6 - 3*x^4 + 3*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x^6 - 3\,x^4 + 3\,x^2}\; dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 - 3*x^4 + x^6)^(1/2),x)
```

```
[Out] int((3*x^2 - 3*x^4 + x^6)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^6 - 3x^4 + 3x^2}\; dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6-3*x**4+3*x**2)**(1/2),x)
```

```
[Out] Integral(sqrt(x**6 - 3*x**4 + 3*x**2), x)
```

3.127 $\int \sqrt{x^2 (3 - 3x^2 + x^4)}\, dx$

Optimal. Leaf size=86

$$-\frac{\sqrt{x^6-3x^4+3x^2}\,(3-2x^2)}{8x}-\frac{3\sqrt{x^6-3x^4+3x^2}\,\sinh^{-1}\left(\frac{3-2x^2}{\sqrt{3}}\right)}{16x\sqrt{x^4-3x^2+3}}$$

```
[Out] -1/8*(-2*x^2+3)*(x^6-3*x^4+3*x^2)^(1/2)/x-3/16*arcsinh(1/3*(-2*x^2+3)*3^(1/
2))*(x^6-3*x^4+3*x^2)^(1/2)/x/(x^4-3*x^2+3)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1996, 1903, 1107, 612, 619, 215}

$$-\frac{\sqrt{x^6-3x^4+3x^2}\,(3-2x^2)}{8x}-\frac{3\sqrt{x^6-3x^4+3x^2}\,\sinh^{-1}\left(\frac{3-2x^2}{\sqrt{3}}\right)}{16x\sqrt{x^4-3x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x^2*(3 - 3*x^2 + x^4)],x]
```

```
[Out] -((3 - 2*x^2)*Sqrt[3*x^2 - 3*x^4 + x^6])/(8*x) - (3*Sqrt[3*x^2 - 3*x^4 + x^
6]*ArcSinh[(3 - 2*x^2)/Sqrt[3]])/(16*x*Sqrt[3 - 3*x^2 + x^4])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1903

```
Int[Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol]
 :> Dist[Sqrt[a*x^q + b*x^n + c*x^(2*n - q)]/(x^(q/2)*Sqrt[a + b*x^(n - q)
+ c*x^(2*(n - q))]), Int[x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))], x
], x] /; FreeQ[{a, b, c, n, q}, x] && EqQ[r, 2*n - q] && PosQ[n - q]
```

Rule 1996

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedTrinomialQ[u, x] &&  !GeneralizedTrinomialMatchQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x^2\left(3-3x^2+x^4\right)}\,dx &= \int \sqrt{3x^2-3x^4+x^6}\,dx\\ &= \frac{\sqrt{3x^2-3x^4+x^6}\int x\sqrt{3-3x^2+x^4}\,dx}{x\sqrt{3-3x^2+x^4}}\\ &= \frac{\sqrt{3x^2-3x^4+x^6}\,\text{Subst}\left(\int\sqrt{3-3x+x^2}\,dx,x,x^2\right)}{2x\sqrt{3-3x^2+x^4}}\\ &= -\frac{\left(3-2x^2\right)\sqrt{3x^2-3x^4+x^6}}{8x}+\frac{\left(3\sqrt{3x^2-3x^4+x^6}\right)\text{Subst}\left(\int\frac{1}{\sqrt{3-3x+x^2}}\,dx,x,x^2\right)}{16x\sqrt{3-3x^2+x^4}}\\ &= -\frac{\left(3-2x^2\right)\sqrt{3x^2-3x^4+x^6}}{8x}+\frac{\left(\sqrt{3}\sqrt{3x^2-3x^4+x^6}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1+\frac{x^2}{3}}}\,dx,x,-3\right.}{16x\sqrt{3-3x^2+x^4}}\\ &= -\frac{\left(3-2x^2\right)\sqrt{3x^2-3x^4+x^6}}{8x}-\frac{3\sqrt{3x^2-3x^4+x^6}\sinh^{-1}\left(\frac{3-2x^2}{\sqrt{3}}\right)}{16x\sqrt{3-3x^2+x^4}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 70, normalized size = 0.81

$$\frac{x\left(4x^6-18x^4+30x^2+3\sqrt{x^4-3x^2+3}\sinh^{-1}\left(\frac{2x^2-3}{\sqrt{3}}\right)-18\right)}{16\sqrt{x^2\left(x^4-3x^2+3\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x^2*(3 - 3*x^2 + x^4)],x]
```

```
[Out] (x*(-18 + 30*x^2 - 18*x^4 + 4*x^6 + 3*Sqrt[3 - 3*x^2 + x^4]*ArcSinh[(-3 + 2
*x^2)/Sqrt[3]]))/(16*Sqrt[x^2*(3 - 3*x^2 + x^4)])
```

fricas [A] time = 0.87, size = 70, normalized size = 0.81

$$-\frac{12\,x\log\left(-\frac{2\,x^3-3\,x-2\,\sqrt{x^6-3\,x^4+3\,x^2}}{x}\right)-8\,\sqrt{x^6-3\,x^4+3\,x^2}\left(2\,x^2-3\right)-9\,x}{64\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2*(x^4-3*x^2+3))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/64*(12*x*log(-(2*x^3 - 3*x - 2*sqrt(x^6 - 3*x^4 + 3*x^2))/x) - 8*sqrt(x^
6 - 3*x^4 + 3*x^2)*(2*x^2 - 3) - 9*x)/x
```

giac [A] time = 0.51, size = 69, normalized size = 0.80

$$\frac{1}{16}\left(2\,\sqrt{x^4-3\,x^2+3}\left(2\,x^2-3\right)-3\,\log\left(-2\,x^2+2\,\sqrt{x^4-3\,x^2+3}+3\right)\right)\text{sgn}(x)+\frac{3}{16}\left(2\,\sqrt{3}+\log\left(2\,\sqrt{3}+3\right)\right)\text{s}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2*(x^4-3*x^2+3))^(1/2),x, algorithm="giac")
```

```
[Out] 1/16*(2*sqrt(x^4 - 3*x^2 + 3)*(2*x^2 - 3) - 3*log(-2*x^2 + 2*sqrt(x^4 - 3*x
^2 + 3) + 3))*sgn(x) + 3/16*(2*sqrt(3) + log(2*sqrt(3) + 3))*sgn(x)
```

maple [A] time = 0.01, size = 81, normalized size = 0.94

$$\frac{\sqrt{\left(x^{4}-3x^{2}+3\right)x^{2}}\,\left(4\sqrt{x^{4}-3x^{2}+3}\,x^{2}+3\,\mathrm{arcsinh}\left(\frac{\sqrt{3}\,\left(2x^{2}-3\right)}{3}\right)-6\sqrt{x^{4}-3x^{2}+3}\right)}{16\sqrt{x^{4}-3x^{2}+3}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4-3*x^2+3)*x^2)^(1/2),x)
```

```
[Out] 1/16*((x^4-3*x^2+3)*x^2)^(1/2)*(4*(x^4-3*x^2+3)^(1/2)*x^2+3*arcsinh(1/3*3^(
1/2)*(2*x^2-3))-6*(x^4-3*x^2+3)^(1/2))/x/(x^4-3*x^2+3)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\left(x^{4}-3\,x^{2}+3\right)x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2*(x^4-3*x^2+3))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt((x^4 - 3*x^2 + 3)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x^{2}\,\left(x^{4}-3\,x^{2}+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(x^4 - 3*x^2 + 3))^(1/2),x)
```

```
[Out] int((x^2*(x^4 - 3*x^2 + 3))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2*(x**4-3*x**2+3))**(1/2),x)
```

```
[Out] Timed out
```

3.128 $\int \sqrt{1-(1-x^2)^3}\,dx$

Optimal. Leaf size=86

$$-\frac{\sqrt{x^6-3x^4+3x^2}\,(3-2x^2)}{8x}-\frac{3\sqrt{x^6-3x^4+3x^2}\,\sinh^{-1}\left(\frac{3-2x^2}{\sqrt{3}}\right)}{16x\sqrt{x^4-3x^2+3}}$$

```
[Out] -1/8*(-2*x^2+3)*(x^6-3*x^4+3*x^2)^(1/2)/x-3/16*arcsinh(1/3*(-2*x^2+3)*3^(1/
2))*(x^6-3*x^4+3*x^2)^(1/2)/x/(x^4-3*x^2+3)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {1996, 1903, 1107, 612, 619, 215}

$$-\frac{\sqrt{x^6-3x^4+3x^2}\,(3-2x^2)}{8x}-\frac{3\sqrt{x^6-3x^4+3x^2}\,\sinh^{-1}\left(\frac{3-2x^2}{\sqrt{3}}\right)}{16x\sqrt{x^4-3x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - (1 - x^2)^3],x]
```

```
[Out] -((3 - 2*x^2)*Sqrt[3*x^2 - 3*x^4 + x^6])/(8*x) - (3*Sqrt[3*x^2 - 3*x^4 + x^
6]*ArcSinh[(3 - 2*x^2)/Sqrt[3]])/(16*x*Sqrt[3 - 3*x^2 + x^4])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1903

```
Int[Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol]
 :> Dist[Sqrt[a*x^q + b*x^n + c*x^(2*n - q)]/(x^(q/2)*Sqrt[a + b*x^(n - q)
+ c*x^(2*(n - q))]), Int[x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))], x
], x] /; FreeQ[{a, b, c, n, q}, x] && EqQ[r, 2*n - q] && PosQ[n - q]
```

Rule 1996

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedTrinomialQ[u, x] &&  !GeneralizedTrinomialMatchQ[u, x]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{1-\left(1-x^2\right)^3}\,dx &= \int \sqrt{3x^2-3x^4+x^6}\,dx\\
&= \frac{\sqrt{3x^2-3x^4+x^6}\int x\sqrt{3-3x^2+x^4}\,dx}{x\sqrt{3-3x^2+x^4}}\\
&= \frac{\sqrt{3x^2-3x^4+x^6}\,\text{Subst}\left(\int \sqrt{3-3x+x^2}\,dx,x,x^2\right)}{2x\sqrt{3-3x^2+x^4}}\\
&= -\frac{\left(3-2x^2\right)\sqrt{3x^2-3x^4+x^6}}{8x}+\frac{\left(3\sqrt{3x^2-3x^4+x^6}\right)\text{Subst}\left(\int \frac{1}{\sqrt{3-3x+x^2}}\,dx,x,x^2\right)}{16x\sqrt{3-3x^2+x^4}}\\
&= -\frac{\left(3-2x^2\right)\sqrt{3x^2-3x^4+x^6}}{8x}+\frac{\left(\sqrt{3}\sqrt{3x^2-3x^4+x^6}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{3}}}\,dx,x,-3+2x^2\right)}{16x\sqrt{3-3x^2+x^4}}\\
&= -\frac{\left(3-2x^2\right)\sqrt{3x^2-3x^4+x^6}}{8x}-\frac{3\sqrt{3x^2-3x^4+x^6}\,\sinh^{-1}\left(\frac{3-2x^2}{\sqrt{3}}\right)}{16x\sqrt{3-3x^2+x^4}}
\end{aligned}
$$

Mathematica [A] time = 0.00, size = 70, normalized size = 0.81

$$\frac{x\left(4x^6-18x^4+30x^2+3\sqrt{x^4-3x^2+3}\,\sinh^{-1}\left(\frac{2x^2-3}{\sqrt{3}}\right)-18\right)}{16\sqrt{x^2\left(x^4-3x^2+3\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - (1 - x^2)^3],x]
```

```
[Out] (x*(-18 + 30*x^2 - 18*x^4 + 4*x^6 + 3*Sqrt[3 - 3*x^2 + x^4]*ArcSinh[(-3 + 2
*x^2)/Sqrt[3]]))/(16*Sqrt[x^2*(3 - 3*x^2 + x^4)])
```

fricas [A] time = 0.59, size = 70, normalized size = 0.81

$$-\frac{12\,x\log\left(-\frac{2\,x^3-3\,x-2\,\sqrt{x^6-3\,x^4+3\,x^2}}{x}\right)-8\,\sqrt{x^6-3\,x^4+3\,x^2}\left(2\,x^2-3\right)-9\,x}{64\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(-x^2+1)^3)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/64*(12*x*log(-(2*x^3 - 3*x - 2*sqrt(x^6 - 3*x^4 + 3*x^2))/x) - 8*sqrt(x^
6 - 3*x^4 + 3*x^2)*(2*x^2 - 3) - 9*x)/x
```

giac [A] time = 0.38, size = 69, normalized size = 0.80

$$\frac{1}{16}\left(2\,\sqrt{x^4-3\,x^2+3}\left(2\,x^2-3\right)-3\,\log\left(-2\,x^2+2\,\sqrt{x^4-3\,x^2+3}+3\right)\right)\text{sgn}(x)+\frac{3}{16}\left(2\,\sqrt{3}+\log\left(2\,\sqrt{3}+3\right)\right)\text{sgn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(-x^2+1)^3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/16*(2*sqrt(x^4 - 3*x^2 + 3)*(2*x^2 - 3) - 3*log(-2*x^2 + 2*sqrt(x^4 - 3*x
^2 + 3) + 3))*sgn(x) + 3/16*(2*sqrt(3) + log(2*sqrt(3) + 3))*sgn(x)
```

maple [A] time = 0.00, size = 81, normalized size = 0.94

$$\frac{\sqrt{x^6-3x^4+3x^2}\,\left(4\sqrt{x^4-3x^2+3}\;x^2+3\,\mathrm{arcsinh}\left(\frac{\sqrt{3}\,(2x^2-3)}{3}\right)-6\sqrt{x^4-3x^2+3}\right)}{16\sqrt{x^4-3x^2+3}\;x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-(-x^2+1)^3)^(1/2),x)
```

```
[Out] 1/16*(x^6-3*x^4+3*x^2)^(1/2)*(4*(x^4-3*x^2+3)^(1/2)*x^2+3*arcsinh(1/3*3^(1/
2)*(2*x^2-3))-6*(x^4-3*x^2+3)^(1/2))/x/(x^4-3*x^2+3)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\left(x^2-1\right)^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(-x^2+1)^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt((x^2 - 1)^3 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\left(x^2-1\right)^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 - 1)^3 + 1)^(1/2),x)
```

```
[Out] int(((x^2 - 1)^3 + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{1-\left(1-x^2\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(-x**2+1)**3)**(1/2),x)
```

```
[Out] Integral(sqrt(1 - (1 - x**2)**3), x)
```

3.129 $\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=38

$$-\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}}$$

```
[Out] -arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/a^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {724, 206}

$$-\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -(ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])]/Sqrt[a])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx &= -\left(2\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx}{\sqrt{a+bx+cx^2}}\right)\right)\\ &= -\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 37, normalized size = 0.97

$$-\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -(ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])]/Sqrt[a])
```

fricas [A] time = 0.91, size = 111, normalized size = 2.92

$$\left[\frac{\log\left(-\frac{8\,abx+\left(b^2+4\,ac\right)x^2-4\,\sqrt{cx^2+bx+a}\,(bx+2\,a)\sqrt{a}+8\,a^2}{x^2}\right)}{2\,\sqrt{a}},\ \frac{\sqrt{-a}\,\arctan\left(\frac{\sqrt{cx^2+bx+a}\,(bx+2\,a)\sqrt{-a}}{2\left(acx^2+abx+a^2\right)}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 + b*x + a)*(b*x + 2*a
)*sqrt(a) + 8*a^2)/x^2)/sqrt(a), sqrt(-a)*arctan(1/2*sqrt(c*x^2 + b*x + a)*
(b*x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a^2))/a]
```

giac [A] time = 0.39, size = 35, normalized size = 0.92

$$\frac{2\,\arctan\left(-\frac{\sqrt{c}\,x-\sqrt{cx^2+bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2*arctan(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))/sqrt(-a))/sqrt(-a)
```

maple [A] time = 0.00, size = 35, normalized size = 0.92

$$-\frac{\ln\left(\frac{bx+2a+2\sqrt{c\,x^2+bx+a}\,\sqrt{a}}{x}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] -1/a^(1/2)*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [B] time = 0.08, size = 34, normalized size = 0.89

$$-\frac{\ln\left(\frac{b}{2}+\frac{a}{x}+\frac{\sqrt{a}\,\sqrt{c\,x^2+b\,x+a}}{x}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x + c*x^2)^(1/2)),x)
```

```
[Out] -log(b/2 + a/x + (a^(1/2)*(a + b*x + c*x^2)^(1/2))/x)/a^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(a + b*x + c*x**2)), x)
```

3.130 $\int \frac{1}{\sqrt{x^2(a+bx+cx^2)}}\,dx$

Optimal. Leaf size=45

$$-\frac{\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{\sqrt{a}}$$

```
[Out] -arctanh(1/2*x*(b*x+2*a)/a^(1/2)/(c*x^4+b*x^3+a*x^2)^(1/2))/a^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1996, 1904, 206}

$$-\frac{\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[x^2*(a + b*x + c*x^2)],x]
```

```
[Out] -(ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])]/Sqrt[a])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1904

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, (x*(2*a + b*x^(n - 2)))/
Sqrt[a*x^2 + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n
 - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1996

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedTrinomialQ[u, x] &&  !GeneralizedTrinomialMatchQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{x^2\left(a+bx+cx^2\right)}}\,dx &= \int \frac{1}{\sqrt{ax^2+bx^3+cx^4}}\,dx\\
&= -\left(2\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{x(2a+bx)}{\sqrt{ax^2+bx^3+cx^4}}\right)\right)\\
&= -\frac{\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{\sqrt{a}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 70, normalized size = 1.56

$$-\frac{x\sqrt{a+bx+cx^2}\,\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[x^2*(a + b*x + c*x^2)],x]
```

```
[Out] -((x*Sqrt[a + b*x + c*x^2]*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*
x^2])])/(Sqrt[a]*Sqrt[x^2*(a + x*(b + c*x))]))
```

fricas [A] time = 0.90, size = 130, normalized size = 2.89

$$\left[\frac{\log\left(-\frac{8\,abx^2+\left(b^2+4\,ac\right)x^3+8\,a^2x-4\,\sqrt{cx^4+bx^3+ax^2}\,(bx+2\,a)\sqrt{a}}{x^3}\right)}{2\,\sqrt{a}},\ \frac{\sqrt{-a}\,\arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}\,(bx+2\,a)\sqrt{-a}}{2\,\left(acx^3+abx^2+a^2x\right)}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2*(c*x^2+b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3 +
 a*x^2)*(b*x + 2*a)*sqrt(a))/x^3)/sqrt(a), sqrt(-a)*arctan(1/2*sqrt(c*x^4 +
 b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x))/a]
```

giac [A] time = 0.47, size = 59, normalized size = 1.31

$$-\frac{2\,\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)\mathrm{sgn}(x)}{\sqrt{-a}}+\frac{2\,\arctan\left(-\frac{\sqrt{c}\,x-\sqrt{cx^2+bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,\mathrm{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2*(c*x^2+b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] -2*arctan(sqrt(a)/sqrt(-a))*sgn(x)/sqrt(-a) + 2*arctan(-(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))/sqrt(-a))/(sqrt(-a)*sgn(x))
```

maple [A] time = 0.01, size = 64, normalized size = 1.42

$$-\frac{\sqrt{c\,x^2+bx+a}\;x\ln\left(\frac{bx+2a+2\sqrt{c\,x^2+bx+a}\;\sqrt{a}}{x}\right)}{\sqrt{\left(c\,x^2+bx+a\right)x^2}\;\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(c*x^2+b*x+a))^(1/2),x)
```

```
[Out] -1/(x^2*(c*x^2+b*x+a))^(1/2)*x*(c*x^2+b*x+a)^(1/2)/a^(1/2)*ln((b*x+2*a+2*(c
*x^2+b*x+a)^(1/2)*a^(1/2))/x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{\left(cx^2+bx+a\right)x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2*(c*x^2+b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt((c*x^2 + b*x + a)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{1}{\sqrt{x^2\,\left(c\,x^2+b\,x+a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x + c*x^2))^(1/2),x)
```

```
[Out] int(1/(x^2*(a + b*x + c*x^2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^2 \left(a + bx + cx^2\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2*(c*x**2+b*x+a))**(1/2),x)
```

```
[Out] Integral(1/sqrt(x**2*(a + b*x + c*x**2)), x)
```

3.131 $\int \frac{1}{\sqrt{x}\sqrt{x(a+bx+cx^2)}}\,dx$

Optimal. Leaf size=47

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{x}(2a+bx)}{2\sqrt{a}\sqrt{ax+bx^2+cx^3}}\right)}{\sqrt{a}}$$

```
[Out] -arctanh(1/2*(b*x+2*a)*x^(1/2)/a^(1/2)/(c*x^3+b*x^2+a*x)^(1/2))/a^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1997, 1913, 206}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{x}(2a+bx)}{2\sqrt{a}\sqrt{ax+bx^2+cx^3}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*Sqrt[x*(a + b*x + c*x^2)]),x]
```

```
[Out] -(ArcTanh[(Sqrt[x]*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x + b*x^2 + c*x^3])]/Sqrt
[a])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1913

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[-2/(n - q), Subst[Int[1/(4*a - x^2), x], x, (x^(m + 1)*
(2*a + b*x^(n - q)))/Sqrt[a*x^q + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, m,
 n, q, r}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && NeQ[b^2 - 4*a*c, 0] && E
qQ[m, q/2 - 1]
```

Rule 1997

```
Int[(u_)^(p_.)*((d_.)*(x_))^(m_.), x_Symbol] :> Int[(d*x)^m*ExpandToSum[u,
x]^p, x] /; FreeQ[{d, m, p}, x] && GeneralizedTrinomialQ[u, x] && !General
izedTrinomialMatchQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{x}\sqrt{x\left(a+bx+cx^2\right)}}\,dx &= \int \frac{1}{\sqrt{x}\sqrt{ax+bx^2+cx^3}}\,dx\\
&= -\left(2\operatorname{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{\sqrt{x}(2a+bx)}{\sqrt{ax+bx^2+cx^3}}\right)\right)\\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{x}(2a+bx)}{2\sqrt{a}\sqrt{ax+bx^2+cx^3}}\right)}{\sqrt{a}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 72, normalized size = 1.53

$$-\frac{\sqrt{x}\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}\sqrt{x(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*Sqrt[x*(a + b*x + c*x^2)]),x]
```

```
[Out] -((Sqrt[x]*Sqrt[a + b*x + c*x^2]*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*
x + c*x^2])])/(Sqrt[a]*Sqrt[x*(a + x*(b + c*x))]))
```

fricas [A] time = 1.05, size = 131, normalized size = 2.79

$$\left[\frac{\log\left(\frac{8abx^2+(b^2+4ac)x^3+8a^2x-4\sqrt{cx^3+bx^2+ax}(bx+2a)\sqrt{a}\sqrt{x}}{x^3}\right)}{2\sqrt{a}}, \frac{\sqrt{-a}\arctan\left(\frac{\sqrt{cx^3+bx^2+ax}(bx+2a)\sqrt{-a}\sqrt{x}}{2(acx^3+abx^2+a^2x)}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(x*(c*x^2+b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log((8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^3 + b*x^2 +
a*x)*(b*x + 2*a)*sqrt(a)*sqrt(x))/x^3)/sqrt(a), sqrt(-a)*arctan(1/2*sqrt(c*
x^3 + b*x^2 + a*x)*(b*x + 2*a)*sqrt(-a)*sqrt(x)/(a*c*x^3 + a*b*x^2 + a^2*x)
)/a]
```

giac [A] time = 0.49, size = 53, normalized size = 1.13

$$\frac{2\arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{2\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(x*(c*x^2+b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] 2*arctan(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))/sqrt(-a))/sqrt(-a) - 2*arctan
(sqrt(a)/sqrt(-a))/sqrt(-a)
```

maple [A] time = 0.01, size = 64, normalized size = 1.36

$$-\frac{\sqrt{cx^2+bx+a}\sqrt{x}\ln\left(\frac{bx+2a+2\sqrt{cx^2+bx+a}\sqrt{a}}{x}\right)}{\sqrt{(cx^2+bx+a)x}\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(x*(c*x^2+b*x+a))^(1/2),x)
```

```
[Out] -x^(1/2)/(x*(c*x^2+b*x+a))^(1/2)*(c*x^2+b*x+a)^(1/2)/a^(1/2)*ln((b*x+2*a+2*
(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{(cx^2+bx+a)x}\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(x*(c*x^2+b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt((c*x^2 + b*x + a)*x)*sqrt(x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{x}\,\sqrt{x\,\left(c\,x^2+b\,x+a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(x*(a + b*x + c*x^2))^(1/2)),x)
```

```
[Out] int(1/(x^(1/2)*(x*(a + b*x + c*x^2))^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(x*(c*x**2+b*x+a))**(1/2),x)
```

```
[Out] Timed out
```

3.132 $\int \frac{\sqrt{x}}{\sqrt{x^3\left(a+bx+cx^2\right)}}\,dx$

Optimal. Leaf size=49

$$-\frac{\tanh^{-1}\left(\frac{x^{3/2}(2a+bx)}{2\sqrt{a}\sqrt{ax^3+bx^4+cx^5}}\right)}{\sqrt{a}}$$

```
[Out] -arctanh(1/2*x^(3/2)*(b*x+2*a)/a^(1/2)/(c*x^5+b*x^4+a*x^3)^(1/2))/a^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1997, 1913, 206}

$$-\frac{\tanh^{-1}\left(\frac{x^{3/2}(2a+bx)}{2\sqrt{a}\sqrt{ax^3+bx^4+cx^5}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/Sqrt[x^3*(a + b*x + c*x^2)],x]
```

```
[Out] -(ArcTanh[(x^(3/2)*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^3 + b*x^4 + c*x^5])]/Sq
rt[a])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1913

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[-2/(n - q), Subst[Int[1/(4*a - x^2), x], x, (x^(m + 1)*
(2*a + b*x^(n - q)))/Sqrt[a*x^q + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, m,
 n, q, r}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && NeQ[b^2 - 4*a*c, 0] && E
qQ[m, q/2 - 1]
```

Rule 1997

```
Int[(u_)^(p_.)*((d_.)*(x_))^(m_.), x_Symbol] :> Int[(d*x)^m*ExpandToSum[u,
x]^p, x] /; FreeQ[{d, m, p}, x] && GeneralizedTrinomialQ[u, x] && !General
izedTrinomialMatchQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{x}}{\sqrt{x^3\left(a+bx+cx^2\right)}}\,dx &= \int \frac{\sqrt{x}}{\sqrt{ax^3+bx^4+cx^5}}\,dx\\
&= -\left(2\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{x^{3/2}(2a+bx)}{\sqrt{ax^3+bx^4+cx^5}}\right)\right)\\
&= -\frac{\tanh^{-1}\left(\frac{x^{3/2}(2a+bx)}{2\sqrt{a}\sqrt{ax^3+bx^4+cx^5}}\right)}{\sqrt{a}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 74, normalized size = 1.51

$$-\frac{x^{3/2}\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}\sqrt{x^3(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/Sqrt[x^3*(a + b*x + c*x^2)],x]
```

```
[Out] -((x^(3/2)*Sqrt[a + b*x + c*x^2]*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*
x + c*x^2])])/(Sqrt[a]*Sqrt[x^3*(a + x*(b + c*x))]))
```

fricas [A] time = 0.98, size = 139, normalized size = 2.84

$$\left[\frac{\log\left(\frac{8\,abx^3+\left(b^2+4\,ac\right)x^4+8\,a^2x^2-4\,\sqrt{cx^5+bx^4+ax^3}\,(bx+2\,a)\sqrt{a}\,\sqrt{x}}{x^4}\right)}{2\,\sqrt{a}},\ \frac{\sqrt{-a}\,\arctan\left(\frac{\sqrt{cx^5+bx^4+ax^3}\,(bx+2\,a)\sqrt{-a}\,\sqrt{x}}{2\left(acx^4+abx^3+a^2x^2\right)}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^3*(c*x^2+b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log((8*a*b*x^3 + (b^2 + 4*a*c)*x^4 + 8*a^2*x^2 - 4*sqrt(c*x^5 + b*x^4
+ a*x^3)*(b*x + 2*a)*sqrt(a)*sqrt(x))/x^4)/sqrt(a), sqrt(-a)*arctan(1/2*sqr
t(c*x^5 + b*x^4 + a*x^3)*(b*x + 2*a)*sqrt(-a)*sqrt(x)/(a*c*x^4 + a*b*x^3 +
a^2*x^2))/a]
```

giac [A] time = 0.49, size = 53, normalized size = 1.08

$$\frac{2\,\arctan\left(-\frac{\sqrt{c}\,x-\sqrt{cx^2+bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{2\,\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^3*(c*x^2+b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] 2*arctan(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))/sqrt(-a))/sqrt(-a) - 2*arctan
(sqrt(a)/sqrt(-a))/sqrt(-a)
```

maple [A] time = 0.01, size = 66, normalized size = 1.35

$$-\frac{\sqrt{c\,x^2+bx+a}\;x^{\frac{3}{2}}\ln\left(\frac{bx+2a+2\sqrt{c\,x^2+bx+a}\,\sqrt{a}}{x}\right)}{\sqrt{\left(c\,x^2+bx+a\right)x^3}\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x^3*(c*x^2+b*x+a))^(1/2),x)
```

```
[Out] -1/(x^3*(c*x^2+b*x+a))^(1/2)*x^(3/2)*(c*x^2+b*x+a)^(1/2)/a^(1/2)*ln((b*x+2*
a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x}}{\sqrt{\left(cx^2+bx+a\right)x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^3*(c*x^2+b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x)/sqrt((c*x^2 + b*x + a)*x^3), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{x}}{\sqrt{x^3 \left(c\,x^2+b\,x+a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x^3*(a + b*x + c*x^2))^(1/2),x)
```

```
[Out] int(x^(1/2)/(x^3*(a + b*x + c*x^2))^(1/2), x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(x**3*(c*x**2+b*x+a))**(1/2),x)
```

```
[Out] Timed out
```

3.133 $\int \frac{1}{x\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=44

$$-\frac{\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}}$$

```
[Out] -1/2*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))/a^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1114, 724, 206}

$$-\frac{\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])]/(2*Sqrt[a])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt{a+bx^2+cx^4}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)\\
&= -\operatorname{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^2}{\sqrt{a+bx^2+cx^4}}\right)\\
&= -\frac{\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 44, normalized size = 1.00

$$-\frac{\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -1/2*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])]/Sqrt[a]
```

fricas [A] time = 0.86, size = 124, normalized size = 2.82

$$\left[\frac{\log\left(-\frac{(b^2+4ac)x^4+8abx^2-4\sqrt{cx^4+bx^2+a}\left(bx^2+2a\right)\sqrt{a}+8a^2}{x^4}\right)}{4\sqrt{a}}, \frac{\sqrt{-a}\arctan\left(\frac{\sqrt{cx^4+bx^2+a}\left(bx^2+2a\right)\sqrt{-a}}{2\left(acx^4+abx^2+a^2\right)}\right)}{2a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2
 + 2*a)*sqrt(a) + 8*a^2)/x^4)/sqrt(a), 1/2*sqrt(-a)*arctan(1/2*sqrt(c*x^4 +
 b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2))/a]
```

giac [A] time = 0.45, size = 38, normalized size = 0.86

$$\frac{\arctan\left(-\frac{\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))/sqrt(-a))/sqrt(-a)
```

maple [A] time = 0.01, size = 39, normalized size = 0.89

$$-\frac{\ln\left(\frac{bx^2+2a+2\sqrt{cx^4+bx^2+a}\,\sqrt{a}}{x^2}\right)}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] -1/2/a^(1/2)*ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/2))/x^2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [B] time = 2.23, size = 44, normalized size = 1.00

$$-\frac{\ln\left(\frac{1}{x^2}\right)}{2\sqrt{a}}-\frac{\ln\left(2a+2\sqrt{a}\sqrt{cx^4+bx^2+a}+bx^2\right)}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] - log(1/x^2)/(2*a^(1/2)) - log(2*a + 2*a^(1/2)*(a + b*x^2 + c*x^4)^(1/2) +
b*x^2)/(2*a^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{a + bx^2 + cx^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(a + b*x**2 + c*x**4)), x)
```

3.134 $\int \frac{1}{\sqrt{x^2(a+bx^2+cx^4)}}\,dx$

Optimal. Leaf size=49

$$-\frac{\tanh^{-1}\left(\frac{x(2a+bx^2)}{2\sqrt{a}\sqrt{ax^2+bx^4+cx^6}}\right)}{2\sqrt{a}}$$

```
[Out] -1/2*arctanh(1/2*x*(b*x^2+2*a)/a^(1/2)/(c*x^6+b*x^4+a*x^2)^(1/2))/a^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1996, 1904, 206}

$$-\frac{\tanh^{-1}\left(\frac{x(2a+bx^2)}{2\sqrt{a}\sqrt{ax^2+bx^4+cx^6}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[x^2*(a + b*x^2 + c*x^4)],x]
```

```
[Out] -ArcTanh[(x*(2*a + b*x^2))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^4 + c*x^6])]/(2*Sqrt
[a])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1904

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, (x*(2*a + b*x^(n - 2)))/
Sqrt[a*x^2 + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n
 - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1996

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedTrinomialQ[u, x] &&  !GeneralizedTrinomialMatchQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{x^2\left(a+bx^2+cx^4\right)}}\,dx &= \int \frac{1}{\sqrt{ax^2+bx^4+cx^6}}\,dx\\
&= -\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{x\left(2a+bx^2\right)}{\sqrt{ax^2+bx^4+cx^6}}\right)\\
&= -\frac{\tanh^{-1}\left(\frac{x(2a+bx^2)}{2\sqrt{a}\sqrt{ax^2+bx^4+cx^6}}\right)}{2\sqrt{a}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 81, normalized size = 1.65

$$-\frac{x\sqrt{a+bx^2+cx^4}\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}\sqrt{x^2\left(a+bx^2+cx^4\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[x^2*(a + b*x^2 + c*x^4)],x]
```

```
[Out] -1/2*(x*Sqrt[a + b*x^2 + c*x^4]*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b
*x^2 + c*x^4])])/(Sqrt[a]*Sqrt[x^2*(a + b*x^2 + c*x^4)])
```

fricas [A] time = 0.95, size = 135, normalized size = 2.76

$$\left[\frac{\log\left(-\frac{\left(b^2+4\,ac\right)x^5+8\,abx^3+8\,a^2x-4\,\sqrt{cx^6+bx^4+ax^2}\left(bx^2+2\,a\right)\sqrt{a}}{x^5}\right)}{4\,\sqrt{a}}, \frac{\sqrt{-a}\arctan\left(\frac{\sqrt{cx^6+bx^4+ax^2}\left(bx^2+2\,a\right)\sqrt{-a}}{2\left(acx^5+abx^3+a^2x\right)}\right)}{2\,a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2*(c*x^4+b*x^2+a))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*log(-((b^2 + 4*a*c)*x^5 + 8*a*b*x^3 + 8*a^2*x - 4*sqrt(c*x^6 + b*x^4 +
 a*x^2)*(b*x^2 + 2*a)*sqrt(a))/x^5)/sqrt(a), 1/2*sqrt(-a)*arctan(1/2*sqrt(c
*x^6 + b*x^4 + a*x^2)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^5 + a*b*x^3 + a^2*x))/a
]
```

giac [A] time = 0.44, size = 62, normalized size = 1.27

$$-\frac{\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)\operatorname{sgn}(x)}{\sqrt{-a}}+\frac{\arctan\left(-\frac{\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,\operatorname{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2*(c*x^4+b*x^2+a))^(1/2),x, algorithm="giac")
```

```
[Out] -arctan(sqrt(a)/sqrt(-a))*sgn(x)/sqrt(-a) + arctan(-(sqrt(c)*x^2 - sqrt(c*x
^4 + b*x^2 + a))/sqrt(-a))/(sqrt(-a)*sgn(x))
```

maple [A] time = 0.01, size = 72, normalized size = 1.47

$$-\frac{\sqrt{c\,x^4+b\,x^2+a}\;x\ln\left(\frac{b\,x^2+2a+2\sqrt{c\,x^4+b\,x^2+a}\;\sqrt{a}}{x^2}\right)}{2\sqrt{\left(c\,x^4+b\,x^2+a\right)x^2}\;\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(c*x^4+b*x^2+a))^(1/2),x)
```

```
[Out] -1/2/(x^2*(c*x^4+b*x^2+a))^(1/2)*x*(c*x^4+b*x^2+a)^(1/2)/a^(1/2)*ln((b*x^2+
2*a+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/2))/x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\left(cx^4+bx^2+a\right)x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2*(c*x^4+b*x^2+a))^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt((c*x^4 + b*x^2 + a)*x^2), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{x^2\,\left(c\,x^4+b\,x^2+a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^2 + c*x^4))^(1/2),x)
[Out] int(1/(x^2*(a + b*x^2 + c*x^4))^(1/2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2*(c*x**4+b*x**2+a))**(1/2),x)
[Out] Timed out
```

3.135 $\int \frac{1}{\sqrt{x}\sqrt{x(a+bx^2+cx^4)}}\,dx$

Optimal. Leaf size=51

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{x}(2a+bx^2)}{2\sqrt{a}\sqrt{ax+bx^3+cx^5}}\right)}{2\sqrt{a}}$$

```
[Out] -1/2*arctanh(1/2*(b*x^2+2*a)*x^(1/2)/a^(1/2)/(c*x^5+b*x^3+a*x)^(1/2))/a^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1997, 1913, 206}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{x}(2a+bx^2)}{2\sqrt{a}\sqrt{ax+bx^3+cx^5}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*Sqrt[x*(a + b*x^2 + c*x^4)]),x]
```

```
[Out] -ArcTanh[(Sqrt[x]*(2*a + b*x^2))/(2*Sqrt[a]*Sqrt[a*x + b*x^3 + c*x^5])]/(2*Sqrt[a])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 1913

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol] :> Dist[-2/(n - q), Subst[Int[1/(4*a - x^2), x], x, (x^(m + 1)*(2*a + b*x^(n - q)))/Sqrt[a*x^q + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, m, n, q, r}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && NeQ[b^2 - 4*a*c, 0] && EqQ[m, q/2 - 1]
```

Rule 1997

```
Int[(u_)^(p_.)*((d_.)*(x_))^(m_.), x_Symbol] :> Int[(d*x)^m*ExpandToSum[u, x]^p, x] /; FreeQ[{d, m, p}, x] && GeneralizedTrinomialQ[u, x] && !GeneralizedTrinomialMatchQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{x}\sqrt{x\left(a+bx^2+cx^4\right)}}\,dx &= \int \frac{1}{\sqrt{x}\sqrt{ax+bx^3+cx^5}}\,dx \\
&= -\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{\sqrt{x}\left(2a+bx^2\right)}{\sqrt{ax+bx^3+cx^5}}\right) \\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{x}(2a+bx^2)}{2\sqrt{a}\sqrt{ax+bx^3+cx^5}}\right)}{2\sqrt{a}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 1.63

$$-\frac{\sqrt{x}\sqrt{a+bx^2+cx^4}\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}\sqrt{x\left(a+bx^2+cx^4\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*Sqrt[x*(a + b*x^2 + c*x^4)]),x]
```

```
[Out] -1/2*(Sqrt[x]*Sqrt[a + b*x^2 + c*x^4]*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt
[a + b*x^2 + c*x^4])])/(Sqrt[a]*Sqrt[x*(a + b*x^2 + c*x^4)])
```

fricas [A] time = 0.93, size = 137, normalized size = 2.69

$$\left[\frac{\log\left(-\frac{(b^2+4ac)x^5+8abx^3+8a^2x-4\sqrt{cx^5+bx^3+ax}\left(bx^2+2a\right)\sqrt{a}\sqrt{x}}{x^5}\right)}{4\sqrt{a}}, \frac{\sqrt{-a}\arctan\left(\frac{\sqrt{cx^5+bx^3+ax}\left(bx^2+2a\right)\sqrt{-a}\sqrt{x}}{2\left(acx^5+abx^3+a^2x\right)}\right)}{2a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(x*(c*x^4+b*x^2+a))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*log(-((b^2 + 4*a*c)*x^5 + 8*a*b*x^3 + 8*a^2*x - 4*sqrt(c*x^5 + b*x^3 +
 a*x)*(b*x^2 + 2*a)*sqrt(a)*sqrt(x))/x^5)/sqrt(a), 1/2*sqrt(-a)*arctan(1/2*
sqrt(c*x^5 + b*x^3 + a*x)*(b*x^2 + 2*a)*sqrt(-a)*sqrt(x)/(a*c*x^5 + a*b*x^3
 + a^2*x))/a]
```

giac [A] time = 0.40, size = 56, normalized size = 1.10

$$\frac{\arctan\left(-\frac{\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(x*(c*x^4+b*x^2+a))^(1/2),x, algorithm="giac")
```

```
[Out] arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))/sqrt(-a))/sqrt(-a) - arctan
(sqrt(a)/sqrt(-a))/sqrt(-a)
```

maple [A] time = 0.01, size = 72, normalized size = 1.41

$$-\frac{\sqrt{c\,x^4+b\,x^2+a}\,\sqrt{x}\,\ln\left(\frac{b\,x^2+2a+2\sqrt{c\,x^4+b\,x^2+a}\,\sqrt{a}}{x^2}\right)}{2\sqrt{\left(c\,x^4+b\,x^2+a\right)x}\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/((c*x^4+b*x^2+a)*x)^(1/2),x)
```

```
[Out] -1/2*x^(1/2)/((c*x^4+b*x^2+a)*x)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/a^(1/2)*ln((b*
x^2+2*a+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/2))/x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{\left(cx^4+bx^2+a\right)x}\,\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(x*(c*x^4+b*x^2+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt((c*x^4 + b*x^2 + a)*x)*sqrt(x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{x}\,\sqrt{x\,\left(c\,x^4+b\,x^2+a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(x*(a + b*x^2 + c*x^4))^(1/2)),x)
```

```
[Out] int(1/(x^(1/2)*(x*(a + b*x^2 + c*x^4))^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(x*(c*x**4+b*x**2+a))**(1/2),x)
```

```
[Out] Timed out
```

3.136 $\int \frac{\sqrt{x}}{\sqrt{x^3\left(a+bx^2+cx^4\right)}}\,dx$

Optimal. Leaf size=53

$$-\frac{\tanh^{-1}\left(\frac{x^{3/2}\left(2a+bx^2\right)}{2\sqrt{a}\sqrt{ax^3+bx^5+cx^7}}\right)}{2\sqrt{a}}$$

```
[Out] -1/2*arctanh(1/2*x^(3/2)*(b*x^2+2*a)/a^(1/2)/(c*x^7+b*x^5+a*x^3)^(1/2))/a^(
1/2)
```

Rubi [A] time = 0.08, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1997, 1913, 206}

$$-\frac{\tanh^{-1}\left(\frac{x^{3/2}\left(2a+bx^2\right)}{2\sqrt{a}\sqrt{ax^3+bx^5+cx^7}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/Sqrt[x^3*(a + b*x^2 + c*x^4)],x]
```

```
[Out] -ArcTanh[(x^(3/2)*(2*a + b*x^2))/(2*Sqrt[a]*Sqrt[a*x^3 + b*x^5 + c*x^7])]/(
2*Sqrt[a])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1913

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[-2/(n - q), Subst[Int[1/(4*a - x^2), x], x, (x^(m + 1)*
(2*a + b*x^(n - q)))/Sqrt[a*x^q + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, m,
 n, q, r}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && NeQ[b^2 - 4*a*c, 0] && E
qQ[m, q/2 - 1]
```

Rule 1997

```
Int[(u_)^(p_.)*((d_.)*(x_))^(m_.), x_Symbol] :> Int[(d*x)^m*ExpandToSum[u,
x]^p, x] /; FreeQ[{d, m, p}, x] && GeneralizedTrinomialQ[u, x] &&  !General
izedTrinomialMatchQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{x}}{\sqrt{x^3\left(a+bx^2+cx^4\right)}}\,dx &= \int \frac{\sqrt{x}}{\sqrt{ax^3+bx^5+cx^7}}\,dx\\
&= -\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{x^{3/2}\left(2a+bx^2\right)}{\sqrt{ax^3+bx^5+cx^7}}\right)\\
&= -\frac{\tanh^{-1}\left(\frac{x^{3/2}\left(2a+bx^2\right)}{2\sqrt{a}\sqrt{ax^3+bx^5+cx^7}}\right)}{2\sqrt{a}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 85, normalized size = 1.60

$$-\frac{x^{3/2}\sqrt{a+bx^2+cx^4}\,\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\,\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}\,\sqrt{x^3\left(a+bx^2+cx^4\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/Sqrt[x^3*(a + b*x^2 + c*x^4)],x]
```

```
[Out] -1/2*(x^(3/2)*Sqrt[a + b*x^2 + c*x^4]*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt
[a + b*x^2 + c*x^4])])/(Sqrt[a]*Sqrt[x^3*(a + b*x^2 + c*x^4)])
```

fricas [A] time = 0.93, size = 145, normalized size = 2.74

$$\left[\frac{\log\left(-\frac{\left(b^2+4\,ac\right)x^6+8\,abx^4+8\,a^2x^2-4\,\sqrt{cx^7+bx^5+ax^3}\left(bx^2+2\,a\right)\sqrt{a}\,\sqrt{x}}{x^6}\right)}{4\,\sqrt{a}},\ \frac{\sqrt{-a}\,\arctan\left(\frac{\sqrt{cx^7+bx^5+ax^3}\left(bx^2+2\,a\right)\sqrt{-a}\,\sqrt{x}}{2\left(acx^6+abx^4+a^2x^2\right)}\right)}{2\,a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^3*(c*x^4+b*x^2+a))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^4 + 8*a^2*x^2 - 4*sqrt(c*x^7 + b*x^5
 + a*x^3)*(b*x^2 + 2*a)*sqrt(a)*sqrt(x))/x^6)/sqrt(a), 1/2*sqrt(-a)*arctan(
1/2*sqrt(c*x^7 + b*x^5 + a*x^3)*(b*x^2 + 2*a)*sqrt(-a)*sqrt(x)/(a*c*x^6 + a
*b*x^4 + a^2*x^2))/a]
```

giac [A] time = 0.55, size = 56, normalized size = 1.06

$$\frac{\arctan\left(-\frac{\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^3*(c*x^4+b*x^2+a))^(1/2),x, algorithm="giac")
```

```
[Out] arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))/sqrt(-a))/sqrt(-a) - arctan
(sqrt(a)/sqrt(-a))/sqrt(-a)
```

maple [A] time = 0.01, size = 74, normalized size = 1.40

$$-\frac{\sqrt{c\,x^4+b\,x^2+a}\;x^{\frac{3}{2}}\ln\left(\frac{b\,x^2+2a+2\sqrt{c\,x^4+b\,x^2+a}\;\sqrt{a}}{x^2}\right)}{2\sqrt{\left(c\,x^4+b\,x^2+a\right)x^3}\;\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x^3*(c*x^4+b*x^2+a))^(1/2),x)
```

```
[Out] -1/2/(x^3*(c*x^4+b*x^2+a))^(1/2)*x^(3/2)*(c*x^4+b*x^2+a)^(1/2)/a^(1/2)*ln((
b*x^2+2*a+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/2))/x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{x}}{\sqrt{\left(cx^4+bx^2+a\right)x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^3*(c*x^4+b*x^2+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x)/sqrt((c*x^4 + b*x^2 + a)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{x}}{\sqrt{x^3\,\left(c\,x^4+b\,x^2+a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x^3*(a + b*x^2 + c*x^4))^(1/2),x)
```

```
[Out] int(x^(1/2)/(x^3*(a + b*x^2 + c*x^4))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(x**3*(c*x**4+b*x**2+a))**(1/2),x)
```

```
[Out] Timed out
```

3.137 $\int \frac{1}{x\sqrt{3-3x^2+x^4}}\,dx$

Optimal. Leaf size=40

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{3}(2-x^2)}{2\sqrt{x^4-3x^2+3}}\right)}{2\sqrt{3}}$$

```
[Out] -1/6*arctanh(1/2*(-x^2+2)*3^(1/2)/(x^4-3*x^2+3)^(1/2))*3^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1114, 724, 206}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{3}(2-x^2)}{2\sqrt{x^4-3x^2+3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[3 - 3*x^2 + x^4]),x]
```

```
[Out] -ArcTanh[(Sqrt[3]*(2 - x^2))/(2*Sqrt[3 - 3*x^2 + x^4])]/(2*Sqrt[3])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt{3-3x^2+x^4}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{3-3x+x^2}}\,dx, x, x^2\right)\\
&= -\text{Subst}\left(\int \frac{1}{12-x^2}\,dx, x, \frac{3(2-x^2)}{\sqrt{3-3x^2+x^4}}\right)\\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{3}(2-x^2)}{2\sqrt{3-3x^2+x^4}}\right)}{2\sqrt{3}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 40, normalized size = 1.00

$$-\frac{\tanh^{-1}\left(\frac{6-3x^2}{2\sqrt{3}\sqrt{x^4-3x^2+3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[3 - 3*x^2 + x^4]),x]
```

```
[Out] -1/2*ArcTanh[(6 - 3*x^2)/(2*Sqrt[3]*Sqrt[3 - 3*x^2 + x^4])]/Sqrt[3]
```

fricas [A] time = 0.79, size = 47, normalized size = 1.18

$$\frac{1}{6}\sqrt{3}\log\left(-\frac{3x^2+2\sqrt{3}\left(x^2-2\right)+2\sqrt{x^4-3x^2+3}\left(\sqrt{3}+2\right)-6}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^4-3*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*log(-(3*x^2 + 2*sqrt(3)*(x^2 - 2) + 2*sqrt(x^4 - 3*x^2 + 3)*(sq
rt(3) + 2) - 6)/x^2)
```

giac [A] time = 0.47, size = 55, normalized size = 1.38

$$\frac{1}{6}\sqrt{3}\log\left(x^2+\sqrt{3}-\sqrt{x^4-3x^2+3}\right)-\frac{1}{6}\sqrt{3}\log\left(-x^2+\sqrt{3}+\sqrt{x^4-3x^2+3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^4-3*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*log(x^2 + sqrt(3) - sqrt(x^4 - 3*x^2 + 3)) - 1/6*sqrt(3)*log(-x
^2 + sqrt(3) + sqrt(x^4 - 3*x^2 + 3))
```

maple [A] time = 0.00, size = 31, normalized size = 0.78

$$-\frac{\sqrt{3}\,\operatorname{arctanh}\left(\frac{\left(-3x^2+6\right)\sqrt{3}}{6\sqrt{x^4-3x^2+3}}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^4-3*x^2+3)^(1/2),x)
```

```
[Out] -1/6*3^(1/2)*arctanh(1/6*(-3*x^2+6)*3^(1/2)/(x^4-3*x^2+3)^(1/2))
```

maxima [A] time = 0.95, size = 20, normalized size = 0.50

$$-\frac{1}{6}\sqrt{3}\,\operatorname{arsinh}\left(-\sqrt{3}+\frac{2\sqrt{3}}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^4-3*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(3)*arcsinh(-sqrt(3) + 2*sqrt(3)/x^2)
```

mupad [B] time = 0.43, size = 33, normalized size = 0.82

$$-\frac{\sqrt{3}\left(\ln\left(x^2-\frac{2\sqrt{3}\sqrt{x^4-3x^2+3}}{3}-2\right)+\ln\left(\frac{1}{x^2}\right)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x^4 - 3*x^2 + 3)^(1/2)),x)
```

```
[Out] -(3^(1/2)*(log(x^2 - (2*3^(1/2)*(x^4 - 3*x^2 + 3)^(1/2))/3 - 2) + log(1/x^2
)))/6
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{x^4 - 3x^2 + 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**4-3*x**2+3)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(x**4 - 3*x**2 + 3)), x)
```

3.138 $\int \frac{1}{\sqrt{x^2(3-3x^2+x^4)}}\,dx$

Optimal. Leaf size=45

$$-\frac{\tanh^{-1}\left(\frac{x(6-3x^2)}{2\sqrt{3}\sqrt{x^6-3x^4+3x^2}}\right)}{2\sqrt{3}}$$

```
[Out] -1/6*arctanh(1/6*x*(-3*x^2+6)*3^(1/2)/(x^6-3*x^4+3*x^2)^(1/2))*3^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1996, 1904, 206}

$$-\frac{\tanh^{-1}\left(\frac{x(6-3x^2)}{2\sqrt{3}\sqrt{x^6-3x^4+3x^2}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[x^2*(3 - 3*x^2 + x^4)],x]
```

```
[Out] -ArcTanh[(x*(6 - 3*x^2))/(2*Sqrt[3]*Sqrt[3*x^2 - 3*x^4 + x^6])]/(2*Sqrt[3])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1904

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, (x*(2*a + b*x^(n - 2)))/
Sqrt[a*x^2 + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n
 - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1996

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedTrinomialQ[u, x] &&  !GeneralizedTrinomialMatchQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x^2\left(3-3x^2+x^4\right)}}\,dx &= \int \frac{1}{\sqrt{3x^2-3x^4+x^6}}\,dx\\ &= -\text{Subst}\left(\int \frac{1}{12-x^2}\,dx, x, \frac{x\left(6-3x^2\right)}{\sqrt{3x^2-3x^4+x^6}}\right)\\ &= -\frac{\tanh^{-1}\left(\frac{x(6-3x^2)}{2\sqrt{3}\sqrt{3x^2-3x^4+x^6}}\right)}{2\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 73, normalized size = 1.62

$$-\frac{x\sqrt{x^4-3x^2+3}\tanh^{-1}\left(\frac{6-3x^2}{2\sqrt{3}\sqrt{x^4-3x^2+3}}\right)}{2\sqrt{3}\sqrt{x^2\left(x^4-3x^2+3\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[x^2*(3 - 3*x^2 + x^4)],x]
```

```
[Out] -1/2*(x*Sqrt[3 - 3*x^2 + x^4]*ArcTanh[(6 - 3*x^2)/(2*Sqrt[3]*Sqrt[3 - 3*x^2
 + x^4])])/(Sqrt[3]*Sqrt[x^2*(3 - 3*x^2 + x^4)])
```

fricas [A] time = 0.71, size = 55, normalized size = 1.22

$$\frac{1}{6}\sqrt{3}\log\left(-\frac{3x^3+2\sqrt{3}\left(x^3-2x\right)+2\sqrt{x^6-3x^4+3x^2}\left(\sqrt{3}+2\right)-6x}{x^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2*(x^4-3*x^2+3))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*log(-(3*x^3 + 2*sqrt(3)*(x^3 - 2*x) + 2*sqrt(x^6 - 3*x^4 + 3*x^
2)*(sqrt(3) + 2) - 6*x)/x^3)
```

giac [A] time = 0.43, size = 60, normalized size = 1.33

$$\frac{\sqrt{3}\log\left(x^2+\sqrt{3}-\sqrt{x^4-3x^2+3}\right)-\sqrt{3}\log\left(-x^2+\sqrt{3}+\sqrt{x^4-3x^2+3}\right)}{6\,\mathrm{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2*(x^4-3*x^2+3))^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*(sqrt(3)*log(x^2 + sqrt(3) - sqrt(x^4 - 3*x^2 + 3)) - sqrt(3)*log(-x^2
+ sqrt(3) + sqrt(x^4 - 3*x^2 + 3)))/sgn(x)
```

maple [A] time = 0.00, size = 58, normalized size = 1.29

$$\frac{\sqrt{x^4-3x^2+3}\,\sqrt{3}\,x\,\mathrm{arctanh}\left(\frac{\left(x^2-2\right)\sqrt{3}}{2\sqrt{x^4-3x^2+3}}\right)}{6\sqrt{\left(x^4-3x^2+3\right)x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^4-3*x^2+3)*x^2)^(1/2),x)
```

```
[Out] 1/6/((x^4-3*x^2+3)*x^2)^(1/2)*x*(x^4-3*x^2+3)^(1/2)*3^(1/2)*arctanh(1/2*(x^
2-2)*3^(1/2)/(x^4-3*x^2+3)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{\left(x^4-3x^2+3\right)x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2*(x^4-3*x^2+3))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt((x^4 - 3*x^2 + 3)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{x^2\left(x^4-3\,x^2+3\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(x^4 - 3*x^2 + 3))^(1/2),x)
```

```
[Out] int(1/(x^2*(x^4 - 3*x^2 + 3))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^2\left(x^4-3x^2+3\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2*(x**4-3*x**2+3))**(1/2),x)
```

```
[Out] Integral(1/sqrt(x**2*(x**4 - 3*x**2 + 3)), x)
```

3.139 $\int \frac{1}{\sqrt{x}\sqrt{x(3-3x+x^2)}}\,dx$

Optimal. Leaf size=43

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{3}(2-x)\sqrt{x}}{2\sqrt{x^3-3x^2+3x}}\right)}{\sqrt{3}}$$

[Out] -1/3*arctanh(1/2*(2-x)*3^(1/2)*x^(1/2)/(x^3-3*x^2+3*x)^(1/2))*3^(1/2)

Rubi [A] time = 0.05, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1997, 1913, 206}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{3}(2-x)\sqrt{x}}{2\sqrt{x^3-3x^2+3x}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[x]*Sqrt[x*(3 - 3*x + x^2)]),x]

[Out] -(ArcTanh[(Sqrt[3]*(2 - x)*Sqrt[x])/(2*Sqrt[3*x - 3*x^2 + x^3])]/Sqrt[3])

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1913

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[-2/(n - q), Subst[Int[1/(4*a - x^2), x], x, (x^(m + 1)*
(2*a + b*x^(n - q)))/Sqrt[a*x^q + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, m,
 n, q, r}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && NeQ[b^2 - 4*a*c, 0] && E
qQ[m, q/2 - 1]
```

Rule 1997

```
Int[(u_)^(p_.)*((d_.)*(x_))^(m_.), x_Symbol] :> Int[(d*x)^m*ExpandToSum[u,
x]^p, x] /; FreeQ[{d, m, p}, x] && GeneralizedTrinomialQ[u, x] && !General
izedTrinomialMatchQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{x}\sqrt{x\left(3-3x+x^2\right)}}\,dx &= \int \frac{1}{\sqrt{x}\sqrt{3x-3x^2+x^3}}\,dx\\
&= -\left(2\,\text{Subst}\left(\int \frac{1}{12-x^2}\,dx, x, \frac{(6-3x)\sqrt{x}}{\sqrt{3x-3x^2+x^3}}\right)\right)\\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{3}(2-x)\sqrt{x}}{2\sqrt{3x-3x^2+x^3}}\right)}{\sqrt{3}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 62, normalized size = 1.44

$$\frac{\sqrt{x}\sqrt{x^2-3x+3}\tanh^{-1}\left(\frac{\sqrt{3}(x-2)}{2\sqrt{x^2-3x+3}}\right)}{\sqrt{3}\sqrt{x(x^2-3x+3)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*Sqrt[x*(3 - 3*x + x^2)]),x]
```

```
[Out] (Sqrt[x]*Sqrt[3 - 3*x + x^2]*ArcTanh[(Sqrt[3]*(-2 + x))/(2*Sqrt[3 - 3*x + x
^2])])/(Sqrt[3]*Sqrt[x*(3 - 3*x + x^2)])
```

fricas [A] time = 0.81, size = 49, normalized size = 1.14

$$\frac{1}{6}\sqrt{3}\log\left(\frac{7x^3+4\sqrt{3}\sqrt{x^3-3x^2+3x}(x-2)\sqrt{x}-24x^2+24x}{x^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(x*(x^2-3*x+3))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*log((7*x^3 + 4*sqrt(3)*sqrt(x^3 - 3*x^2 + 3*x)*(x - 2)*sqrt(x)
- 24*x^2 + 24*x)/x^3)
```

giac [A] time = 0.33, size = 47, normalized size = 1.09

$$\frac{1}{3}\sqrt{3}\log\left(x+\sqrt{3}-\sqrt{x^2-3x+3}\right)-\frac{1}{3}\sqrt{3}\log\left(-x+\sqrt{3}+\sqrt{x^2-3x+3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(x*(x^2-3*x+3))^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*log(x + sqrt(3) - sqrt(x^2 - 3*x + 3)) - 1/3*sqrt(3)*log(-x + s
qrt(3) + sqrt(x^2 - 3*x + 3))
```

maple [A] time = 0.01, size = 50, normalized size = 1.16

$$\frac{\sqrt{x^2-3x+3}\sqrt{3}\sqrt{x}\operatorname{arctanh}\left(\frac{(x-2)\sqrt{3}}{2\sqrt{x^2-3x+3}}\right)}{3\sqrt{(x^2-3x+3)x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(x*(x^2-3*x+3))^(1/2),x)
```

```
[Out] 1/3*x^(1/2)/(x*(x^2-3*x+3))^(1/2)*(x^2-3*x+3)^(1/2)*3^(1/2)*arctanh(1/2*(x-
2)*3^(1/2)/(x^2-3*x+3)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{(x^2-3x+3)x}\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(x*(x^2-3*x+3))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt((x^2 - 3*x + 3)*x)*sqrt(x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{x}\,\sqrt{x\,\left(x^2-3\,x+3\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(x*(x^2 - 3*x + 3))^(1/2)),x)
[Out] int(1/(x^(1/2)*(x*(x^2 - 3*x + 3))^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(x*(x**2-3*x+3))**(1/2),x)
[Out] Timed out
```

3.140 $\int \frac{x^{-1+\frac{q}{2}}}{\sqrt{bx^n+cx^{2n-q}+ax^q}}\,dx$

Optimal. Leaf size=70

$$-\frac{\tanh^{-1}\left(\frac{x^{q/2}(2a+bx^{n-q})}{2\sqrt{a}\sqrt{ax^q+bx^n+cx^{2n-q}}}\right)}{\sqrt{a}\,(n-q)}$$

```
[Out] -arctanh(1/2*x^(1/2*q)*(2*a+b*x^(n-q))/a^(1/2)/(b*x^n+c*x^(2*n-q)+a*x^q)^(1
/2))/(n-q)/a^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1913, 206}

$$-\frac{\tanh^{-1}\left(\frac{x^{q/2}(2a+bx^{n-q})}{2\sqrt{a}\sqrt{ax^q+bx^n+cx^{2n-q}}}\right)}{\sqrt{a}\,(n-q)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + q/2)/Sqrt[b*x^n + c*x^(2*n - q) + a*x^q],x]
```

```
[Out] -(ArcTanh[(x^(q/2)*(2*a + b*x^(n - q)))/(2*Sqrt[a]*Sqrt[b*x^n + c*x^(2*n -
q) + a*x^q])]/(Sqrt[a]*(n - q)))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1913

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[-2/(n - q), Subst[Int[1/(4*a - x^2), x], x, (x^(m + 1)*
(2*a + b*x^(n - q)))/Sqrt[a*x^q + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, m,
 n, q, r}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && NeQ[b^2 - 4*a*c, 0] && E
qQ[m, q/2 - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1+\frac{q}{2}}}{\sqrt{bx^n+cx^{2n-q}+ax^q}}\,dx &= -\frac{2\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{x^{q/2}(2a+bx^{n-q})}{\sqrt{bx^n+cx^{2n-q}+ax^q}}\right)}{n-q}\\ &= -\frac{\tanh^{-1}\left(\frac{x^{q/2}(2a+bx^{n-q})}{2\sqrt{a}\sqrt{bx^n+cx^{2n-q}+ax^q}}\right)}{\sqrt{a}\,(n-q)}\end{aligned}$$

Mathematica [F] time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{x^{-1+\frac{q}{2}}}{\sqrt{bx^n+cx^{2n-q}+ax^q}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^(-1 + q/2)/Sqrt[b*x^n + c*x^(2*n - q) + a*x^q],x]
```

```
[Out] Integrate[x^(-1 + q/2)/Sqrt[b*x^n + c*x^(2*n - q) + a*x^q], x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/2*q)/(b*x^n+c*x^(2*n-q)+a*x^q)^(1/2),x, algorithm="fricas
")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{1}{2}q-1}}{\sqrt{cx^{2n-q} + bx^n + ax^q}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/2*q)/(b*x^n+c*x^(2*n-q)+a*x^q)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^(1/2*q - 1)/sqrt(c*x^(2*n - q) + b*x^n + a*x^q), x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{q}{2}-1}}{\sqrt{a\,x^q + b\,x^n + c\,x^{2n-q}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+1/2*q)/(b*x^n+c*x^(2*n-q)+a*x^q)^(1/2),x)
```

```
[Out] int(x^(-1+1/2*q)/(b*x^n+c*x^(2*n-q)+a*x^q)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{1}{2}q-1}}{\sqrt{cx^{2n-q} + bx^n + ax^q}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/2*q)/(b*x^n+c*x^(2*n-q)+a*x^q)^(1/2),x, algorithm="maxima
")
```

```
[Out] integrate(x^(1/2*q - 1)/sqrt(c*x^(2*n - q) + b*x^n + a*x^q), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{\frac{q}{2}-1}}{\sqrt{b\,x^n + a\,x^q + c\,x^{2n-q}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(q/2 - 1)/(b*x^n + a*x^q + c*x^(2*n - q))^(1/2),x)
```

```
[Out] int(x^(q/2 - 1)/(b*x^n + a*x^q + c*x^(2*n - q))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+1/2*q)/(b*x**n+c*x**(2*n-q)+a*x**q)**(1/2),x)
```

```
[Out] Timed out
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
```

```
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
          print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do
   not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
```

```
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
```

```
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn
   )))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or
   type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
   ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9
```

```
#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
```

```
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
    hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
    sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
    sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
    :
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer
    )
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(
    expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn
    .func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
```

```
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```